Formal Semantics of Natural Language

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ESSLLI 2022, Galway

Class 1

Basic notions and tools

- NOTIONS

What is Semantics?

Semantics: the study of meaning

Formal Semantics: the study of the (logical) relations between form and meaning

Sad-eyed lady of the lowlands,
Where the sad-eyed prophet
says that no man comes

Course Topics

Classes 1-2:

Logical approach to meaning in NL

- *Model-theoretic semantics in a post-PTQ fashion*

Classes 3-5:

Selected problems in current research

- *Plurals*
- *Events and modification*
- *Presupposition*

Examples (classes 1-2)

Treat inferences using higher-order logical operators.

Sue admires <u>herself</u> ➔ *Sue admires Sue*

Dan <u>is</u> tall <u>and</u> thin ➔ *Dan <u>is</u> thin*

<u>Some</u> <u>Dutch</u> man <u>is</u> thin ➔ *<u>Some</u> man <u>is</u> <u>Dutch</u>*

Underlined words denote **functions**, so that sentences can be treated using simple structures.

Sue [admires herself] - (*HERSELF*(**admire**))(**sue**)

Dan [is [tall [and thin]] - (*IS*((*AND*(**thin**))(**tall**))(**dan**)

[Some [Dutch man]][is thin] -
(*SOME*(**dutch**(**man**)))(*IS*(**thin**))

Examples – plurals (class 3)

Find logical regularities that connect singular descriptions to plural descriptions.

[[Sue and Mary]] = **sue** + **mary**

[[the girls]] = $\mathbf{girl_1} + \mathbf{girl_2} + \mathbf{girl_3} + \ldots$

What is "+"? What is the structure of individuals?

Use these regularities to explain:

Sue and Mary ran ←→ Sue ran and Mary ran

Sue and Mary met ←→ **?** (#Sue met and Mary met)

The girls and the boys were separated ←?→
The children were separated

Examples – events (class 4)

Nominal modification:

Sue's Ferrari is a beautiful car
➔ Sue's Ferrari is beautiful

Verbal modification:

Sue danced beautifully **?**➔ Sue is beautiful

Sue danced beautifully ➔ Sue's dancing was beautiful

What kind of entity is "Sue's dancing"?
- An event!

How do events figure with verbs?
- Like other entities figure with nouns!

Examples – presuppositions (class 5)

The Russell-Strawson debate:

The king of France is bald – false or undefined?

The projection perspective on this debate:

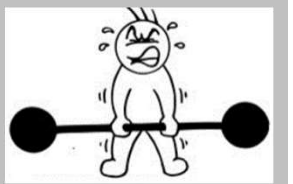
If Sue met the Burgadan astronaut, she must be excited

➔ *There's a Burgadan astronaut*

Is Sue met a Burgadan astronaut, she must be excited

?➔ *There's a Burgadan astronaut*

Can Kleene truth tables shed light on this projection behavior?



IF	*	0	1
*	*	*	*
0	*	1	1
1	*	0	1

IF	*	0	1
*	*	*	1
0	1	1	1
1	*	0	1

Prerequisite: Naive Set Theory

- membership, equality, subset
- set specification
- empty set and set construction
- set union, intersection, complement, difference
- powersets
- ordered pairs, cartesian products
- relations, domain, range
- properties of relations: symmetry, transitivity...
- functions
- inverse functions, function composition
- injection, surjection, bijection

Exercises: Chapter 1, Winter (2016)

Non-Prerequisites

Class 1 – Basic notions and tools

Entailment as a core semantic intuition
The truth-conditionality criterion
Equivalence, tautology, contradiction, contingency
Comparison to philosophical and mathematical logic
Compositionality
Structural ambiguity; ambiguity vs. vagueness
Types and domains
Characteristic functions
Currying
Arbitrary, combinatorial and logical denotations

Reading: chapters 1 and 2 of [24], including exercises

Class 2 – Simple meaning composition

Using Lambda notation
Reflexive pronouns in variable-free semantics
Simple intersective modifiers
Cross-categorial coordination and negation
Simple quantifiers
Word meaning and intended models
Function application
Syntax-semantics interface
Category-to-type matching

Reading: chapter 3 of [24], including exercises

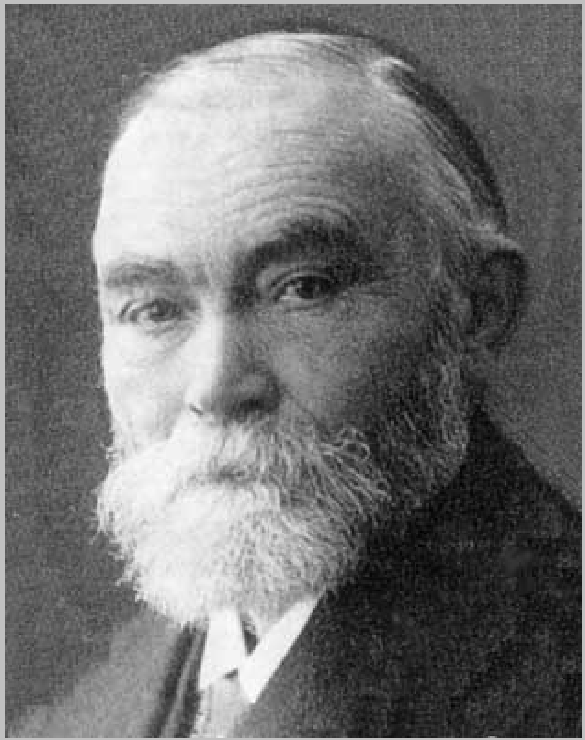
Reading Material

See online materials –

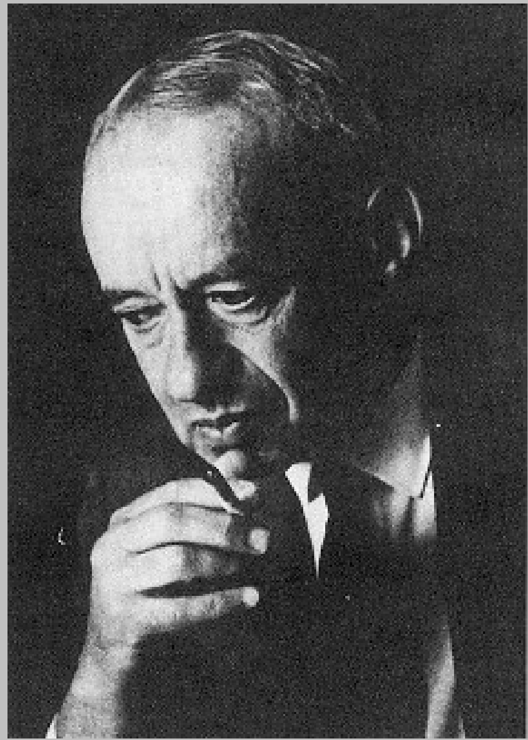
Classes 1-2: chapters 1-3 from Winter (2016)

Classes 3-5: selection of review articles

Logicians on meaning



Gottlob Frege (1848-1925)



Alfred Tarski (1902-1983)

Frege: Meanings are *composed to each other*.

Tarski: Meanings can be described as objects in a *mathematical world*, external to language itself.

Meanwhile in Cognitive Science

"It seems clear, then that undeniable, though only imperfect correspondences hold between formal and semantic features in language."

