Scope Dominance with Upward Monotone Quantifiers

To appear in *Journal of Logic, Language and Information*

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Abstract

We give a complete characterization of the class of upward monotone generalized quantifiers Q_1 and Q_2 over countable domains that satisfy the scheme $Q_1x\ Q_2y\ \phi \rightarrow Q_2y\ Q_1x\ \phi$. This generalizes the characterization of such quantifiers over *finite* domains, according to which the scheme holds iff Q_1 is $\exists$ or Q_2 is $\forall$ (excluding trivial cases). Our result shows that in infinite domains, there are more general types of quantifiers that support these entailments.

1 Introduction

A type 1 *generalized quantifier* over a domain E is a set $Q \subseteq \wp(E)$. We henceforth refer to such sets more briefly as *quantifiers*. For instance, over a domain E and some $X \subseteq E$, the following are the quantifiers that are more traditionally written as $\exists x \in X$ and $\forall x \in X$, respectively:

$$EXIST(X) \stackrel{def}{=} \{A \subseteq E : X \cap A \neq \emptyset\}.$$

$$UNIV(X) \stackrel{def}{=} \{A \subseteq E : X \subseteq A\}.$$

We call such quantifiers *EXIST* ("existential") and *UNIV*, respectively.[1] The quantifiers Q that are both *EXIST* and *UNIV* are of the form $\{A \subseteq E : x \in A\}$ for some $x \in E$,

[1] We do not simply say that $EXIST(X)$ is *existential* to avoid confusion with the larger class of quantifiers that Keenan and Westerståhl (1996) call *intersective*, and which are often referred to as *existential*.

which are precisely the principal ultrafilters over E.

When Q_1 and Q_2 are quantifiers and R a binary relation, the formula $Q_1x\ Q_2y\ R(x,y)$ is often written Q_1Q_2R, which is interpreted in E as follows.

(1) $\{x \in E : R_x \in Q_2\} \in Q_1$,

where $R_x = \{y \in E : R(x,y)\}$. Henceforth we will also use the notation R^y for $\{x \in E : R(x,y)\}$, considering the following equivalence:

(2) $Q_2Q_1R^{-1} \Leftrightarrow \{y \in E : R^y \in Q_1\} \in Q_2$.

Previous studies of generalized quantifiers have characterized various *scope commutativity* properties of quantifiers in constructions with multiple quantification. Notably, Westerståhl (1996) characterizes the class of *self-commuting* quantifiers – those quantifiers Q that satisfy the following equivalence:

(3) For all $R \subseteq E^2$: $QQR \Leftrightarrow QQR^{-1}$.

Zimmermann (1993) characterizes the class of *scopeless* quantifiers – those quantifiers Q that satisfy the following equivalence.

(4) For all $Q_1 \subseteq \wp(E)$, for all $R \subseteq E^2$: $QQ_1R \Leftrightarrow Q_1QR^{-1}$.

He shows that the scopeless quantifiers over E are precisely the ultrafilters over E.

Westerståhl (1986) studies the more general problem of characterizing the quantifiers Q_1, Q_2 that satisfy the following unidirectional entailment.

(5) For all $R \subseteq E^2$: $Q_1Q_2R \Rightarrow Q_2Q_1R^{-1}$.

When this entailment holds, we say that Q_1 is (scopally) *dominant* over Q_2.

We denote the *complement* of a quantifier Q over E by $\overline{Q} \stackrel{def}{=} \wp(E)\backslash Q$. Keenan (1993) defines the *postcomplement* of a quantifier Q over E as the set $Q- \stackrel{def}{=} \{A \subseteq E : E \setminus A \in Q\}$. The *dual* Q^d (cf. Barwise and Cooper (1981)) of a quantifier Q is the complement of Q's postcomplement:

$$Q^d \stackrel{def}{=} \overline{(Q-)} = (\overline{Q})- = \{A \subseteq E : E \setminus A \notin Q\}.$$

Note that for any quantifier Q: $(Q^d)^d = Q$ and Q is *EXIST* iff Q^d is *UNIV*. Further, over a domain E the two *trivial* quantifiers – $\emptyset$ and $\wp(E)$ – are each other's duals. As the following simple fact shows, there is a close relation between quantifier duality and scope dominance.

