Course: Modeltheoretic Semantics and the Syntax-Semantics Interface

Eddy Ruys and Yoad Winter

Lecture 5: Generalized Quantifiers (2)

Topics: quantifier coordination, boolean coordination vs. conjunction reduction, *there* sentences and "weak" determiners, partitives and "definite" determiners, negative polarity items and monotonicity

1 Generalized Quantifiers as Boolean Objects

Fact: Coordination applies freely to proper names and other NPs alike.

(1) Mary and/or John, neither Mary nor John, every woman or every man, most women and most men, many students but few teachers, one student and five teachers, the teacher and every student etc.

The denotation of these NPs is easily derived using GQs and the boolean treatment of coordination.

(2) a. Mary and John smiled.
$\mathbf{smile'} \in \{A \subseteq E : \mathbf{m'} \in A\} \cap \{A \subseteq E : \mathbf{j'} \in A\}$

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$\overline{\mathbf{smile'}} \in \{A \subseteq E : \mathbf{m'} \in A\} \cap \overline{\mathbf{smile'}} \in \{A \subseteq E : \mathbf{j'} \in A\}$

Analyzing what these propositions amount to easily shows that (2a) is treated as equivalent to (2b), and similarly for (3) and (4). This is of course correct.

The same hold for *all* NP coordinations:
Prediction: NP_1 and/or/nor NP_2 VP $\Leftrightarrow$ NP_1 VP and/or/nor NP_2 VP
Reason: $A \in Q_1 \cap Q_2$ iff $A \in Q_1$ and $A \in Q_2$
$A \in Q_1 \cup Q_2$ iff $A \in Q_1$ or $A \in Q_2$
$A \in \overline{Q_1} \cap \overline{Q_2}$ iff neither $A \in Q_1$ holds nor does $A \in Q_2$

This is in agreement with the old (and ill-defined) transformational rule of *conjunction reduction* (CR). However, consider the following:

(5) a. NP sang and danced $\not\Leftrightarrow$ NP sang and NP danced
NP = some man, no man, not every man, Mary or John, at least/most five women, exactly five women, most women

b. NP sang and danced $\Leftrightarrow$ NP sang and NP danced
NP = every man, Mary, Mary and John

(6) a. NP sang or danced $\not\Leftrightarrow$ NP sang or NP danced
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Boolean semantics and GQs account for these (non-)equivalences. For example:

(7) Some man danced and sang.
$\mathbf{dance}' \cap \mathbf{sing}' \in \{A \subseteq E : \mathbf{man}' \cap A \neq \emptyset\}$
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This can be false when both $\mathbf{man}' \cap \mathbf{dance}'$ and $\mathbf{man}' \cap \mathbf{sing}'$ are non-empty.

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This holds *iff* $\mathbf{man}' \subseteq \mathbf{dance}'$ and $\mathbf{man}' \subseteq \mathbf{sing}'$.

Conclusion: The boolean semantics of GQs is much more fine-grained than any syntactic account of the semantics of coordination.

2 Semantic answers to syntactic questions

A preliminary issue: partially defined determiners.

(9) The cat slept.

$$\mathbf{the}'_{sg}(A)(B) = \begin{cases} \mathbf{every}'(A)(B) & \text{if } |A| = 1 \\ \text{undefined} & \text{otherwise} \end{cases}$$

(10) The cats slept.

$$\mathbf{the}'_{pl}(A)(B) = \begin{cases} \mathbf{every}'(A)(B) & \text{if } |A| \geq 2 \\ \text{undefined} & \text{otherwise} \end{cases}$$

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Question 1: Which NPs can appear in *there* sentences and why?

(12) There is/are *a cat/some cat/no cat/three cats/less/more than ten cats/between five and ten cats/many cats/few cats* in the garden.

(13) *There is/are *every cat/most cats/all cats/the cat(s)/neither cat* in the garden.

Barwise and Cooper: The answer has to do with the semantics of *there* sentences. A determiner is allowed to appear in *there* sentences if and only if it does not make them tautological or contradictory.

(14) A sentence *there is/are NP* is true iff E is in the quantifier denoted by the NP.

For instance:

(15) There are three cats.
$\mathbf{three}'(\mathbf{cat}')(E)$
$\Leftrightarrow |\mathbf{cat}' \cap E| \geq 3$

(16) A determiner D is called *positive strong* iff for every $A \subseteq E$: $D(A)(E)$ is true whenever it is defined.
A determiner D is called *negative strong* iff for every $A \subseteq E$: $D(A)(E)$ is false whenever it is defined.
A determiner that is neither positive strong nor negative strong is called *weak*.