(*Syntactic Structures*, 1957)

Noam Chomsky (1928)

Towards a Synthesis

Richard Montague (1930-1971)

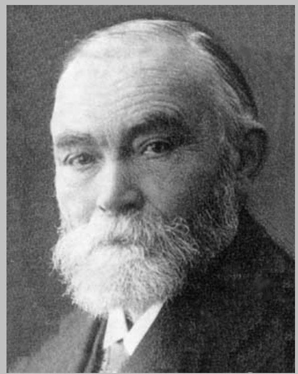
"There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of language within a single natural and mathematically precise theory. On this point I differ from a number of philosophers, but agree, I believe, with Chomsky and his associates."
(*Universal Grammar*, 1970)

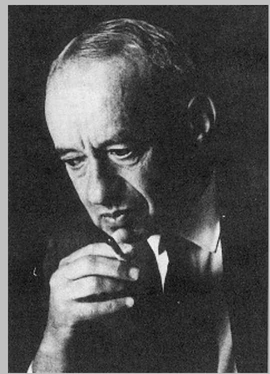
The Key to Montague's Program

Frege's Principle of Compositionality

The meaning of a compound expression is a function of the meanings of its parts, and the ways they combine with each other.

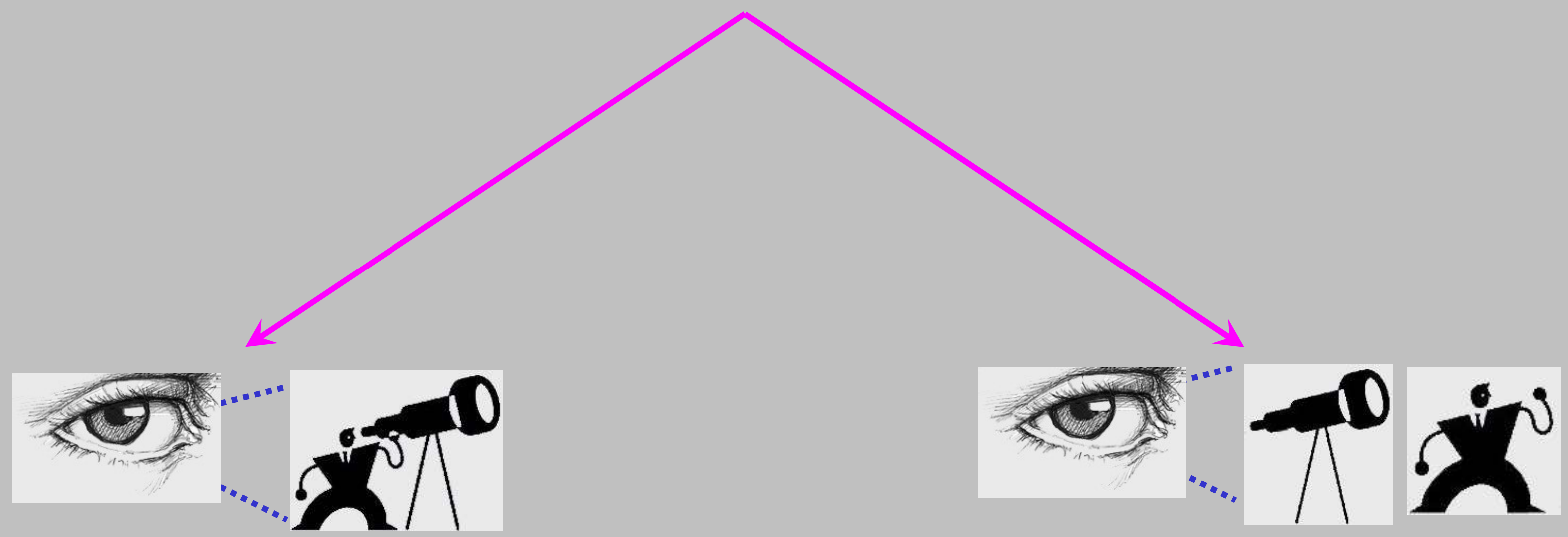
Form ⟷ Compositionality ⟷ **Meaning**

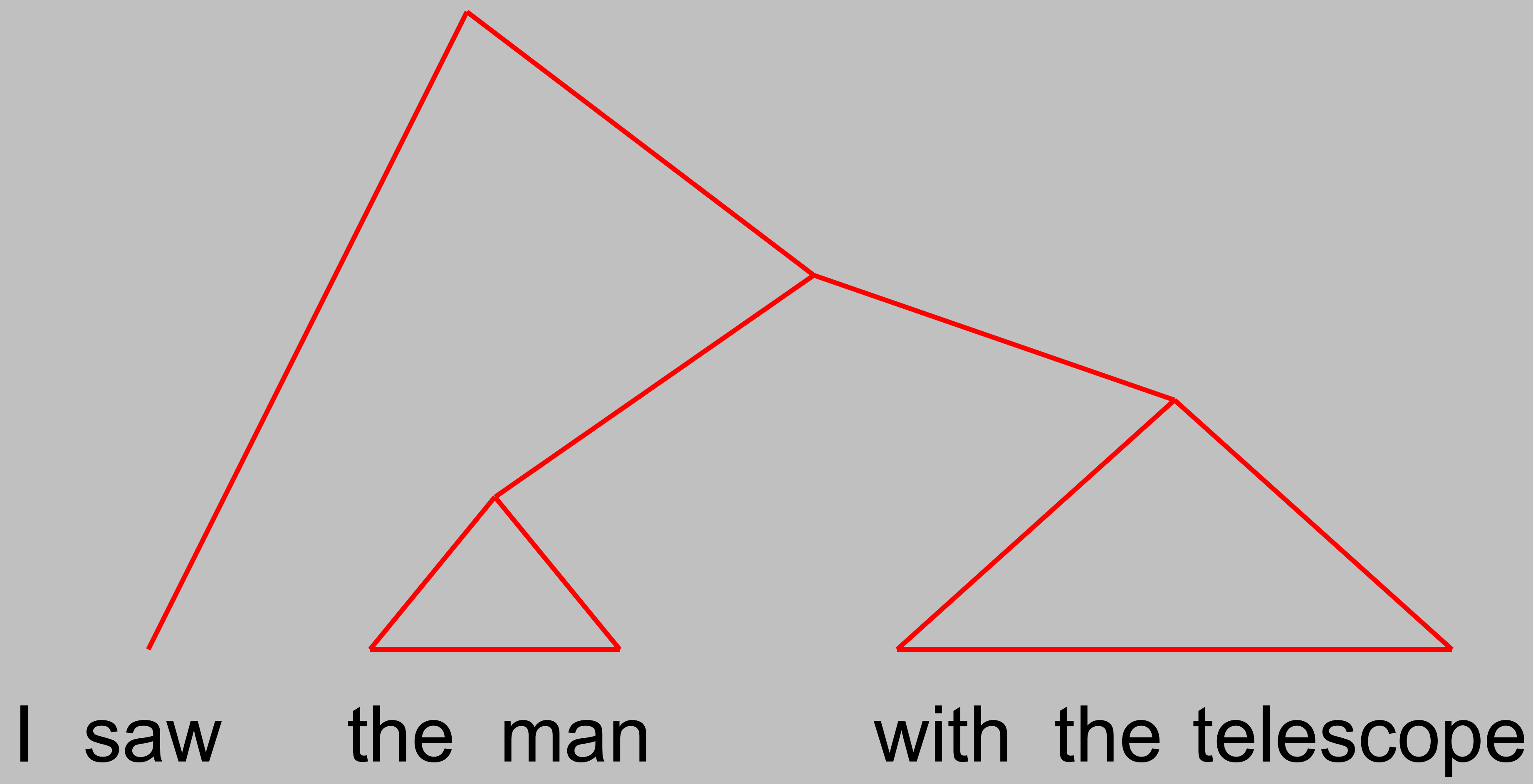


Ambiguous Expressions

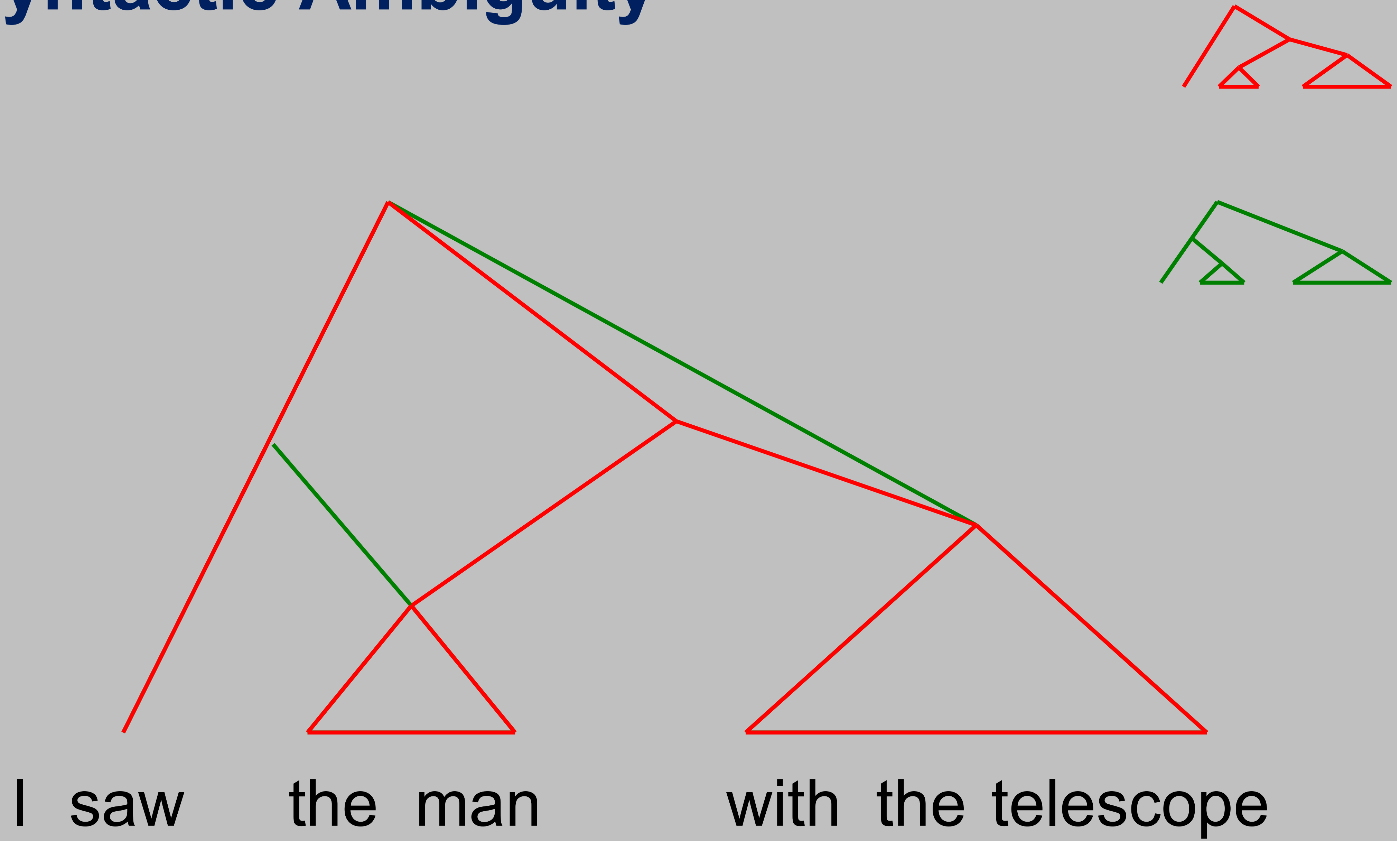
I saw the man with the telescope



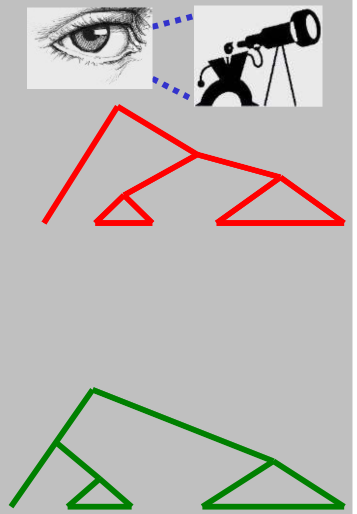
Syntactic Ambiguity

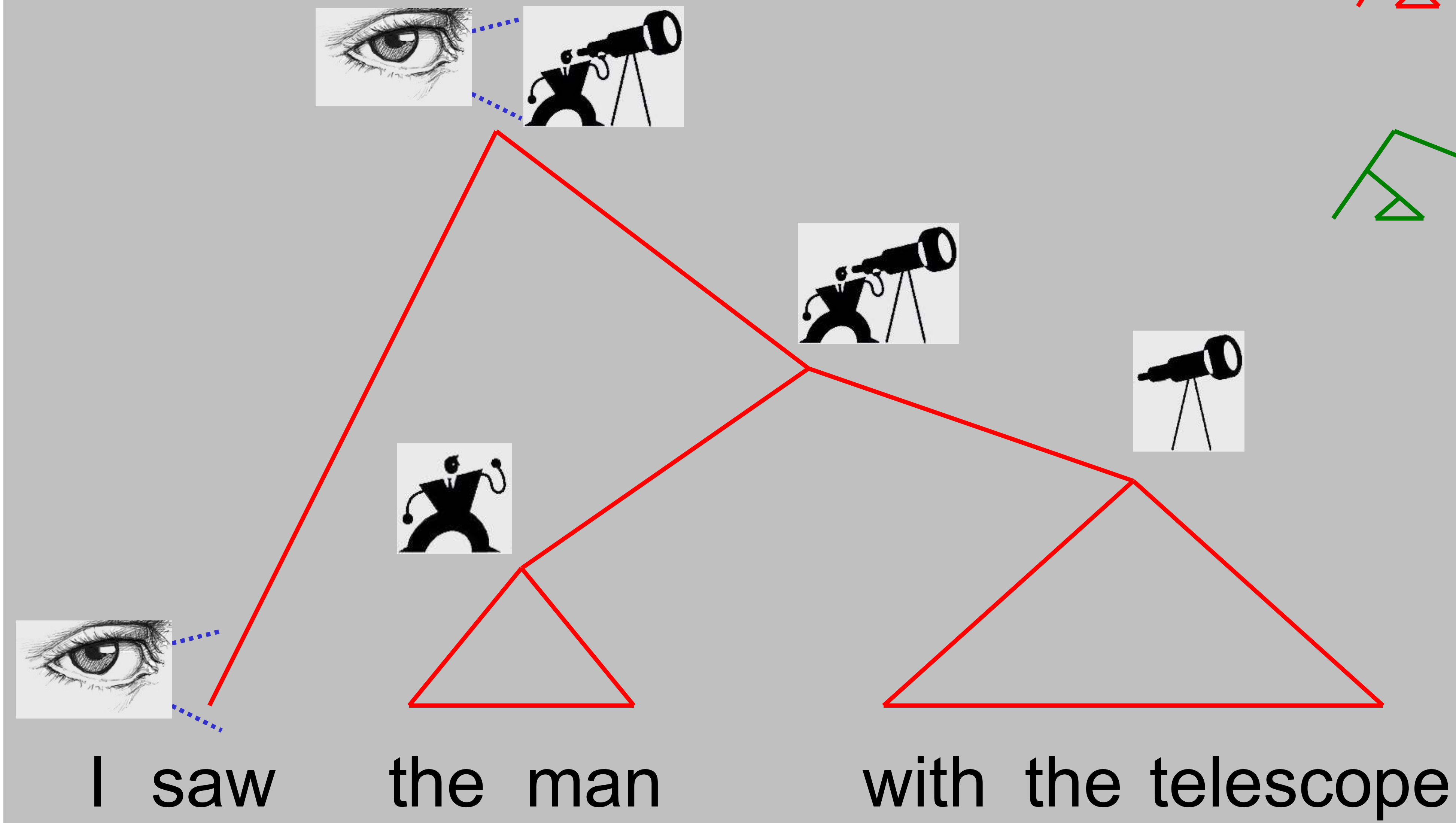


Syntactic Ambiguity



Syntactic-Semantic Ambiguity





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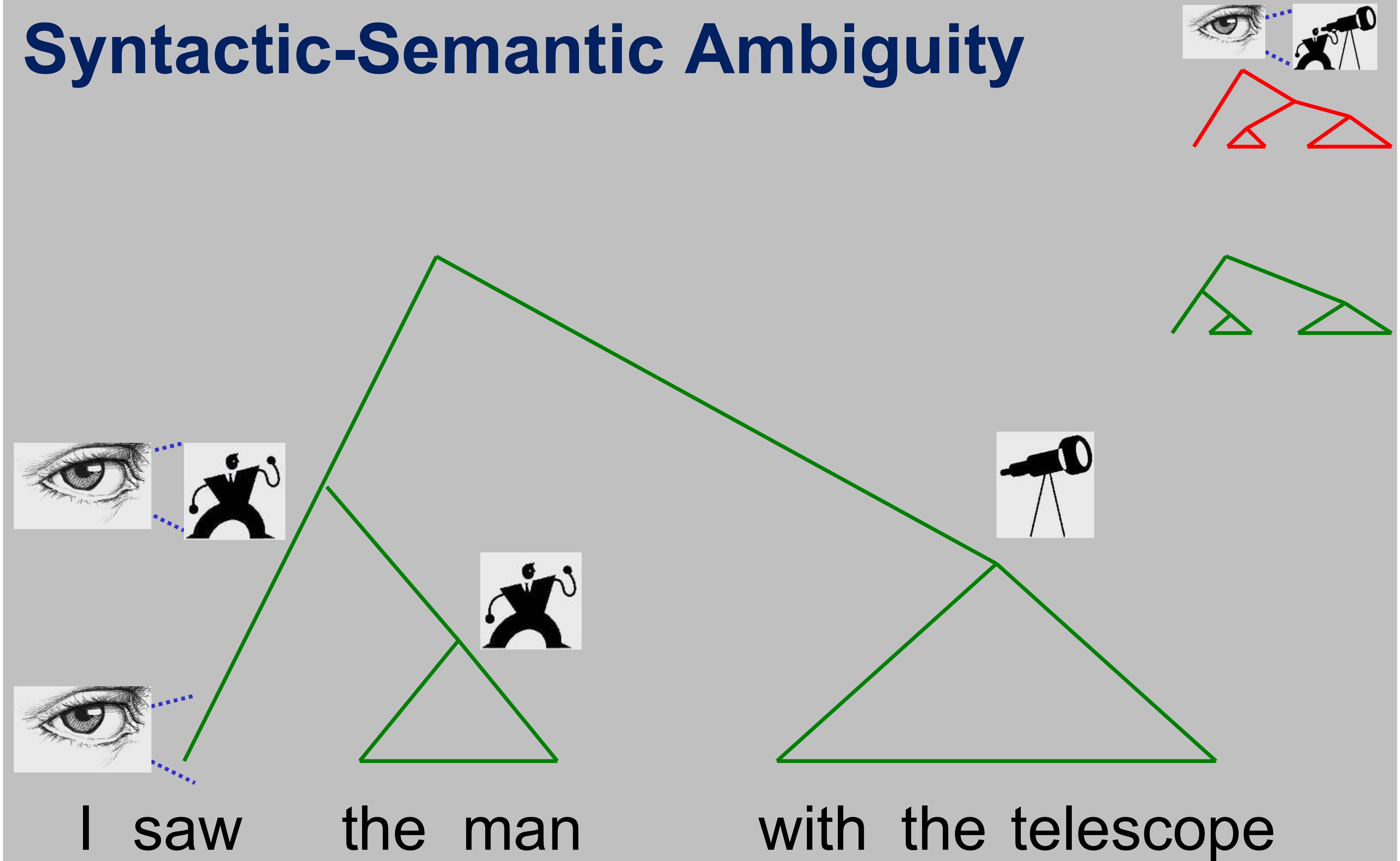


Meaning and Form

Entailment

(6) Tina is tall and thin.

From this premise, any speaker of English is able to draw the conclusion in sentence (7).

(7) Tina is thin.

We say that sentence (6) *entails* (7), and denote it (6)⇒(7). In this entailment, we call sentence (6) the *premise*, or *antecedent*. Sentence (7) is called the *conclusion*, or *consequent*,

(8) a. A dog entered the room ⇒ An animal entered the room
b. John picked a blue card from this pack ⇒ John picked a card from this pack
c. I met my only living grandmother yesterday ⇒ I met my grandmother yesterday

(9) Tina is a bird.

(10) Tina can fly.

(11) Tina is a bird, but she cannot fly, because... (she is too young to fly, a penguin, an ostrich, etc.)

