Course: Modeltheoretic Semantics and the Syntax-Semantics Interface
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Lecture 4:
Generalized Quantifiers (1)

Topics: NP denotations as sets of sets, NP monotonicity, determiner denotations as relations between sets, determiner monotonicity

Reading: Keenan, E. L. 1996: The Semantics of Determiners, in S. Lappin (ed.), *The Handbook of Contemporary Semantic Theory*, Blackwell.

(1) Every man ran.

Let *every man* denote a *set of sets*: the set of subsets of E that include the set of men:

(2) $\{B \subseteq E : \mathbf{man}' \subseteq B\}$

In type-theoretical terms: *every man* denotes an $(et)t$ function.
Application of this function to the VP denotation:

(3) $\mathbf{run}' \in \{B \subseteq E : \mathbf{man}' \subseteq B\}$
$\Leftrightarrow \mathbf{man}' \subseteq \mathbf{run}'$
"every member of the set $\mathbf{man}'$ is a member of the set $\mathbf{run}'$"

(4) Some man ran.

(5) $\mathbf{run}' \in \{B \subseteq E : \mathbf{man}' \cap B \neq \emptyset\}$
$\Leftrightarrow \mathbf{man}' \cap \mathbf{run}' \neq \emptyset$
"there is an entity that is a member of both $\mathbf{man}'$ and $\mathbf{run}'$"

(6) No man ran.

(7) $\mathbf{run}' \in \{B \subseteq E : \mathbf{man}' \cap B = \emptyset\}$
$\Leftrightarrow \mathbf{man}' \cap \mathbf{run}' = \emptyset$

(8) *exactly five men*: $\{B \subseteq E : |\mathbf{man}' \cap B| = 5\}$

(9) *most men*: $\{B \subseteq E : |\mathbf{man}' \cap B| > |\mathbf{man}' \setminus B|\}$

Terminology: We call $Q \in \wp(\wp(E))$ (a set of subsets of E) a *generalized quantifier* (GQ) over E.
Also proper names like *Tina* denote GQs from now on:

(10) Tina ran.

(11) $\mathbf{run}' \in \{B \subseteq E : \mathbf{tina}' \in B\}$
"the set of runners is in the set of subsets of E that contain $\mathbf{tina}'$"
$\Leftrightarrow \mathbf{tina}' \in \mathbf{run}'$

(12) a. Some man ran quickly $\Rightarrow$ Some man ran

b. Some man ran quickly $\not\Leftarrow$ Some man ran

(13) a. No man ran quickly $\not\Rightarrow$ No man ran

b. No man ran quickly $\Leftarrow$ No man ran

(14) a. Exactly five men ran quickly $\not\Rightarrow$ Exactly five men ran

b. Exactly five men ran quickly $\not\Leftarrow$ Exactly five men ran

Some man is called an *upward monotone* (mon$\uparrow$) noun phrase.
No man is called a *downward monotone* (mon$\downarrow$) noun phrase.
Exactly five men is called a *non-monotone* (neither mon$\uparrow$ nor mon$\downarrow$) noun phrase.

Definition 1 (quantifier monotonicity) *A generalized quantifier* $Q \in \wp(\wp(E))$ *is:*

1. *mon*$\uparrow$ *iff for all* $A \subseteq B \subseteq E$: *if* $A \in Q$ *then* $B \in Q$.
2. *mon*$\downarrow$ *iff for all* $A \subseteq B \subseteq E$: *if* $B \in Q$ *then* $A \in Q$.

Determiners like the ones considered above denote functions from sets to GQs:

(15) $\mathbf{every}'(A) \;=\; \{B \subseteq E : A \subseteq B\}$

Thus, determiners are functions of type $(et)((et)t)$.
Alternatively, we can view them as *relations* between subsets of E:

(16) $\mathbf{every}'(A)(B)$ iff $A \subseteq B$

(17) $\mathbf{some}'(A)(B)$ iff $A \cap B \neq \emptyset$

(18) $\mathbf{most}'(A)(B)$ iff $|A \cap B| > |A \setminus B|$

Terminology: We call $D \in \wp(\wp(E) \times \wp(E))$ (a relation between subsets of E) a *determiner* over E.

