

The Semantics of Intensionalization

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Sensitivity to intensions

- Some words are **intension-sensitive** (IS):
seek a lawyer, *fake* diamond, *believe* that...
- Other words are **intension-insensitive** (INS):
kiss a lawyer, *shiny* diamond
- IS words and expressions lead to **intensional** phenomena:
propositional attitudes, privative modification.
- INS words and expressions only require **extensional** semantics.

Syntactic compatibility of IS and INS expressions

IS and INS expressions may share syntactic categories and appear in the same constructions.

Especially – in the case of transitive verbs:

John needed and inherited a house.

Mary sought, found and ate a fish.

Sue ordered and got a new PC.

“Modularity” of intensional phenomena

Intensional phenomena may appear due to mechanisms that are also relevant for purely extensional effects.

The Quine-Montague Hypothesis: *De dicto/de re* ambiguity as scope ambiguity:

A queen kissed every king.

A queen looked for a king.

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A queen kissed every king.

A queen looked for a king. (vs. *A queen kissed a king*)

Keenan and Faltz 1985, p. 274:

“Our general task will be to create a system of model-theoretic semantic interpretation for our logical language which will preserve the advantages and insights revealed by our extensional system while allowing properly intensional facts to be represented.”

Our aim – a modular architecture of intensional semantics

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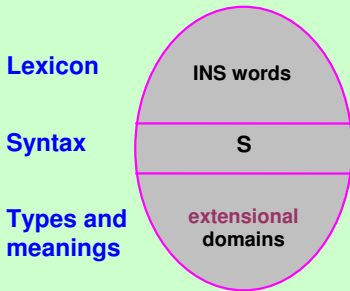
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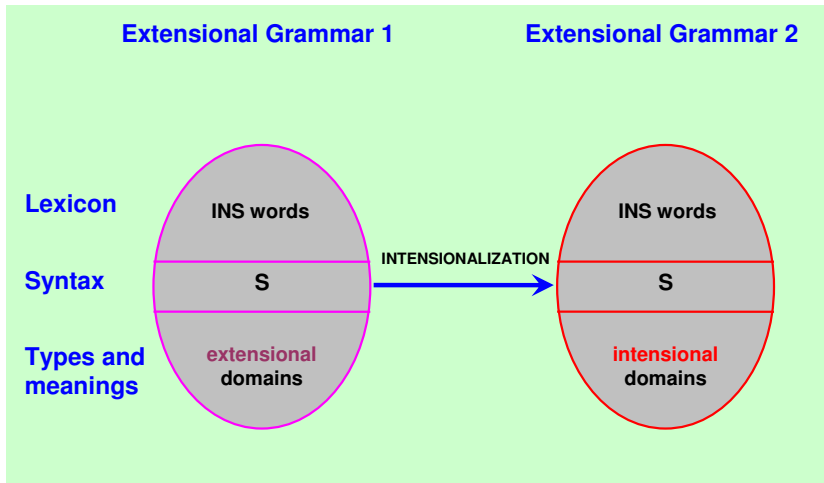
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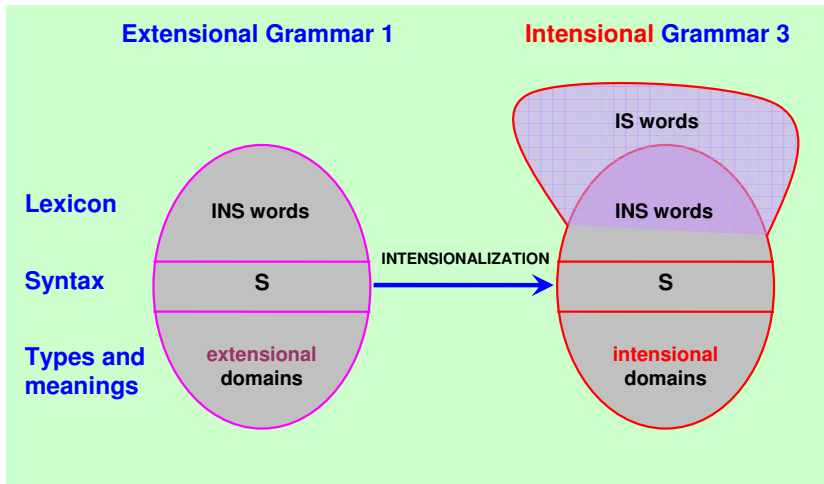
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 - and especially – preserving truth-conditional behavior.
- **Add IS items** to the lexicon.