(12) #Tina is tall and thin, but she is not thin, because...

Entailment *is the indefeasible relation, denoted* $S_1 \Rightarrow S_2$*, between a premise* S_1 *and a valid conclusion* S_2 *expressed as natural language sentences.*

Mentalist vs. Linguistic Meaning Relations

(1) a. What is common to the objects that people categorize as being *red*?

b. How do people react when they are addressed with the request *please pick a blue card from this pack*?

c. What emotions are invoked by expressions like *my sweetheart*, *my grandmother* or *my boss*?

(2) a. How do speakers identify relations between pairs of words like *red-color*, *dog-animal* and *chair-furniture*?

b. What are the relations between the use of the imperative sentence *please pick a <u>blue</u> card from this pack* and the use of the similar sentence *please pick a card from this pack*?

c. How are the descriptions *my grandmother* and *my only living grandmother* related to each other in language use?

(3) Red is a color / ?Red is an animal

(4) The color red annoys me / ?The animal red annoys me

(5) Every red thing has a color / ?Every red thing has an animal

Models and Entailments

Let exp *be a language expression, and let* M *be a model. We write* $[[\mathrm{exp}]]^M$ *when referring to the denotation of* exp *in the model* M.

A semantic theory T *is said to satisfy the* **truth-conditionality criterion** (TCC) *if for all sentences* S_1 *and* S_2*, the following two conditions are equivalent:*

I. *Sentence* S_1 *intuitively entails sentence* S_2.

II. *For all models* M *in* T: $[[S_1]]^M \leq [[S_2]]^M$.

	$y = 0$	$y = 1$
$x = 0$	**yes**	**yes**
$x = 1$	**no**	**yes**

ENTAILMENT

$$S_1 \Rightarrow S_2$$

TCC $\longleftrightarrow$

MODELS

$$[[S_1]]^{M_1} \leq [[S_2]]^{M_1}$$
$$[[S_1]]^{M_2} \leq [[S_2]]^{M_2}$$
