Skolem Functions in Linguistics

Yoad Winter

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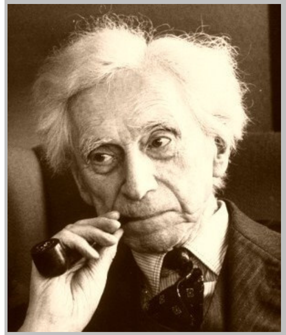
Syntax, Semantics, and Discourse: the Theory of the Interface
Workshop in memory of Tanya Reinhart

Indefinites: existential quantifiers?



Bertnard Russell
(1872-1970)

"The great majority of logicians who have dealt with this question were misled by grammar."
(Russell 1919)

My understanding: "indefinite descriptions may behave as if they were 'referential' like proper names, but let syntax not confuse us gentlemen – their meaning is that of existential quantifiers".

What's wrong about existential quantification?

The Epsilon Calculus

(Hilbert 1920)

$$\exists x\, [A(x)] \Leftrightarrow A(\varepsilon x.A(x))$$

$$\forall x\, [A(x)] \Leftrightarrow A(\varepsilon x.\neg A(x))$$

Motivation: provide witness for every existential claim.

(Meyer-Viol 1995)

David Hilbert
(1862-1943)

Modern Natural Language Semantics
1970s-1980s: Quantifiers Everywhere

Richard Montague
(1930-1971)

Syntax as a guide for theories of meaning:

All noun phrases denote generalized quantifiers

Montague (1973)

Russell's distinctions – left for philosophy of language

Hilbert's concerns – left for proof theory

Modern Natural Language Semantics

1980s-1990s: A Dynamic Turn

Empirical problems for Montagovian uniformity**:**

Every farmer who owns a donkey beats it.
(Kamp 1981, Heim 1982)

If a friend of mine from Texas had died in the fire, I would have inherited a fortune.
(Fodor and Sag 1982, Farkas 1981)

Hilbert strikes back – perhaps indefinites are (discourse) "referential" after all?

Early signs of SFs – branching

(historical observation by Schlenker 2006)

Henkin (1961): non-linear quantifier scope?

Branching quantifiers: $\begin{matrix}\forall x \exists z \\ \forall y \exists u\end{matrix} > \Phi(x,y,z,u)$

Henkin's Semantics involves *Skolem Functions* (next slide).

Hintikka (1973): branching in natural language –

Some book by every author is referred to in some essay by every critic.

$\begin{matrix}[\forall x{:}\mathbf{author}(x)]\ [\exists z{:}\mathbf{book\text{-}by}(z,x)] \\ [\forall y{:}\mathbf{critic}(y)]\ [\exists u{:}\mathbf{essay\text{-}by}(u,y)]\end{matrix} > \mathbf{referred\text{-}to\text{-}in}(z,u)$

What are Skolem Functions?

In the logical tradition:

Functions from (tuples of) *n* entities to entities.

For example:

$f: \quad \langle a,a\rangle \mapsto b \quad \langle a,b\rangle \mapsto a \quad \langle b,a\rangle \mapsto a \quad \langle b,b\rangle \mapsto b$

SF from pairs (2-tuples) over a simple domain with elements *a* and *b*.

Skolemization (higher-order Hilbertization)

Removing existential quantifiers from formulas in Predicate Calculus.

Example:

(1) *Everyone gave everyone something.*

➔ For every two people *x* and *y* we can find a thing *f*(*x*,*y*) that *x* gave *y.*

The function *f* is an Skolem Function of arity 2 that witnesses (1).

Skolemization (cont.)

Everyone gave everyone something.

(1) (2)

$$\forall x \forall y \exists z [R(x,y,z)] \rightsquigarrow \forall x \forall y [R(x,y,f(x,y)]$$

Suppose that *R* satisfies:

$$R(a,a,b) \wedge R(a,b,a) \wedge R(b,a,a) \wedge R(b,b,b)$$

Such an *R* satisfies (1) and with *f* they satisfy (2):

$$f: \quad \langle a,a \rangle \mapsto b \quad \langle a,b \rangle \mapsto a \quad \langle b,a \rangle \mapsto a \quad \langle b,b \rangle \mapsto b$$

In linguistics: restricted quantifiers

Everyone gave everyone some present.

$$\forall x \forall y [\exists z : A(z)][R(x,y,z)] \rightsquigarrow \forall x \forall y [R(x,y,f(x,y,A))]$$

In the linguistic practice:

Skolem Functions are functions from *n-tuples of entities and non-empty sets A* to entities *in A*.

When n=0 (no entity arguments) the function is a ***choice function***: it chooses a fixed element from A.

SF semantics for Hintikka's examples?

(Henkin/Hintikka)

Some book by every author is referred to in some essay by every critic.

∃f∃g [∀x:**author**(x)] [∀y:**critic**(y)]
referred-to-in(f(x,λz.**book-by**(z,x)), g(y,λu.**essay-by**(u,y)))

But the status of branching has remained undecided in the logical-linguistic literature:

- Branching generalized quantifiers (Barwise 1979, Westerstähl 1987, Van Benthem 1989, Sher 1991)
- Doubts about evidence for branching (Fauconnier 1975, Beghelli et al. 1997)
- Intermediate positions (Schlenker 2006).

More signs of SFs – functional questions

(1) Which woman does every man love?
His mother.

(2) Which woman does no man love?
His mother-in-law.