Extensional Grammar 1







Architectural benefits:

- Extensional treatments have exact parallels in intensional systems.
- Treatments of intensional phenomena are fully lexicalized.
- No *ad hoc* type shifts for intensionality as in Partee & Rooth (1983).

Benefits for treating concrete phenomena:

- *De dicto* *de re* – manifestations of extensional scope shifting principles.
- Avoiding PTQ-style meaning postulates for high types of INS words.
- IS TVs (e.g. *seek*) and INS TVs (e.g. *kiss*) are naturally coordinated.

Pedagogical benefit:

- We don't have to teach intensional systems “from scratch” – we can rely to a large extent on the understanding of extensional systems.

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An **intensionalization procedure** \mathcal{I} is a mapping of an extensional grammar G to a grammar $\mathcal{I}(G)$ that satisfies:

- **Strong syntactic equivalence** between G and $\mathcal{I}(G)$.
- **Truth-conditional soundness:** G and $\mathcal{I}(G)$ support the same entailments.
- **Extendability:** by only adding IS lexical items, $\mathcal{I}(G)$ can be extended to an adequate intensional grammar.

- An explicit intensionalization procedure in Keenan and Faltz (1985).
- Insights on type-theoretical frameworks in Van Benthem (1988).
- A limited intensionalization procedure can be inferred from Heim and Kratzer (1998,ch.12).
- Shan (2001) formalizes a similar intensionalization procedure using *monads*, following Barker's (2002) modular treatment of quantification using *continuations*.

Modify components of a grammar G as follows:

- **Types** – add a basic type s .
- **Frame** – add a nonempty domain D_s of **possible worlds**.
- **Typing** – modify types of lexical items.
- **Meanings** – modify meanings of lexical items.

Sentences in $\mathcal{I}(G)$ are of type st .

The following should be equivalent, given two derivations $\mathcal{D}(S_1)$ and $\mathcal{D}(S_2)$ in G of sentences S_1 and S_2 :

- For every model \mathcal{M} of G :

$$\llbracket \mathcal{D}(S_1) \rrbracket^{\mathcal{M}} = 1 \Rightarrow \llbracket \mathcal{D}(S_2) \rrbracket^{\mathcal{M}} = 1$$

- For every model \mathcal{M} of $\mathcal{I}(G)$ and for every $w \in D_s$:

$$\llbracket \mathcal{D}(S_1) \rrbracket^{\mathcal{M}}(w) = 1 \Rightarrow \llbracket \mathcal{D}(S_2) \rrbracket^{\mathcal{M}}(w) = 1$$

- Following Van Benthem (1988):
 - For all extensional types: type t (of truth-values) is replaced by type st (of propositions).
 - Only relational types are used (inessential).
- Heim and Kratzer's distinction between logical constants and non-logical constants is preserved.
- A sophisticated mapping is now needed for logical constants, which may have many s 's in their intensionalized type.
- Words for boolean operators are treated syncategorematically.

A proposal by Van Benthem (1988), attributed to Reinhard Muskens:

*“a **relational** rather than a **functional** type theory may prove the proper setting for this investigation...”*

We restrict the set of possible types to \mathcal{T}_{ext} , which is the least set s.t.

- $t \in \mathcal{T}_{\text{ext}}$.
- If $\sigma_1 \in \{e\} \cup \mathcal{T}_{\text{ext}}$ and $\sigma_2 \in \mathcal{T}_{\text{ext}}$ then $(\sigma_1\sigma_2) \in \mathcal{T}_{\text{ext}}$.

Note that if $\sigma \in \mathcal{T}_{\text{ext}}$ then:

- $\sigma = (\sigma_1 \dots (\sigma_n t) \dots)$ for some $n \geq 0$ and $\sigma_1, \dots, \sigma_n \in \{e\} \cup \mathcal{T}_{\text{ext}}$.
- D_σ is isomorphic to $\wp(D_{\sigma_1} \times \dots \times D_{\sigma_n})$.

Intensionalizing types

- For every $\sigma \in \mathcal{T}_{\text{ext}}$, let $\lceil \sigma \rceil$ the type that results from replacing each occurrence of t within σ by st .
(This is adopted from Van Benthem (1988).)
- Let $\mathcal{T}_{\text{int}} \stackrel{\text{def}}{=} \{\lceil \sigma \rceil \mid \sigma \in \mathcal{T}_{\text{ext}}\}$.
- Denote by $\lfloor \cdot \rfloor$ the inverse of $\lceil \cdot \rceil$.