$$[[S_1]]^{M_3} \leq [[S_2]]^{M_3}$$
$$\dots$$

Assumptions about our models

1. In every model M, in addition to the two truth-values, we also have an arbitrary non-empty set E_M of *entities in* M, which contains the simplest objects in this model.
2. In any model M, the proper name *Tina* denotes an arbitrary entity in E_M.
3. In any model M, the adjectives *tall* and *thin* denote arbitrary <u>sets</u> of entities in E_M.

$$\textsc{is}(x, A) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

$\textsc{and}(A, B) = A \cap B$ = the set of all members of E that are both in A and in B

Thus:

$$[[\textit{Tina is thin}]]^M = \textsc{is}(\mathbf{tina}, \mathbf{thin})$$

$$[[\textit{Tina is tall and thin}]]^M = \textsc{is}(\mathbf{tina}, \textsc{and}(\mathbf{tall}, \mathbf{thin}))$$

Convention:

Let blik *be a word in a language. When the denotation* $[[\text{blik}]]^M$ *of* blik *is arbitrary, we mark it* **blik**, *and when it is constant across models we mark it* BLIK. *In both notations the model* M *is implicit.*

TCC - example

Expression	**Cat.**	**Type**	**Abstract denotation**	**Denotations in example models with $E = \{a, b, c, d\}$**		
				M_1	M_2	M_3
Tina	PN	entity	**tina**	a	b	b
tall	A	set of entities	**tall**	$\{b, c\}$	$\{b, d\}$	$\{a, b, d\}$
thin	A	set of entities	**thin**	$\{a, b, c\}$	$\{b, c\}$	$\{a, c, d\}$
tall and thin	AP	set of entities	AND(**tall**, **thin**)	$\{b, c\}$	$\{b\}$	$\{a, d\}$
Tina is thin	S	truth-value	IS(**tina**, **thin**)	1	1	0
Tina is tall and thin	S	truth-value	IS(**tina**, AND(**tall**, **thin**))	0	1	0

Categories: PN = proper name; A = adjective; AP = adjective phrase; S = sentence

Table 2.2: Denotations for expressions in the entailment (6)⇒(7)

Compositionality

(15) a. All pianists are composers, and Tina is a pianist.

b. All composers are pianists, and Tina is a pianist.