Fact 1 *For all quantifiers Q_1 and Q_2: Q_1 is dominant over Q_2 iff Q_2^d is dominant over Q_1^d.*

This fact follows directly from the definition of scope dominance and duality, and the observation that for any $R \subseteq E^2$ we have:

$$Q_1 Q_2 R \;\Leftrightarrow\; \neg(Q_1^d\, Q_2^d\, (E^2 \setminus R)).$$

For the sake of completeness we give in section 2 a simple proof of Westerståhl's characterization of dominance between quantifiers in *finite* domains. The main part of the paper is section 3, where this characterization is extended to *countable* domains. Section 4 concludes with some remarks about scope commutativity, finiteness and monotonicity in natural language semantics.

2 Finite domains

Westerståhl's characterization is restricted to *upward monotone* quantifiers over *finite domains*. Standardly, by saying that a quantifier Q over E is upward monotone we mean that Q is closed under supersets: $A \in Q$ and $A \subseteq B$ implies $B \in Q$. Note that Q is upward monotone iff Q^d is. Under upward monotonicity and finiteness of the domain, Westerståhl's claim can be stated as follows.[2]

Fact 2 *Let Q_1 and Q_2 be upward monotone quantifiers over a finite domain E. Q_1 is dominant over Q_2 iff these quantifiers fall under at least one of the following cases.*

(i) Q_1 is EXIST or Q_2 is UNIV.

(ii) $Q_1 = \wp(E)$ and $Q_2 \neq \emptyset$, or $Q_2 = \emptyset$ and $Q_1 \neq \wp(E)$.

[2] Westerståhl characterizes scope entailments for *determiners* – functions from sets to generalized quantifiers. The following is a simpler statement of the result for generalized quantifiers.

Proof The "if" direction of the proof is easy, and does not require finiteness of the domain. For the "only if" direction, assume that Q_1 is dominant over Q_2. First it is easy to see that if $Q_1 = \wp(E)$ then $Q_2 \neq \emptyset$ and (dually) that if $Q_2 = \emptyset$ then $Q_1 \neq \wp(E)$. Assume for contradiction that neither *(i)* nor *(ii)* holds. Then by finiteness of E there is a minimal set $A \in Q_1$ such that $|A| \geq 2$ (otherwise by upward monotonicity, $Q_1 = \wp(E)$ or $Q_1 = EXIST(\bigcup_{\{x\} \in Q_1} \{x\})$). By the dual consideration, there are $B_1, B_2 \in Q_2$ such that $B_1 \cap B_2 \notin Q_2$. Given the sets A, B_1 and B_2, and an arbitrary $a \in A$, it is easy to verify that the relation $(\{a\} \times B_1) \cup ((A \setminus \{a\}) \times B_2)$ contradicts our assumption that Q_1 is dominant over Q_2. □

Westerståhl (1996) calls two quantifiers $Q_1, Q_2 \subseteq \wp(E)$ *independent* if they satisfy the following equivalence.

(6) For all $R \subseteq E^2$: $Q_1 Q_2 R \Leftrightarrow Q_2 Q_1 R^{-1}$.

Using Fact 2 it is easy to establish the following corollary.

Corollary 3 *Let Q_1 and Q_2 be upward monotone quantifiers over a finite domain E. Then Q_1 and Q_2 are independent iff Q_1 and Q_2 fall under at least one of the following cases.*

(i) *Q_1 and Q_2 are EXIST, or Q_1 and Q_2 are UNIV.*

(ii) *Q_1 or Q_2 are principal ultrafilters.*

(iii) *Q_1 or Q_2 are trivial, and $Q_1 \neq \overline{Q_2}$.*

Recall that the *trivial* quantifiers over a domain E are $\wp(E)$ and $\emptyset$.

Examples: For illustrating scope dominance in simple natural language sentences, consider first a well-known type of example.

(7) Some priest visited every city.

Let us assume that the nouns *priest* and *city* are denoted by the sets $P, C \subseteq E$ respectively, and that the verb *visited* is denoted by the binary relation $V \subseteq E^2$. Sentence (7) has two readings, depending on the order in which the quantifiers operate on the arguments of the relation V:

(8) a. $EXIST(P)\ UNIV(C)\ V$

b. $UNIV(C)\ EXIST(P)\ V^{-1}$

The statement in (8a) is called the *object narrow scope* (ONS) reading of sentence (7), whereas the the statement in (8b) is called the *object wide scope* (OWS) reading of the sentence. As a matter of first-order logic, (8a) entails (8b) but not vice versa. Thus, the quantifier $EXIST(P)$ is dominant over the quantifier $UNIV(C)$ for any $P, C \subseteq E$, but the opposite does not hold.