Engdahl (1980,1986), Groenendijk and Stokhof (1984), Jacobson (1999):

(1) = *what is the Skolem function f such that the following holds?*

$$\forall x\, [\mathbf{man}(x) \rightarrow \boldsymbol{love}(x, f(x, \boldsymbol{woman}))]$$

Early 90s – the plot thickens

Reinhart (1992), early drafts of Reinhart (1997) and Kratzer (1998)

Choice functions derive the special scope properties of indefinites and *wh*-in-situ:

"Quantification over choice functions is a crucial linguistic device and its precise formal properties should be studied in much greater depth than what I was able to do here."
Reinhart (1992)

Tanya Reinhart (1943-2007)

Hilbert strikes harder: CFs (SFs) as a general semantics for indefinites and *wh*-elements.

Reinhart's CF thesis

Exceptional scope of indefinites belongs in the semantics – neither (logical) syntax nor pragmatics (Fodor and Sag) are responsible.

If a friend of mine from Texas had died in the fire, I would have inherited a fortune.

Reinhart's analysis, with DRT-style closure:

$$\exists f[CH(f) \wedge [\mathbf{die}(f(\mathbf{friend})) \rightarrow \mathbf{fortune}]]$$

Precursors semantic scope mechanisms:
Cooper (1975), Hendriks (1993)

Summary: short history of SFs in linguistics

– 1960s	logico-philosophical foundations
1970s	branching quantification
1980s	functional questions
1990s –	scope of indefinites, and more…

Caveat: more researchers have studied epsilon-terms and their possible relations to anaphora, predating current attempts – see Slater (1986), Egli (1991).

Mid 90s: new questions

- Formalizing CFs/SFs in linguistics
- CFs vs. general SFs
- Empirical consequences of attributing the scope of indefinites to semantics
- Functional pronouns
- General role of CFs/SFs within the DP: definites, numerals, anaphoric pronouns

Precise use of CFs/SFs

Empty set problem:
Some fortuneteller from Utrecht arrived.

$$\exists f[CH(f) \wedge [\mathbf{arrive}(f(\mathbf{fortuneteller}))]]$$

Winter (1997): $\exists f[CH_{\mathbf{Q}}(f) \wedge [(f(\mathbf{fortuneteller}))(\mathbf{arrive})]]$

Montague-style

Do away with existential closure of CFs?

Kratzer (1998): $\mathbf{arrive}(f(\mathbf{fortuneteller}))$

Hilbert / Fodor & Sag-style

CFs or general SFs?

The problem of "intermediate scope":

(1) *Every professor will rejoice if a student of mine/his cheats on the exam.*

Is there a contrast in cases like (1)?
Fodor and Sag – Yes.
Wide agreement nowdays – No.
(Farkas, Abusch, Ruys, Reinhart, Chierchia)

Kratzer: Evidence for "referential" general SFs
Reinhart: Evidence for intermediate existential closure
Chierchia: Evidence for both

CFs or general SFs? (cont.)

Winter (2001) – uses general SFs to block undesired effects with CFs.
Every child loves a woman he knows.

$$\cancel{\exists f[CH(f) \wedge \forall x[\mathbf{child}'(x) \rightarrow \mathbf{love}'(f(\lambda y.\mathbf{woman}'(y) \wedge \mathbf{know}'(y)(x)))(x)]]]}$$

Rather – the arity of the SK matches the number of bound variables within the indefinite's restriction:

a woman – SK_0 = *CF*
a woman he knows – SK_1
a woman who told it to him – SK_2

Advantages of "semantic scope"

Ruys' problem of numeral indefinites:

(1) *If three workers in our staff have a baby soon we will have to face hard organizational problems.* Winter (1997)

Double scope:

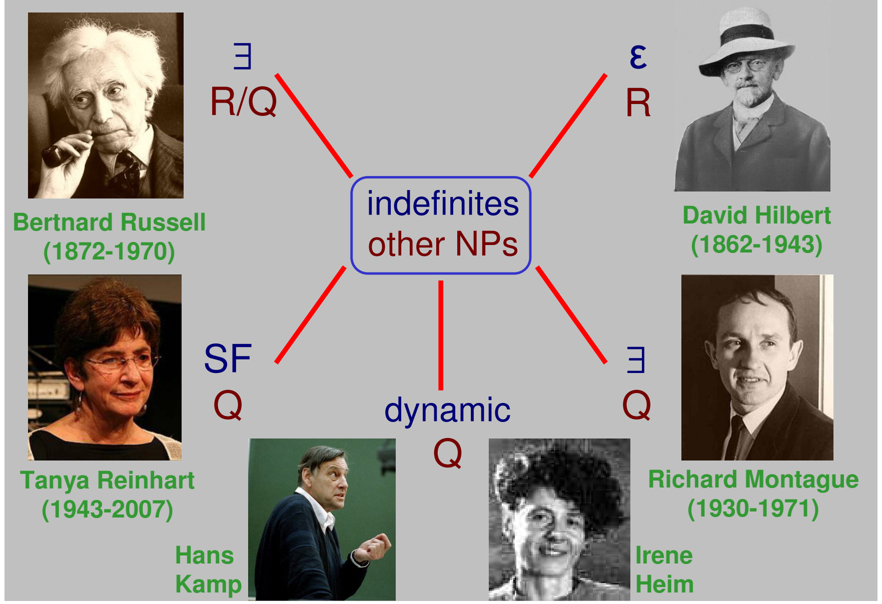
1- Existential scope – island insensitive
2- Distribution scope – island sensitive

Explained by CF **semantic** strategy.

On-going work on SFs in Linguistics

- **Indefinites/functional readings**
  (Winter 2004)
- **Branching and indefinites**
  (Schlenker 2006)
- **Donkey anaphora and SFs**
  Peregrin and von Heusinger 2004
  Elbourne 2005 → Brennan 2008

Indefinites and Quantification – pictures



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