Compositionality: *The denotation of a complex expression is determined by the denotations of its immediate parts and the ways they combine with each other.*

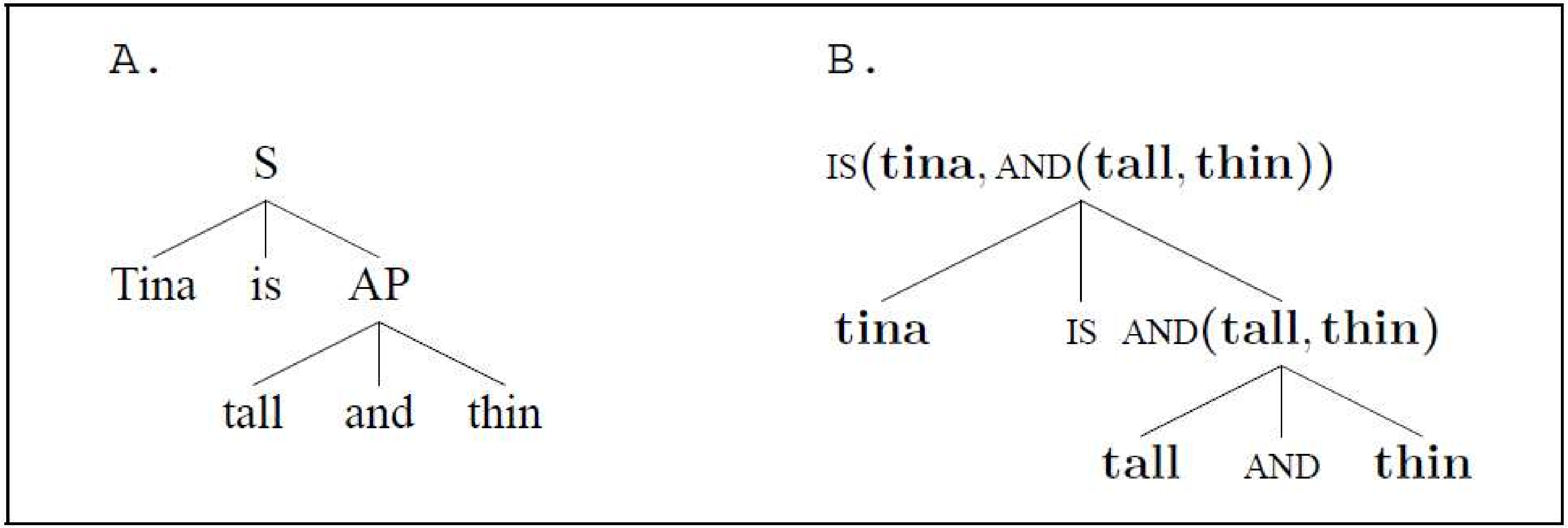


Figure 2.2: compositional derivation of denotations

Structural ambiguity (1)

(16) Tina is not tall and thin.

(18) AP ⟶ *tall*, *thin*, ...

AP ⟶ AP *and* AP

AP ⟶ *not* AP

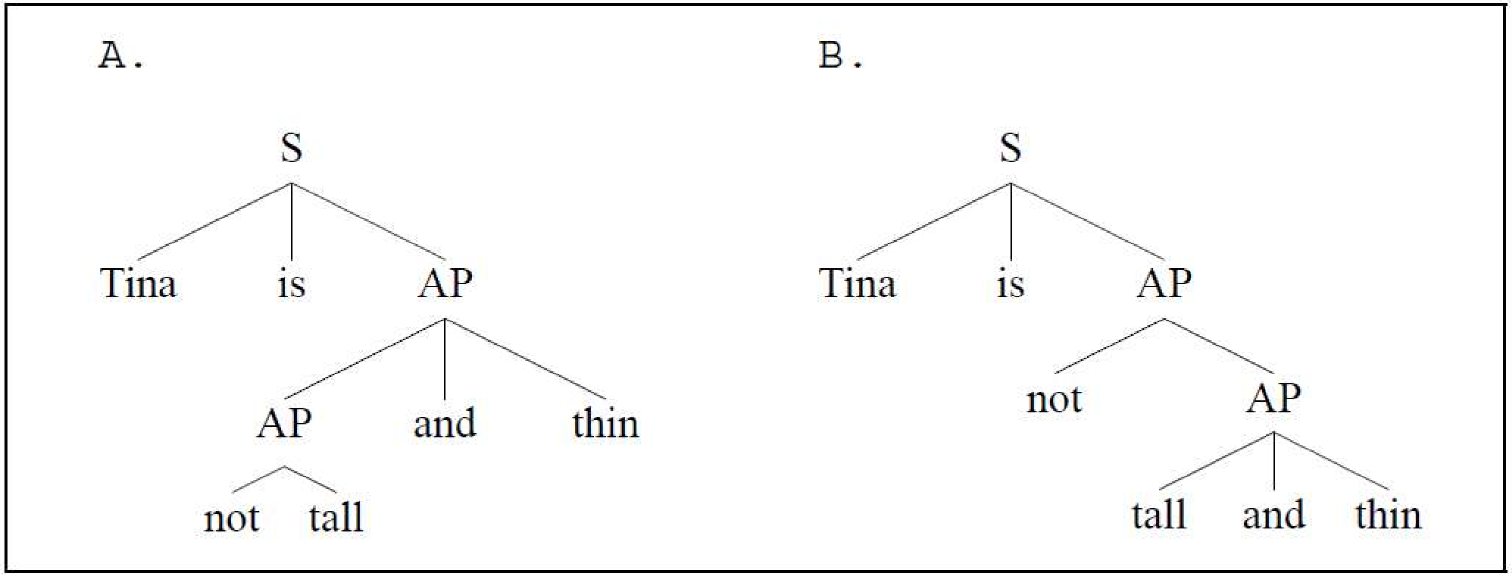


Figure 2.3: structural ambiguity

Structural ambiguity (2)

$\textsc{not}(A) = \overline{A} = E \setminus A$ = the set of all the members of E that are not in A