The determiners in (12) are weak. Those in (13) are strong. Generally: strong determiners are those that make *there* sentences semantically trivial (=tautological or contradictory whenever defined). Consequently they are ruled out.

Question 2: Which NPs can appear in partitives and why?

(17) All of *the book/the books/the five books/these books* is/are interesting.

(18) *All of *a book/no book/five books/most books* is/are interesting.

Barwise and Cooper/Ladusaw: The answer is again semantic. The denotation of *of* retrieves a set from the denotation of the complement NP. If this set cannot be retrieved, the partitive becomes ill-formed.

(19) The constituent *of NP* is defined and denotes the set A iff NP denotes the quantifier $\{B \subseteq E : A \subseteq B\}$.

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Consequently, *all of the cats*, whenever defined, denotes the same quantifier as *every cat/all cats*.

(21) A determiner is called *definite* iff for every $A \subseteq E$: whenever the quantifier $D(A)$ is defined A is non-empty and $D(A) = \{B \subseteq E : A \subseteq B\}$.

The determiners in (17) are definite. Those in (18) are not. Generally, only definite determiners guarantee, whenever defined, that the *of* in the partitive can extract from them a non-empty set. Hence, only these NPs are allowed in partitive constructions.

A general strategy: Reduction of syntactic anomaly to semantic anomaly *that naturally arises from the semantics of the construction.*

Question 3: What licenses negative polarity items?

Examples (22)-(28) from Keenan (1996).
The Ladusaw-Fauconnier Generalization: Negative polarity items occur within arguments of monotonic decreasing functions but not within arguments of monotonic increasing functions.

Exercises

1. Write down the quantifiers denoting the following NPs: *every woman or every man*, *Mary and most students*, *John or five girls*, *neither many women nor many men*, *John and the girls*, *more than five girls but less than five boys*.

2. Show that given these denotations you can account for the corresponding equivalences with sentential coordinations as in (2)-(4).

3. Derive the semantics of the determiner *not every* using the boolean complement in the determiner domain. Hint: recall that determiners are relations between sets, and hence subsets of $\wp(E) \times \wp(E)$.
   Can you think of other possibly grammatical constructions with boolean operations in the determiner domain?

4. Consider the familiar sentence: *Tina is not tall and thin.* What readings are expected by a rule of CR? Compare these predictions with those of the boolean treatment.

5. Consider the sentence *there are cats in the garden*, as well as the questionable sentence ?*there is John in the garden.* What problem do these cases pose to the proposal made in class? Can you think of a modification that would improve the situation?

6. Consider sentences like *both/the two children are smiling.* Define *both* and *the two* as determiners. Hint: see the definitions of *the* and *neither.* Contrast now the cases in *all of the two/*both books are interesting.* What is the problem these two examples pose for Barwise and Cooper's proposal? Can you improve the situation?

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Lecture 5: Generalized Quantifiers (2)

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This is in agreement with the old (and ill-defined) transformational rule of *conjunction reduction* (CR). However, consider the following:

(5) a. NP sang and danced $\not\Leftrightarrow$ NP sang and NP danced
NP = some man, no man, not every man, Mary or John, at least/most five women, exactly five women, most women

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Conclusion: The boolean semantics of GQs is much more fine-grained than any syntactic account of the semantics of coordination.

2 Semantic answers to syntactic questions

A preliminary issue: partially defined determiners.

(9) The cat slept.

$$\mathbf{the}'_{sg}(A)(B) = \begin{cases} \mathbf{every}'(A)(B) & \text{if } |A| = 1 \\ \text{undefined} & \text{otherwise} \end{cases}$$

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Question 1: Which NPs can appear in *there* sentences and why?

(12) There is/are *a cat/some cat/no cat/three cats/less/more than ten cats/between five and ten cats/many cats/few cats* in the garden.

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For instance:

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The determiners in (12) are weak. Those in (13) are strong. Generally: strong determiners are those that make *there* sentences semantically trivial (=tautological or contradictory whenever defined). Consequently they are ruled out.

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6. Consider sentences like *both/the two children are smiling.* Define *both* and *the two* as determiners. Hint: see the definitions of *the* and *neither.* Contrast now the cases in *all of the two/*both books are interesting.* What is the problem these two examples pose for Barwise and Cooper's proposal? Can you improve the situation?

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Lecture 5: Generalized Quantifiers (2)

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1 Generalized Quantifiers as Boolean Objects

Fact: Coordination applies freely to proper names and other NPs alike.

(1) Mary and/or John, neither Mary nor John, every woman or every man, most women and most men, many students but few teachers, one student and five teachers, the teacher and every student etc.