(19) a. Some blue car arrived $\Rightarrow$ Some car arrived

b. Some blue car arrived $\not\Leftarrow$ Some car arrived

(20) a. Every blue car arrived $\not\Rightarrow$ Every car arrived

b. Every blue car arrived $\Leftarrow$ Every car arrived

(21) a. Exactly five blue cars arrived $\not\Rightarrow$ Exactly five cars arrived

b. Exactly five blue cars arrived $\not\Leftarrow$ Exactly five cars arrived

Definition 2 (determiner monotonicity) *A determiner* $D \in \wp(\wp(E) \times \wp(E))$ *is:*

1. $\uparrow$*mon iff for all* $A_1 \subseteq A_2 \subseteq E$ *and* $B \subseteq E$: *if* $D(A_1)(B)$ *then* $D(A_2)(B)$.
2. $\downarrow$*mon iff for all* $A_1 \subseteq A_2 \subseteq E$ *and* $B \subseteq E$: *if* $D(A_2)(B)$ *then* $D(A_1)(B)$.

3. *mon↑ iff for all $A \subseteq E$ and $B_1 \subseteq B_2 \subseteq E$: if $D(A)(B_1)$ then $D(A)(B_2)$.*

4. *mon↓ iff for all $A \subseteq E$ and $B_1 \subseteq B_2 \subseteq E$: if $D(A)(B_2)$ then $D(A)(B_1)$.*

Fact: A determiner D is mon↑ (mon↓) iff for every $A \subseteq E$: $D(A)$ is a mon↑ (mon↓) *quantifier.*
Some is called an *upward left-monotone* (↑mon) determiner (it is also mon↑).
Every is called a *downward left-monotone* (↓mon) determiner (but it is mon↑).
Exactly five men is not left-monotone (nor is it right-monotone).

(22) Every man ran ⇔ Every man is a man who ran

(23) Some man ran ⇔ Some man is a man who ran

(24) Exactly five men ran ⇔ Exactly five men are men who ran

... and so on for all determiners!

Definition 3 (conservativity) *A determiner $D \in \wp(\wp(E) \times \wp(E))$ is* conservative *iff for all $A, B \subseteq E$: $D(A)(B) \Leftrightarrow D(A)(A \cap B)$.*

Universal: All natural language determiners (simple and complex) are conservative.

Exercises

1. Give denotations for the following NPs: *more than four women, at most five children, between four and ten cows, all but five students, less than half the men, not every goat, Tina and Mary.*

2. For each NP in 1 say whether it is mon↑, mon↓ or non-monotone.

3. Give a relational denotation for the determiners in 1: *more than four, at most five, between four and ten, all but five, less than half the, not every.*

4. For each determiner in 3 specify its left and right monotonicity.

5. Arrange the determiners mentioned in the lecture notes and in exercise 3 in a table according to the nine possibilities for monotonicity in both arguments ($9 = 3$ possibilities for L-monotonicity $\times$ 3 for R-monotonicity). Try to fill in the "holes" in the table (=logical possibilities that no determiner realizes) with natural language determiners, however syntactically artificial these may be. You may consult Keenan's paper for examples.

6. Show that the following artificial determiner is *not* conservative:
   $NC(A)(B)$ iff $B \subseteq A$
   Hint: give A and B such that $NC(A)(B)$ and $NC(A)(A \cap B)$ do not have the same truth-value.
   Can you think of a syntactic element with the meaning of NC in a sentence of the form __ *students ran*? After you find such an element: give syntactic arguments for whether it is of the same category of the word *all* or not. Conflicting arguments are welcome – you don't have to decide on this matter.