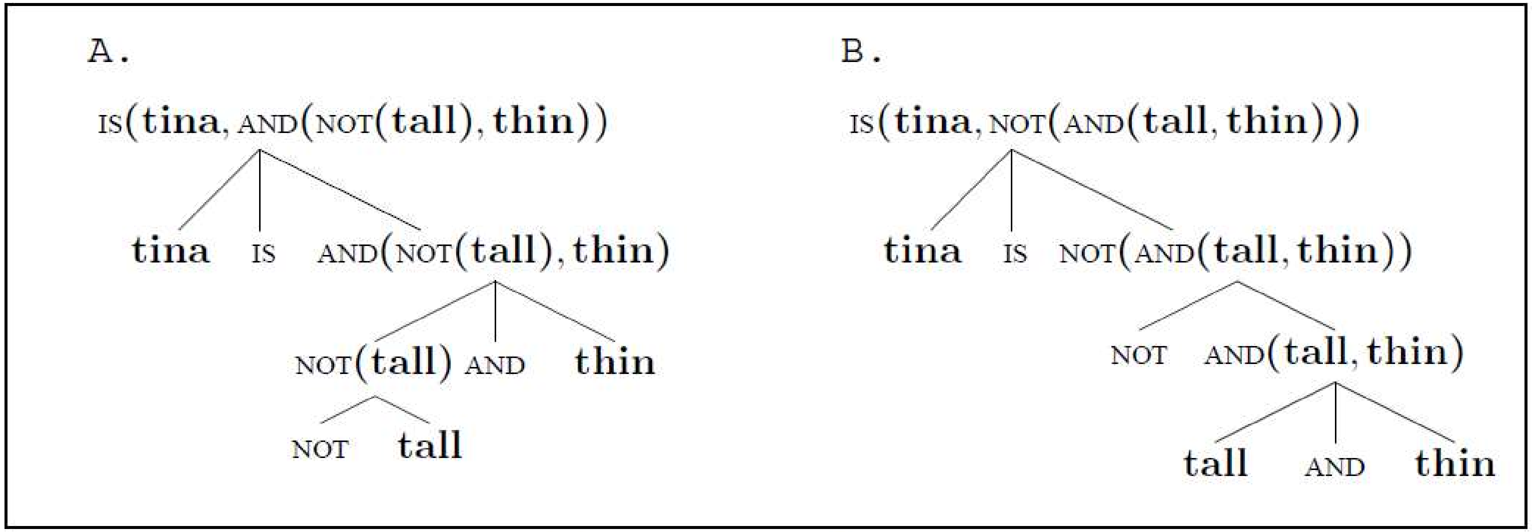


Figure 2.4: compositionality and ambiguity

(19) a. $\textsc{is}(\mathbf{tina}, \textsc{and}(\textsc{not}(\mathbf{tall}), \mathbf{thin})) = 1$ iff $\mathbf{tina} \in \overline{\mathbf{tall}} \cap \mathbf{thin}$

b. $\textsc{is}(\mathbf{tina}, \textsc{not}(\textsc{and}(\mathbf{tall}, \mathbf{thin}))) = 1$ iff $\mathbf{tina} \in \overline{\mathbf{tall} \cap \mathbf{thin}}$

Note: Ambiguity vs. vagueness

Class 1 (cont.)

Basic notions and tools

- NOTIONS
- TOOLS

Functions

Given two sets A and B, a function from A to B is a rule or procedure that "inputs" elements of A and "outputs" elements of B.

Notation:

$$f : A \to B$$

Every element $a \in A$ gets "sent to" some element of B which is called $f(a)$.

Functions are also called mappings.

A is called the domain of f, and B the codomain or range.

Sets of functions

For any two sets A and B, the set of all functions from A to B is denoted

$$B^A \qquad \text{alternatively, } A \to B$$

Example:

$\{a, b, c\}^{\{1,2\}}$ = the following functions:

$1 \mapsto a$	$2 \mapsto a$
$1 \mapsto a$	$2 \mapsto b$
$1 \mapsto a$	$2 \mapsto c$
$1 \mapsto b$	$2 \mapsto a$
$1 \mapsto b$	$2 \mapsto b$
$1 \mapsto b$	$2 \mapsto c$
$1 \mapsto c$	$2 \mapsto a$
$1 \mapsto c$	$2 \mapsto b$
$1 \mapsto c$	$2 \mapsto c$

Basic/Complex Types and Domains

A **type** is a label for part of a model that is called a **domain**.

Basic types and domains:

e	:	D_e	- *arbitrary*	- *of entities*
t	:	D_t	= {0,1}	- *of truth-values*

Complex types and domains: defined inductively from basic types and domains.

Example

$E = D_e$ = the set of entities {t,j,m}

[[*thin*]] = T = {t,j}

We can also define T as a *function* from D_e to D_t:

t→1

j→1

m→0

This function **characterizes** T in $E = D_e$.

D_{et} of the complex type ***et*** is the domain of such functions.

Characteristic functions over {t,j,m}

Subset of D_e	Function in D_{et}			
$\varnothing$	f_1 :	t $\mapsto$ 0	j $\mapsto$ 0	m $\mapsto$ 0
{m}	f_2 :	t $\mapsto$ 0	j $\mapsto$ 0	m $\mapsto$ 1
{j}	f_3 :	t $\mapsto$ 0	j $\mapsto$ 1	m $\mapsto$ 0
{j, m}	f_4 :	t $\mapsto$ 0	j $\mapsto$ 1	m $\mapsto$ 1
{t}	f_5 :	t $\mapsto$ 1	j $\mapsto$ 0	m $\mapsto$ 0
{t, m}	f_6 :	t $\mapsto$ 1	j $\mapsto$ 0	m $\mapsto$ 1
{t, j}	f_7 :	t $\mapsto$ 1	j $\mapsto$ 1	m $\mapsto$ 0
{t, j, m}	f_8 :	t $\mapsto$ 1	j $\mapsto$ 1	m $\mapsto$ 1

Table 2.1: Subsets of D_e and their characteristic functions in D_{et}

Characteristic Functions

Let X be any set.

Every subset $A \subseteq X$ gives us a function $f_A : X \to \{0, 1\}$ called the characteristic function of A.

It is defined as follows:

$$f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

Example: $X = \{a, b, c, d\}$, $A = \{a, c\}$

Here are some examples of how this function f_A works:

- $f_A(a) = 1$.
- $f_A(b) = 0$.
- $f_A(c) = 1$.
- $f_A(d) = 0$.

Definition: Types and domains

Definition 1. The set of **types** over the basic types e and t is the smallest set $\mathcal{T}$ that satisfies:

(i) $\{e, t\} \subseteq \mathcal{T}$

(ii) If τ and σ are types in $\mathcal{T}$ then $(\tau\sigma)$ is also a type in $\mathcal{T}$.

$$e,\ t,$$

$$ee,\ tt,\ et,\ te,$$

$$e(ee),\ e(tt),\ e(et),\ e(te),\ t(ee),\ t(tt),\ t(et),\ t(te),$$

$$(ee)e,\ (tt)e,\ (et)e,\ (te)e,\ (ee)t,\ (tt)t,\ (et)t,\ (te)t,$$

$$(ee)(ee),\ (ee)(tt),\ (ee)(et),\ (ee)(te),\ (tt)(ee),\ (tt)(tt),\ (tt)(et),\ (tt)(te)$$

Definition 2. For all types τ and σ in $\mathcal{T}$, the **domain** $D_{\tau\sigma}$ of the type $(\tau\sigma)$ is the set $D_\sigma^{D_\tau}$ – the functions from D_τ to D_σ.

Intransitive verbs

Tina smiled.

$$\mathbf{smile}_{et}(\mathbf{tina}_e)$$

Function Application

From types e and et, FA gives t (as we have seen above).

From types $(e(et))(et)$ and $e(et)$, FA gives et.

Types $(e(et))(et)$ and et cannot combine using FA: neither of these types is a prefix of the other.

Function Application (FA):

(ab) + a = b

f + x = f(x)

Intransitive and Transitive verbs

Tina smiled.

Tina [praised Mary].

$$\text{smile}_{et}(\text{tina}_e)$$

$$(\text{praise}_{e(et)}(\text{mary}_e))(\text{tina}_e)$$

or

$$\text{praise}(\text{mary})(\text{tina})$$

"Curried" Relations

$$U = \{\langle t, m\rangle, \langle m, t\rangle, \langle m, j\rangle, \langle m, m\rangle\}$$

$$\begin{array}{llll} f_U : & t & \mapsto & [t \mapsto 0 \quad j \mapsto 0 \quad m \mapsto 1] \\ & j & \mapsto & [t \mapsto 0 \quad j \mapsto 0 \quad m \mapsto 1] \\ & m & \mapsto & [t \mapsto 1 \quad j \mapsto 0 \quad m \mapsto 1] \end{array}$$

– f_U maps the entity t to the function characterizing the set $\{m\}$.