The situation is similar in cases where (exactly) one of the existential/universal quantifiers is replaced by another upward monotone quantifier, not necessarily first-order. The sentences in (9) below illustrate some cases like that, where the ONS reading entails the OWS reading. The corresponding quantifiers we assume are given in (10).

(9) a. At least half/at least two/all but at most five of the priests visited every city.

b. Some priest visited at least half/at least two/all but at most five of the cities.

(10)
$$\begin{array}{lcl} \mathsf{at\_least\_half\_of\_the}(X) & = & \{A \subseteq E : |X \cap A| \geq |X \setminus A|\} \\ \mathsf{at\_least\_}n(X) & = & \{A \subseteq E : |X \cap A| \geq n\} \\ \mathsf{all\_but\_at\_most\_}n\mathsf{\_of\_the}(X) & = & \{A \subseteq E : |A \setminus X| \leq n\} \end{array}$$

Note that the quantifier $\mathsf{at\_least\_half\_of\_the}(X)$ is not first-order definable.

Westerståhl's result shows that over finite domains, the *EXIST* quantifiers (for Q_1) and the *UNIV* quantifiers (for Q_2) are the only non-trivial upward monotone quantifiers that lead to entailments as in (5). Thus, the sentences in (7) and (9) are representative of the cases where upward monotone quantifiers lead to an entailment from the ONS reading to the OWS reading on finite domains.

3 Countable domains

As Westerståhl observes, his characterization of scope dominance over finite domains in Fact 2 does not hold for infinite domains. Thus, over infinite domains there are non-trivial upward monotonic quantifiers besides the *EXIST* and *UNIV* quantifiers that give rise to scope dominance. Consider the following example (following Westerståhl), where E is assumed to be countable.

(11) Infinitely many dots are contained in at least one of the three circles.

$$Q_1 = \{A \subseteq E : |D \cap A| = \aleph_0\}$$
$$Q_2 = \{A \subseteq E : C \cap A \neq \emptyset\}, \text{ where } |C| = 3$$

It is easy to verify that Q_1 is dominant over Q_2, but Q_1 and Q_2 are upward monotone and the conditions in Fact 2 do not hold. Incidentally, since Q_2 is *EXIST*, it is dominant over Q_1. In this section we characterize such cases of scope dominance in the class of upward monotone quantifiers over countable domains.

Let us define some properties of quantifiers that will be useful for characterizing scope dominance. First, we say that a quantifier Q satisfies the *union property* (U) when Q^d is closed under finite intersections. Thus, for all $A_1, A_2 \subseteq E$: if $A_1 \cup A_2 \in Q$ then $A_1 \in Q$ or $A_2 \in Q$. For example, any *EXIST* quantifier satisfies (U), while a *UNIV* quantifier $UNIV(X)$ satisfies (U) if and only if X is either a singleton or the empty set. The set of all infinite subsets of E satisfies (U) as well.

Further, we say that a quantifier Q satisfies the *Descending Chain Condition* (DCC) if for every descending sequence $A_1 \supseteq A_2 \supseteq \ldots A_n \supseteq \ldots$ in Q, the intersection $\bigcap_i A_i$ is in Q as well. For example, any *UNIV* quantifier satisfies (DCC). A quantifier $EXIST(X)$ satisfies (DCC) if and only if X is finite. Another quantifier that satisfies (DCC) is the following, where the domain $E = \mathbb{N}$ is the set of natural numbers:

$$\{A \subseteq \mathbb{N} : \forall n \in \mathbb{N}\, [2n \in A \vee 2n+1 \in A]\}.$$

If every set in a quantifier Q contains a finite subset that is also in Q, we say that Q satisfies (FIN). The following fact shows that for upward monotone quantifiers over countable domains, the (FIN) property is dual to (DCC).

Fact 4 *For any upward monotone quantifier Q over a countable domain E: Q satisfies* (DCC) *iff Q^d satisfies* (FIN).