Examples:

- $\lceil t \rceil = st$ (**propositions**).
- $\lceil et \rceil = e(st)$ (**properties**).
Note: $D_{e(st)}$ is isomorphic to $D_{s(et)}$ – the standard domain of properties.
- $\lceil e(et) \rceil = e(e(st))$

- **Logical constants:** If a word α has a constant denotation $f \in D_\sigma$ (e.g., *every*), then in the intensionalized grammar α denotes a constant $L(f) \in D_{\neg\sigma}$.
- **Non-logical constants:** If a word α can denote any $f \in D_\sigma$, then in the intensionalized grammar α can denote any $f \in D_{\neg\sigma}$.
For instance: words that denote arbitrary one-place predicates in D_{et} are mapped to words that denote arbitrary elements in $D_{e(st)}$.

But how to define $L(\cdot)$?

Intensionalizing *every*

- **every** = $\lambda A_{et} \lambda B_{et} . \forall x_e [A(x) \rightarrow B(x)]$.

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- We would like to arrive at the PTQ-like denotation:

$$L(\mathbf{every}) = \lambda A_{e(st)} \lambda B_{e(st)} \lambda w_s \cdot \forall x_e [A(x)(w) \rightarrow B(x)(w)]$$

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- Note that if we define the **extension** of $A \in D_{e(st)}$ in $w \in D_s$ as

$$A^w \stackrel{def}{=} \lambda x_e . A(w)(x) \text{ then:}$$

$$L(\mathbf{every})(A)(B)(w) = \mathbf{every}(A^w)(B^w)$$

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And furthermore – τ has to be extensional.

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- Thus, standardly: it is implicitly assumed that φ has to have a type $s\tau$ in order to have an extension.
And furthermore – τ has to be extensional.
- This is a too strong assumption for obtaining a general extensionalization procedure.

We therefore generalize our observation about intensional determiners in PTQ.

Definition (extension F^w of F in a world w)

Let $\sigma \in \mathcal{T}_{\text{int}} \cup \{e\}$, $F \in D_\sigma$ and $w \in D_s$.

- 1 if $\sigma = e$ then $F^w = F$;
- 2 if $\sigma = (\sigma_1 \cdots (\sigma_n(st)) \cdots)$, $n \geq 0$, then for all $x_1 \in D_{\lfloor \sigma_1 \rfloor}, \dots, x_n \in D_{\lfloor \sigma_n \rfloor}$:

$$F^w(x_1) \cdots (x_n) = 1 \Leftrightarrow \exists Y_1 \cdots Y_n \left[\bigwedge_{i=1}^n (Y_i^w = x_i) \wedge f(Y_1) \cdots (Y_n)(w) = 1 \right]$$

In words: A tuple x_1, \dots, x_n is in the w -extension of a relation F iff there is a tuple Y_1, \dots, Y_n in F whose w -extensions are x_1, \dots, x_n .

Intensionalizing logical constants

The L operator: Let $\sigma \in \mathcal{T}_{\text{ext}} \cup \{e\}$ and $f \in D_\sigma$.

- 1 if $\sigma = e$ then $L(f) = f$;
- 2 if $\sigma = (\sigma_1 \cdots (\sigma_n t) \cdots)$, $n \geq 0$, then for every $w \in D_s$ and for all $X_1 \in D_{\neg\sigma_1}, \dots, X_n \in D_{\neg\sigma_n}$:
$$(L(f))(X_1) \cdots (X_n)(w) = f(X_1^w) \cdots (X_n^w).$$

In words: A tuple X_1, \dots, X_n and w are in the intension of a relation f iff the w -extensions of X_1, \dots, X_n are in f .

If a word α has a constant denotation $f \in D_\sigma$, then in the intensionalized grammar α denotes the constant $L(f) \in D_{\neg\sigma}$.

Note: Applying L to extensional det's gives intensional PTQ-style det's.

Theorem

The intensionalization procedure described above is sound for any extensional grammar with:

- *Logical constants – of constant denotation.*
- *Non-logical constants (only n -ary predicates over e -type entities) – of arbitrary denotation.*
- *Syncategorematic boolean operators.*

$$F^w(x_1) \cdots (x_n) = F(L(x_1)) \cdots (L(x_n))(w)$$

Expected benefits:

- No restriction to relational types.
- No restriction over the types of non-logical constants.
- Simpler soundness proof (De Groote, Kanazawa and Muskens).
- Similar – or wider – linguistic coverage (work in progress).