– f_U maps the entity j to the function characterizing the same set, $\{m\}$.

– f_U maps the entity m to the function characterizing the set $\{t, m\}$.

When the function f_U is the denotation of the verb *praise*, and the entities t, j and m are the denotations of the respective names, this is the situation where:

– Mary is the only one who praised Tina.

– Mary is the only one who praised John.

– Tina and Mary, but not John, praised Mary.

Currying

F: $(M \times W) \rightarrow [0,1]$

F gives any pair of man and woman (m,w) a score $F(m,w)$ indicating matching

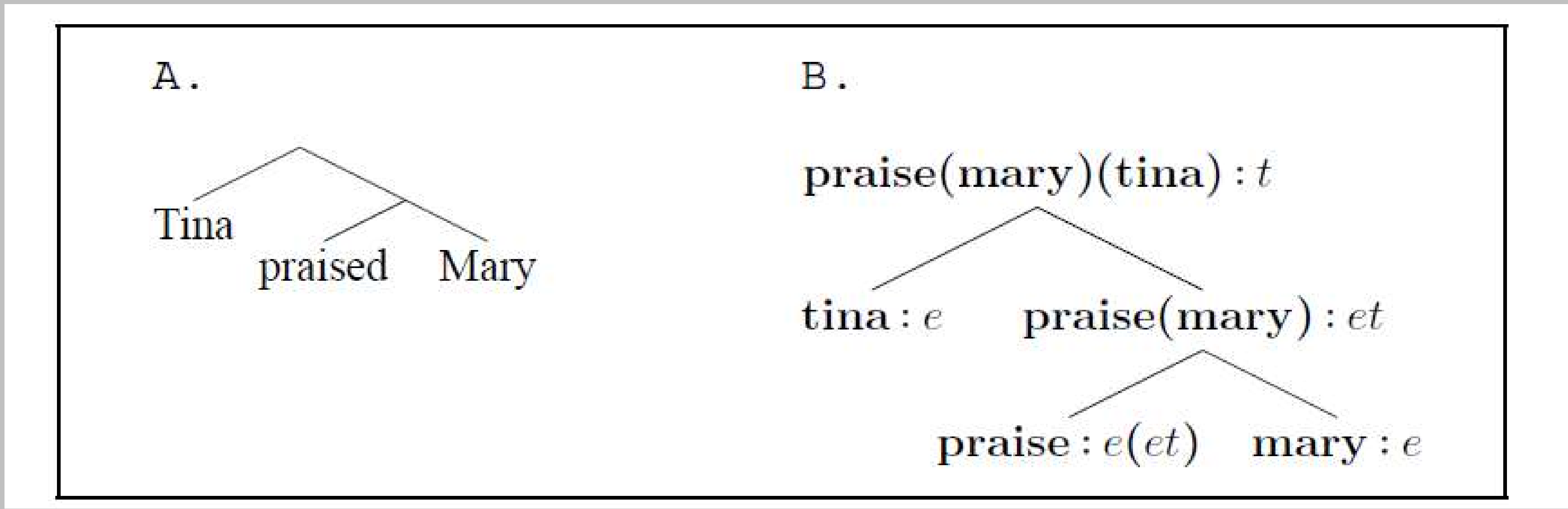
G: $M \rightarrow (W \rightarrow [0,1])$

G gives any man m a function $G(m)$ mapping any woman w to a score $(G(m))(w)$.

Thus, we can define: $(G(m))(w) = F(m,w)$

We say that G is the Curried version of F, and that F is the deCurried version of G.

Use of Currying for compositional interpretation of binary structures



Modifiers

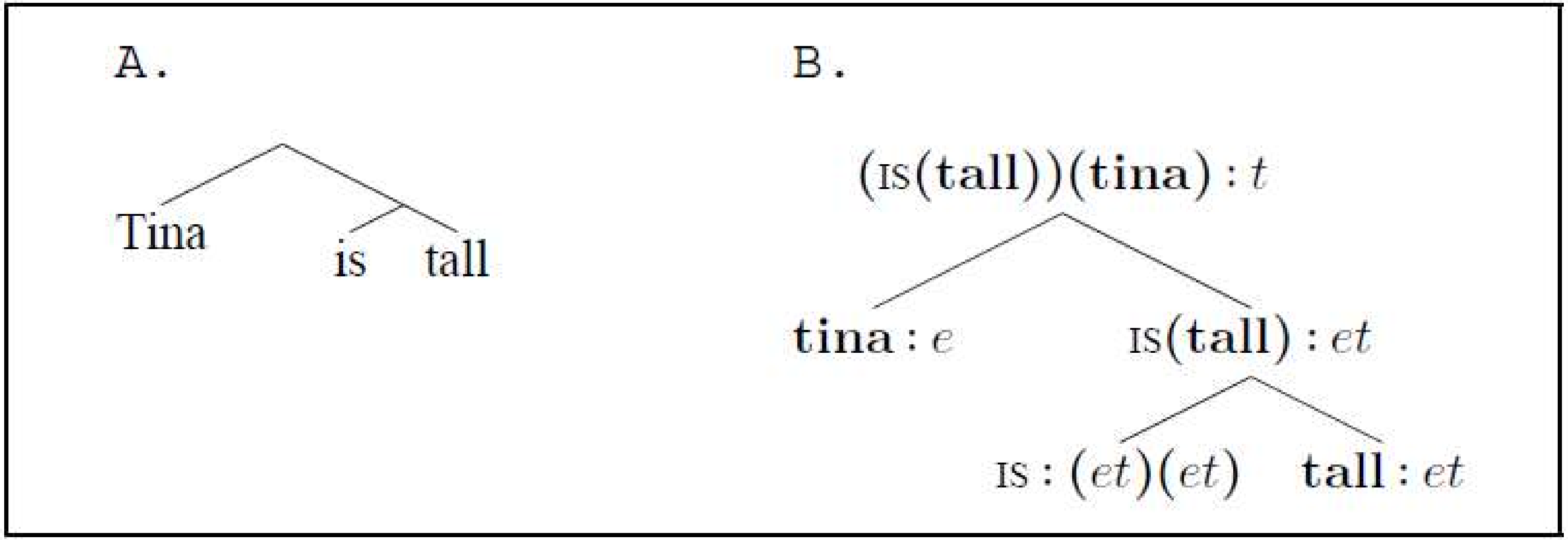
Mary [walked quickly]

➔

Mary walked

Non-arbitrary Denotations: IS

For every function f in D_{et}: $\text{IS}(f) = f$.

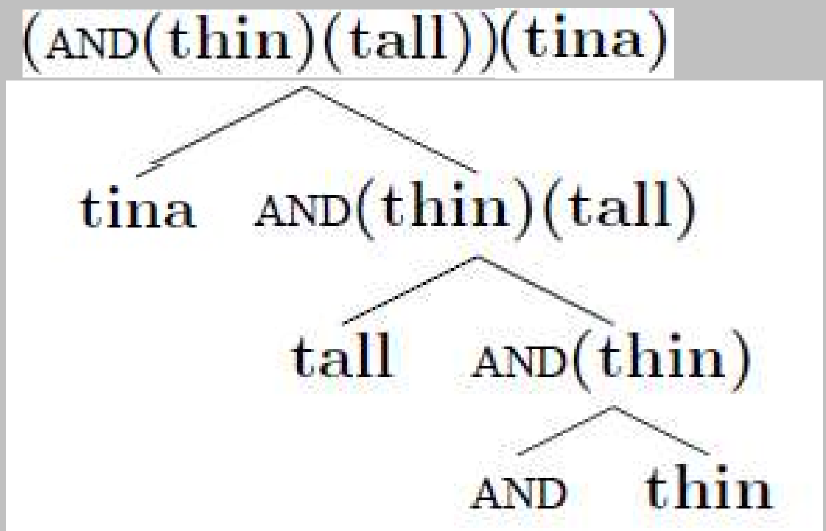


Alternative structures – alternative types?