Proof Assume that Q satisfies (DCC) and assume for contradiction that there is $A \in Q^d$ such that for all $B \subseteq A$: if $B \in Q^d$ then B is infinite. Let $B_0 \subset A$ be a finite set.

Hence $B_0 \notin Q^d$, and $E \setminus B_0 \in Q$. By countability of E, we can denote $A \setminus B_0 = \{a_i\}_{i=1}^{\infty}$. Let $B_{i+1} = B_i \cup \{a_{i+1}\}$, for any $i \geq 0$. By our assumption on A we have $B_i \notin Q^d$ for any $i \geq 0$, hence $E \setminus B_i \in Q$ for any $i \geq 0$. But $\bigcap_i (E \setminus B_i) = E \setminus A \notin Q$, in contradiction to Q satisfying (DCC).

Conversely, assume that Q^d satisfies (FIN). Let $B_1 \supseteq B_2 \supseteq \ldots$ be a descending chain in Q, so $E \setminus B_i \notin Q^d$ for any $i \geq 1$. Assume leading to a contradiction that $B = \bigcap_i B_i \notin Q$, thus $E \setminus B = \bigcup_i (E \setminus B_i) \in Q^d$. By (FIN) there is a finite $A' \in Q^d$ s.t. $A' \subseteq \bigcup_i (E \setminus B_i)$. Hence for some n, $A' \subseteq E \setminus B_n$, and from the upward monotonicity of Q, and hence of Q^d, $E \setminus B_n \in Q^d$, a contradiction. $\Box$

These two pairs of dual properties will be used in the proof of the following theorem, which is the main result of this paper.

Theorem 5 *Let Q_1 and Q_2 be upward monotone quantifiers over a countable domain E. Then Q_1 is dominant over Q_2 if and only if all of the following requirements hold:*

(i) Q_1^d or Q_2 are closed under finite intersections;

(ii) Q_1^d or Q_2 satisfy (DCC)*;*

(iii) Q_1^d or Q_2 are not empty.

Proof

For the "if" direction, assume that requirements (i)-(iii) hold. Consider first the case where Q_1^d is closed under finite intersections and Q_2 satisfies (DCC), where both Q_1^d and Q_2 are non-trivial.

Assume that $A \stackrel{def}{=} \{x \in E : R_x \in Q_2\}$ is in Q_1, and let $B \stackrel{def}{=} \{y \in E : R^y \in Q_1\}$. We need to show that $B \in Q_2$. Since E is countable and Q_2 satisfies (DCC) it is sufficient to prove that for every finite $F \subseteq E \setminus B$, we have $E \setminus F \in Q_2$.

For every $b \notin B$, $R^b \notin Q_1$. Since Q_1 has property (U) (by assumption about Q_1^d) and $E \in Q_1$ (by upward monotonicity and non-triviality), we have $E \setminus R^b \in Q_1$. Thus, by the definition of R^b and R_x, the set $A_b \stackrel{def}{=} \{x : b \notin R_x\}$ is in Q_1.

Now for any $F = \{b_1, \ldots, b_n\} \subseteq E \setminus B$, the sets $A_{b_1}, \ldots, A_{b_n}$ are all in Q_1, and since Q_1 is closed under finite intersections, we have $A \cap A_{b_1} \cap \ldots \cap A_{b_n} \in Q_1$.

Since Q_1 is non-trivial, this last set is non-empty, and hence there is $x \in E$ such that $R_x \in Q_2$ and also $F \cap R_x = \emptyset$. By the upward monotonicity of Q_2 it follows that $E \setminus F \in Q_2$.

For the other cases in requirements (i)-(iii), dominance of Q_1 over Q_2 now follows directly from Fact 1 about duality, and from the observation that for any non-empty quantifier Q over a countable domain: if Q is closed under finite intersections and satisfies (DCC), then Q is *UNIV*.

For the "only if" direction, assume that Q_1 is dominant over Q_2.