A simple extensional lexicon

word α	type	denotes in	λ -term
<i>Mary, John,...</i>	e	D_e	
<i>king, queen,...</i>	et	D_{et}	
<i>smile,...</i>	et	D_{et}	
<i>kiss,...</i>	$e(et)$	$D_{e(et)}$	
<i>every</i>	$(et)((et)t)$	{ every }	$\lambda A_{et} \lambda B_{et}. \forall x_e [A(x) \rightarrow B(x)]$
<i>a</i>	$(et)((et)t)$	{ some }	$\lambda A_{et} \lambda B_{et}. \exists x_e [A(x) \wedge B(x)]$

Zero morphology items and syncategorematic operations

word α	type	denotes in	λ -term
ϵ_{ONS}	$(e(et))(((et)t)(et))$	{ons}	$\lambda R_{e(et)} \lambda F_{(et)t} \lambda x_e. F(\lambda y_e. R(y)(x))$
ϵ_{OWS}	$((((et)t)(et))$ $((((et)t)((et)t)t))$	{ows}	$\lambda R_{(((et)t)(et))} \lambda F_{(et)t} \lambda Q_{(et)t}.$ $F(\lambda y_e. Q(\lambda x_e. R(\lambda A_{et}. A(y))(x)))$
ϵ_{lift}	$e((et)t)$	{lift}	$\lambda x_e \lambda A_{et}. A(x)$

In addition: boolean conjunction, disjunction and negation are treated syncategorematically.

Example

The sentence *Every king kissed a queen* has (at least) the two derivations in (1) and (2).

(1) $[[\text{Every king}][[\epsilon_{\text{ONS}} \text{ kissed}][\text{a queen}]]]$

(2) $[[\text{Every king}][[\epsilon_{\text{OWS}} [\epsilon_{\text{ONS}} \text{ kissed}]]][\text{a queen}]]]$

- $[[\text{(1)}]]^{\mathcal{M}} = 1$ iff

$$\forall x \in D_e[[\text{king}]]^{\mathcal{M}}(x) \rightarrow \exists y \in D_e[[\text{queen}]]^{\mathcal{M}}(y) \wedge [[\text{kiss}]]^{\mathcal{M}}(y)(x)]]$$

- $[[\text{(2)}]]^{\mathcal{M}} = 1$ iff

$$\exists y \in D_e[[\text{queen}]]^{\mathcal{M}}(y) \wedge \forall x \in D_e[[\text{king}]]^{\mathcal{M}}(x) \rightarrow [[\text{kiss}]]^{\mathcal{M}}(y)(x)]]$$

- Standardly, (2) (extensionally) entails (1).

After intensionalization

word α	type	denotes in	λ -term
<i>Mary, John,...</i>	e	D_e	
<i>king, queen,...</i>	$e(st)$	$D_{e(st)}$	
<i>smile,...</i>	$e(st)$	$D_{e(st)}$	
<i>kiss,...</i>	$e(e(st))$	$D_{e(e(st))}$	
<i>every</i>	$(e(st))((e(st))(st))$	$\{L(\mathbf{every})\}$	(3)
<i>a</i>	$(e(st))((e(st))(st))$	$\{L(\mathbf{some})\}$	(4)

$$(3) L(\mathbf{every}) = \lambda\mathcal{A}_{e(st)}\lambda\mathcal{B}_{e(st)}\lambda w_s.\forall x_e[\mathcal{A}(x)(w) \rightarrow \mathcal{B}(x)(w)]$$

$$(4) L(\mathbf{some}) = \lambda\mathcal{A}_{e(st)}\lambda\mathcal{B}_{e(st)}\lambda w_s.\exists x_e[\mathcal{A}(x)(w) \wedge \mathcal{B}(x)(w)]$$

After intensionalization (cont.)