The denotation of these NPs is easily derived using GQs and the boolean treatment of coordination.

(2) a. Mary and John smiled.
$\mathbf{smile}' \in \{A \subseteq E : \mathbf{m}' \in A\} \cap \{A \subseteq E : \mathbf{j}' \in A\}$
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$\overline{\mathbf{smile}'} \in \{A \subseteq E : \mathbf{m}' \in A\} \cap \overline{\mathbf{smile}'} \in \{A \subseteq E : \mathbf{j}' \in A\}$

Analyzing what these propositions amount to easily shows that (2a) is treated as equivalent to (2b), and similarly for (3) and (4). This is of course correct.

The same hold for *all* NP coordinations:
Prediction: NP_1 and/or/nor NP_2 VP $\Leftrightarrow$ NP_1 VP and/or/nor NP_2 VP
Reason: $A \in Q_1 \cap Q_2$ iff $A \in Q_1$ and $A \in Q_2$
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This is in agreement with the old (and ill-defined) transformational rule of *conjunction reduction* (CR). However, consider the following:

(5) a. NP sang and danced $\not\Leftrightarrow$ NP sang and NP danced
NP = some man, no man, not every man, Mary or John, at least/most five women, exactly five women, most women

b. NP sang and danced $\Leftrightarrow$ NP sang and NP danced
NP = every man, Mary, Mary and John

(6) a. NP sang or danced $\not\Leftrightarrow$ NP sang or NP danced
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Boolean semantics and GQs account for these (non-)equivalences. For example:

(7) Some man danced and sang.
$\mathbf{dance}' \cap \mathbf{sing}' \in \{A \subseteq E : \mathbf{man}' \cap A \neq \emptyset\}$
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This holds *iff* $\mathbf{man}' \subseteq \mathbf{dance}'$ and $\mathbf{man}' \subseteq \mathbf{sing}'$.

Conclusion: The boolean semantics of GQs is much more fine-grained than any syntactic account of the semantics of coordination.

2 Semantic answers to syntactic questions

A preliminary issue: partially defined determiners.

(9) The cat slept.

$$\mathbf{the}'_{sg}(A)(B) = \begin{cases} \mathbf{every}'(A)(B) & \text{if } |A| = 1 \\ \text{undefined} & \text{otherwise} \end{cases}$$

(10) The cats slept.

$$\mathbf{the}'_{pl}(A)(B) = \begin{cases} \mathbf{every}'(A)(B) & \text{if } |A| \geq 2 \\ \text{undefined} & \text{otherwise} \end{cases}$$

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$$\mathbf{neither}'(A)(B) = \begin{cases} \mathbf{no}'(A)(B) & \text{if } |A| = 2 \\ \text{undefined} & \text{otherwise} \end{cases}$$

Question 1: Which NPs can appear in *there* sentences and why?

(12) There is/are *a cat/some cat/no cat/three cats/less/more than ten cats/between five and ten cats/many cats/few cats* in the garden.

(13) *There is/are *every cat/most cats/all cats/the cat(s)/neither cat* in the garden.

Barwise and Cooper: The answer has to do with the semantics of *there* sentences. A determiner is allowed to appear in *there* sentences if and only if it does not make them tautological or contradictory.

(14) A sentence *there is/are NP* is true iff E is in the quantifier denoted by the NP.

For instance:

(15) There are three cats.
$\mathbf{three}'(\mathbf{cat}')(E)$
$\Leftrightarrow |\mathbf{cat}' \cap E| \geq 3$

(16) A determiner D is called *positive strong* iff for every $A \subseteq E$: $D(A)(E)$ is true whenever it is defined.
A determiner D is called *negative strong* iff for every $A \subseteq E$: $D(A)(E)$ is false whenever it is defined.
A determiner that is neither positive strong nor negative strong is called *weak*.

The determiners in (12) are weak. Those in (13) are strong. Generally: strong determiners are those that make *there* sentences semantically trivial (=tautological or contradictory whenever defined). Consequently they are ruled out.

Question 2: Which NPs can appear in partitives and why?

(17) All of *the book/the books/the five books/these books* is/are interesting.

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Barwise and Cooper/Ladusaw: The answer is again semantic. The denotation of *of* retrieves a set from the denotation of the complement NP. If this set cannot be retrieved, the partitive becomes ill-formed.

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Consequently, *all of the cats*, whenever defined, denotes the same quantifier as *every cat/all cats*.

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Question 3: What licenses negative polarity items?

Examples (22)-(28) from Keenan (1996).
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