To show that (i) holds, assume that Q_1^d is not closed under finite intersections, so Q_1 does not satisfy (U). Hence, by upward monotonicity of Q_1, it contains a set $A = A_1 \cup A_2$ where A_1 and A_2 are disjoint and neither of them is in Q_1. To show that Q_2 is closed under finite intersections, let us denote for any $B_1, B_2 \in Q_2$:

$$R = (A_1 \times B_1) \cup (A_2 \times B_2).$$

We have $\{x : R_x \in Q_2\} \in Q_1$ and therefore $B \stackrel{def}{=} \{y : R^y \in Q_1\} \in Q_2$. But, since $A_1, A_2 \notin Q_1$, we have $B = B_1 \cap B_2$, hence $B_1 \cap B_2 \in Q_2$.

To show that (ii) holds, assume that Q_1^d does not satisfy (DCC), so from Fact 4 it follows that there is a set $A \in Q_1$ such that every subset of A that is also in Q_1 is infinite. To show that Q_2 satisfies (DCC), let $B_1 \supseteq B_2 \ldots \supseteq B_n \supseteq \ldots$ be a sequence of sets in Q_2, and let B be their intersection. We again assume that E is the set of natural numbers, and enumerate $A = \{a_1, a_2, \ldots, \}$. Consider the relation

$$R = \bigcup_{n=1}^{\infty} (\{a_n\} \times B_n).$$

Then the set $\{x : R_x \in Q_2\} = A \in Q_1$, or $Q_1 Q_2 R$, and by our assumption it follows that $Q_2 Q_1 R^{-1}$ holds, or $\{y : R^y \in Q_1\} \in Q_2$. We claim that this last set equals B, so Q_2 satisfies (DCC). Clearly, for every $y \in B$: $R^y = A \in Q_1$. However, if $y \notin B$ then there is n such that $y \notin B_m$ for all $m \geq n$, and therefore $R^y \subseteq A$ is finite. By our previous observation, such R^y is not in Q_1.

Clause (iii) is easily seen to hold. $\Box$

From this theorem it is easy to conclude the following, more direct, classification of the upward monotone quantifiers Q_1, Q_2 that support scope dominance over countable domains. These are precisely the pairs of quantifiers Q_1 and Q_2 that satisfy *at least one* of the following requirements.

(12) (i) Q_1 is *EXIST* or Q_2 is *UNIV*.

(ii) Q_1 satisfies (U), $Q_2 \neq \emptyset$ and Q_2 satisfies (DCC), or
Q_2 is closed under finite intersections, $Q_1 \neq \wp(E)$ and Q_1 satisfies (FIN).

(iii) $Q_1 = \wp(E)$ and $Q_2 \neq \emptyset$, or $Q_2 = \emptyset$ and $Q_1 \neq \wp(E)$.

That Theorem 5 is a generalization of Fact 2 for countable domains is obvious from clauses (i) and (iii) in this statement of the theorem. Clause (12)(ii) becomes redundant over finite domains, since over such domains the upward monotone quantifiers that satisfy (U) are exactly the *EXIST* quantifiers and $\wp(E)$, and the (DCC) requirement for Q_2 is trivially satisfied. Dually, over finite domains the upward monotone quantifiers that are closed under finite intersections are the *UNIV* quantifiers and $\emptyset$, and the "finiteness" requirement for Q_2 is trivially satisfied. However, as we shall exemplify below, over infinite domains (also countably infinite), there are non-trivial non-*EXIST* upward monotone quantifiers that satisfy (U), and (dually) there are non-trivial non-*UNIV* upward monotone quantifiers that are closed under finite intersections. Thus, clause (12)(ii) is where Theorem 5 generalizes Fact 2.

As in the case of scope dominance over finite domains (cf. Corollary 3), Theorem 5 allows us to characterize the pairs of *independent* quantifiers. To do so, let us first prove the following two lemmas.

Lemma 6 *An upward monotone quantifier Q over a countable domain E satisfies both* (U) *and* (DCC) *iff $Q = \wp(E)$ or $Q = EXIST(X)$ for some finite $X \subseteq E$.*

Proof The proof of the "if" direction is easy. For the "only if" direction, assume that Q satisfies (U) and $Q \neq \wp(E)$. We will show that there are no minimal sets in Q other than singletons. Let A be some arbitrary set in Q. If A is finite then it must contain a singleton in Q. If A is infinite, then either it contains a singleton in Q, or by (U) and the countability of E, we can form a descending chain of subsets of A, all in Q, whose intersection is empty. From (DCC) it follows that $\emptyset \in Q$ and by upward monotonicity $Q = \wp(E)$, in contradiction to our assumption. Thus, every set in Q contains a singleton in Q, and if $X = \{x \in E : \{x\} \in Q\}$ then by upward monotonicity $Q = EXIST(X)$. Suppose for contradiction that X is infinite, then again by (DCC), we conclude that $\emptyset \in Q$, contradiction. □