word α	type	denotes in	λ -term
ϵ_{ONS}	$(e(e(st))(((e(st))(st))(e(st))))$	$\{L(\mathbf{ons})\}$	(5)
ϵ_{OWS}	$((((e(st))(st))(e(st))))$ $((((e(st))(st))(((e(st))(st))(st))))$	$\{L(\mathbf{ows})\}$	(6)
ϵ_{lift}	$e((e(st))(st))$	$\{L(\mathbf{lift})\}$	(7)

$$(5) L(\mathbf{ons}) = \lambda \mathcal{R}_{e(e(st))} \lambda \mathcal{F}_{(e(st))(st)} \lambda x_e \lambda w_s. \mathcal{F}^w(\lambda y_e. \mathcal{R}(y)(x)(w))$$

$$(6) L(\mathbf{ows}) = \lambda \mathcal{R}_{(((e(st))(st))(e(st)))} \lambda \mathcal{F}_{(e(st))(st)} \lambda \mathcal{Q}_{(e(st))(st)} \lambda w_s. \\ \mathcal{F}^w(\lambda y_e. \mathcal{Q}^w(\lambda x_e. \mathcal{R}^w(\lambda A_{et}. A(y))(x)))$$

$$(7) L(\mathbf{lift}) = \lambda x_e \lambda A_{e(st)}. A(x)$$

Example

The sentence *Every king kissed a queen* has (at least) the two derivations:

(8) $[[\text{Every king}][[\epsilon_{\text{ONS}} \text{ kissed}][\text{a queen}]]]$

(9) $[[\text{Every king}][[\epsilon_{\text{OWS}} [\epsilon_{\text{ONS}} \text{ kissed}]]][\text{a queen}]]]$

Given an intensional model \mathcal{M} and $w \in D_s$:

- $[(8)]^{\mathcal{M}}(w) = 1$ iff

$$\forall x_e [[king]^{\mathcal{M}}(x)(w) \rightarrow \exists y_e [[queen]^{\mathcal{M}}(y)(w) \wedge [kiss]^{\mathcal{M}}(y)(x)(w)]]$$

- $[(9)]^{\mathcal{M}}(w) = 1$ iff

$$\exists y_e [[queen]^{\mathcal{M}}(y)(w) \wedge \forall x_e [[king]^{\mathcal{M}}(x)(w) \rightarrow [kiss]^{\mathcal{M}}(y)(x)(w)]]$$

- Now, the extensional entailment is preserved after intensionalization: (9) (intensionally) entails (8).

- We can add a transitive verb like *seek* as a nonlogical constant of type $((e(st))(st))(e(st))$.
- Thus, the object of *seek* is an intensional quantifier like in PTQ.

Example

In a model \mathcal{M} , the derivation (10) is interpreted as the proposition in (11).

(10) [Mary [sought [a king]]]

(11) $\llbracket seek \rrbracket^{\mathcal{M}}(\lambda \mathcal{B}_{e(st)} \lambda w_s. \exists y_e [\llbracket king \rrbracket^{\mathcal{M}}(y)(w) \wedge \mathcal{B}(y)(w)])(\llbracket Mary \rrbracket^{\mathcal{M}})$

Deriving the *de re* interpretation extensionally

- The interpretation in (11) is the *de dicto* interpretation of (10).
- We can also derive the *de re* interpretation, using the *intensionalized* version of the *extensional* scope mechanism.

Example

In a model \mathcal{M} , the derivation (12) is interpreted as the proposition in (13).

(12) $[[\epsilon_{\text{lift}} \text{ Mary}]] [[\epsilon_{\text{OWS}} \text{ sought}] [\text{a king}]]]$

(13) $\lambda w_s \exists y_e [[\text{king}]^{\mathcal{M}}(y)(w) \wedge ([\text{seek}]^{\mathcal{M}})^w(\lambda A_{et}.A(y))([\text{Mary}]^{\mathcal{M}})]$

Example

(14) [Mary [[sought [and [ϵ_{ONS} kissed]]] [a king]]]

The denotation of (14) in a model \mathcal{M} is:

$$\lambda w_s. \exists y_e [\mathbf{king}(y)(w) \wedge \mathbf{kiss}(y)(\mathbf{m})(w)] \wedge \\ \mathbf{seek}(\lambda \mathcal{B}_{e(st)} \lambda w_s. \exists y_e [\mathbf{king}(y)(w) \wedge \mathcal{B}(y)(w)])(\mathbf{m})$$

= “Mary sought a king and kissed a king”

- A *de dicto* reading of *a king* relative to *seek*.
- The existential import of *Mary kissed a king* is preserved thanks to the intensionalization technique.

- Intensionalization glues an *extensional grammar* to *intensional lexical entries*.
- Implication 1: Extensional scope mechanisms allow to derive intensional *de dicto/de re* ambiguities – the Quine-Montague Hypothesis.
- Implication 2: Extensional composition of objects with transitive verbs allows to derive conjunctions of extensional TVs with intensional TVs.
- *Hope*: Such modular architectures will be found useful for different grammatical frameworks and various linguistic phenomena, and especially for teaching them.