Lemma 7 *If a quantifier Q satisfies* (DCC) *and* (FIN) *then there are finitely many minimal sets in Q, all of them finite.*

Proof By (FIN) it follows that the minimal sets in Q are all finite. Assume for contradiction that there are infinitely many (finite) minimal sets in Q, and denote this collection of sets by $\mathcal{X}$. It follows that for any $A, B \in \mathcal{X}$ such that $A \neq B$, $A \cap B$ is a proper subset of both A and B. Let F_1 be in $\mathcal{X}$. Because F_1 is finite and $\mathcal{X}$ is infinite, there must be some $F_1' \subsetneq F_1$ such that the collection of sets $\mathcal{X}_1 = \{F \in \mathcal{X} : F \cap F_1 = F_1'\}$ is infinite. We can continue this process by defining F_i, F_i' and $\mathcal{X}_i$ for every $i \geq 1$ as follows:

F_{i+1} is some set in $\mathcal{X}_i$.

F_{i+1}' is some proper subset of F_{i+1} such that $\{F \in \mathcal{X}_i : F \cap F_{i+1} = F_{i+1}'\}$ is infinite.

$\mathcal{X}_{i+1} \stackrel{def}{=} \{F \in \mathcal{X}_i : F \cap F_{i+1} = F_{i+1}'\}$

We obtain an infinite sequence $F_1, \ldots, F_i, \ldots$ of finite minimal sets in Q, together with $F_1', \ldots, F_i', \ldots$, such that for every i, $F_i' \subsetneq F_i$ and for every $m > n$, $F_m \cap F_n = F_n'$.

We now let $A_n = \bigcup_{m \geq n} F_m$. This is a decreasing sequence of sets, all in Q, so by (DCC), $A = \bigcap_{n=1}^{\infty} A_n$ is in Q. We claim that $A = \bigcup_{n=1}^{\infty} F_n'$. Indeed, note that A consists of all elements which belong to infinitely many sets F_n. Let x be some element in $\bigcup_{n=1}^{\infty} F_n'$, thus $x \in F_n'$ for some n. For every $m > n$, $x \in F_m$ because $F_m \in \mathcal{X}_n$. Thus x is in infinitely many sets F_m, and therefore $x \in A$. For the opposite direction, assume that $x \in A$, thus belongs to infinitely many F_n. In particular it belongs to some F_m, F_n for $m > n$. But then $x \in (F_m \cap F_n) = F_n'$, thus belonging to $\bigcup_{n=1}^{\infty} F_n'$.

By our assumption on Q, the set A contains a finite subset $B \in Q$. The set B is then contained in the union of finitely many sets F_n', which implies that for some m

(larger than all these n's): $B \subsetneq F_m$. Because both F_m and B are in Q, this contradicts the minimality of F_m. $\square$

Using Theorem 5 and the two lemmas above, the proof of the following claim is by a simple enumeration of cases.

Corollary 8 *Let Q_1 and Q_2 be upward monotone quantifiers over a countable domain E. Then Q_1 and Q_2 are independent iff these two quantifiers or their duals Q_1^d and Q_2^d constitute a pair S_1, S_2, not necessarily in this order, which falls under at least one of the following cases.*

(i) $S_1 = EXIST(X)$ *for some* $X \neq \emptyset$*, s.t.* X *is finite and* S_2 *satisfies* (U)*, or* X *is infinite and* S_2 *is EXIST.*

(ii) S_1 *or* S_2 *are principal ultrafilters.*

(iii) For some finite collection $\mathcal{X} \subseteq \wp(E)$ *of finite sets,* $S_1 = \bigcup_{X \in \mathcal{X}} UNIV(X)$*, and* S_2 *is an ultrafilter.*

(iv) $S_1 = \emptyset$ *and* $S_2 \neq \wp(E)$*.*

Remark: Since we assume here the Axiom of Choice, non-principal ultrafilters exist over E, so (iii) is not subsumed by (ii).

Examples: First let us note that in example (11) above, $Q_2 = EXIST(C)$ for a finite C ($|C| = 3$). Q_1 satisfies (U), hence Q_1 and Q_2 fall under clause (i) in Corollary 8, and the ONS reading of the sentence is equivalent to the OWS reading. The following example illustrates the dual case covered by clause (i) in Corollary 8, where $Q_1 = UNIV(C)$ for a finite C, and Q_2 is closed under finite intersections.

(13) Each of the three circles contains all but finitely many dots.

$$Q_1 = \{A \subseteq E : C \subseteq A\}, \text{ where } |C| = 3$$

$$Q_2 = \{A \subseteq E : |D \setminus A| < \aleph_0\}$$

To illustrate non-trivial usages of clause (iii) in Corollary 8, we would have to use non-principal ultrafilters, which we here omit.

As for dominance between quantifiers without independence, the quantifiers in (14) and (15) below satisfy clause (ii) of (12). Hence, in these cases the ONS reading entails the OWS reading, but not vice versa (assuming a finite $n > 0$).

(14) Infinitely many dots are contained in all but at most n circles.

(15) At least n circles contain all but finitely many dots.

Similarly, consider the following examples.

(16) Infinitely many dots are contained in circle 1 or [circles 2 and 3].

(17) Circle 1 and [circles 2 or 3] contain all but finitely many dots.

We assume that the object of sentence (16) and the subject of sentence (17) denote the following quantifiers respectively, for three different circles c_1, c_2 and c_3.

$$\{A \subseteq E : c_1 \in A \vee (c_2 \in A \wedge c_3 \in A)\}$$

$$\{A \subseteq E : c_1 \in A \wedge (c_2 \in A \vee c_3 \in A)\}$$

Also the quantifiers in these sentences satisfy clause (ii) of (12), hence the scope dominance, but the two quantifiers in each sentence are not independent.

4 Concluding remarks

In this paper we have characterized scope dominance and independence for upward monotone quantifiers over countable domains. This is a natural extension of the results by Westerståhl and Zimmermann about self-commuting and scopeless quantifiers. This characterization directly extends a previous result by Altman et al. (2001), which concentrated on a subclass of quantifiers on countable domains, called *finitely based* quantifiers. Our results are still partial in some obvious respects. First, we did not characterize scope dominance for uncountable domains. Theorem 5 does not hold for such domains, for a similar reason to the reason that Fact 2 about finite domains does not hold for countable domains. Consider for instance the following sentence and quantifiers, parallel to (11) above over countable domains.

(18) Uncountably many dots are contained in at least one of the countably many circles.

$$Q_1 = \{A \subseteq E : |D \cap A| = \aleph_1\}$$
$$Q_2 = \{A \subseteq E : C \cap A \neq \emptyset\}, \text{ where } |C| = \aleph_0$$

The quantifier Q_1 is dominant over Q_2, but these quantifiers do not satisfy the conditions of Theorem 5. Thus, a further generalization of our result is called for.

It is also natural to look for a characterization of dominance with *non-upward monotone* quantifiers. One recent result in this area is the characterization in Ben-Avi and Winter (2004) of scope dominance with downward monotone quantifiers over finite domains. One can also add further requirements on the relation R in (5), and obtain more quantifiers Q_1 and Q_2 that exhibit scope dominance for this restricted class of relations. Such more refined characterizations are relevant for natural language, where there are often logical restrictions on the possible denotations of binary relations. For instance, in the sentence *every priest is taller than some peasant*, where the relation *be taller than* is transitive, the ONS reading and the OWS reading are equivalent over finite domains, in contrast to the case with general R's.

Characterizations of scope independence are useful for reducing ambiguity in computational representations of natural language sentences. One system that goes in this direction, using the results that were obtained in the present paper, is described in Altman and Winter (2003). Another system with the same motivations, based on slightly different formal assumptions, is described in Chaves (2003).

Acknowledgements

The first and third authors were partly supported by grant no. 1999210 ("Extensions and Implementations of Natural Logic") from the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel. The third author is grateful for the UiL OTS of Utrecht University, where part of this research was conducted. We are indebted to two anonymous *JLLI* referees for their useful remarks on a previous draft.

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