Non-arbitrary Denotations: AND

For every two functions f_A and f_B in D_{et}, characterizing the subsets A and B of D_e: $(\text{AND}(f_A))(f_B)$ is defined as the function $f_{A \cap B}$, characterizing the intersection of A and B.

Explain:
Tina is tall and thin ==> Tina is thin

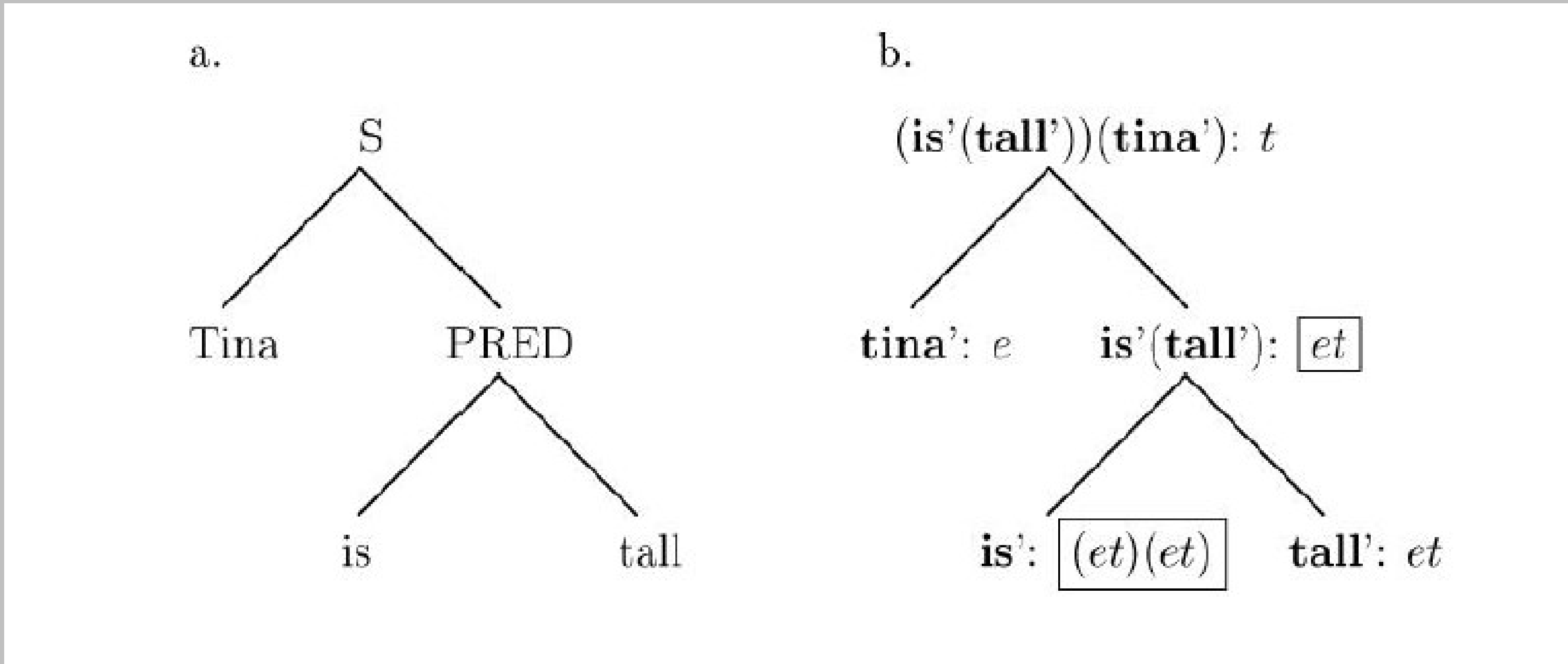


Types?

Summary of useful types

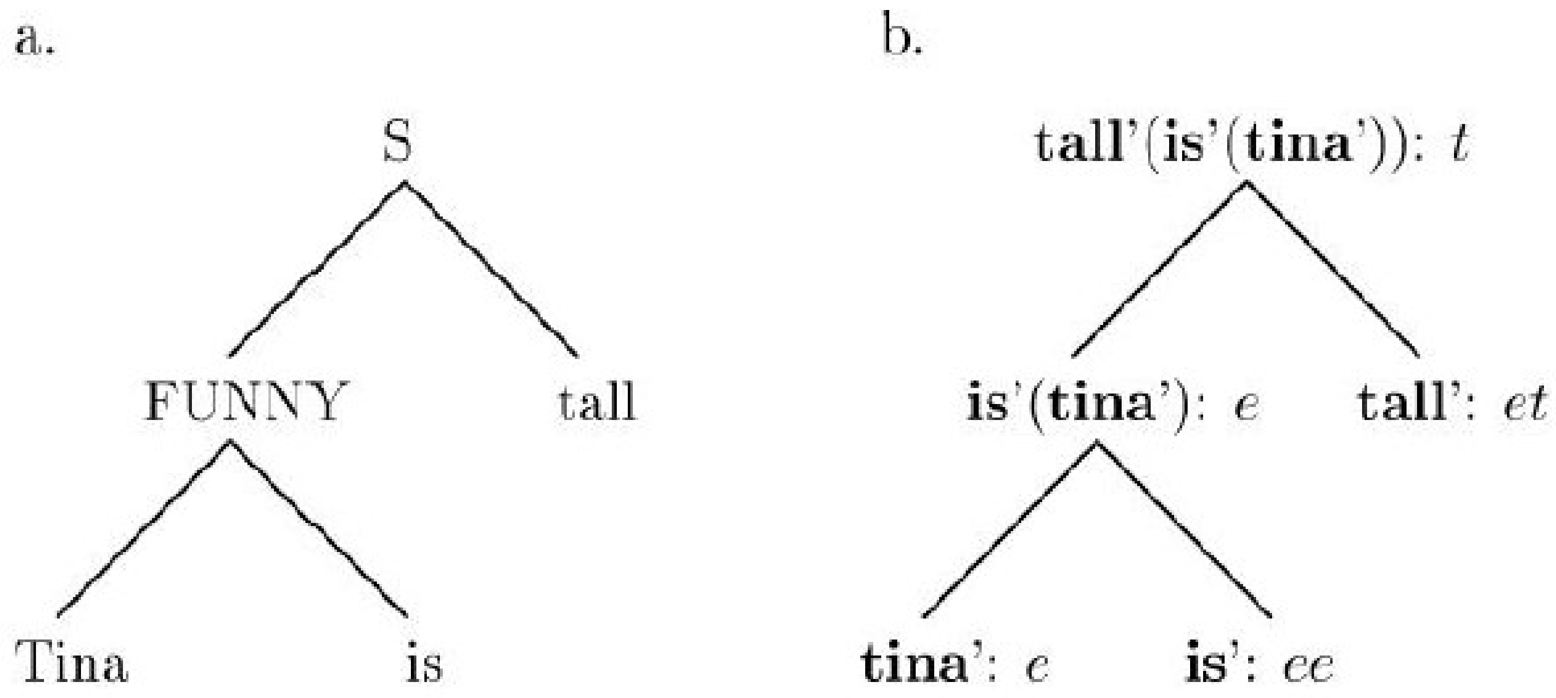
t	sentences
e	proper names, referential noun phrases
et	intransitive verbs and common nouns
$e(et)$	transitive verbs
$(et)t$	quantificational noun phrases
$(et)e$	articles (*the*, *a*)
$(et)((et)t)$	determiners (*some*, *every*)
$\tau\tau$	modifiers (adjectives, adverbs, prepositional phrases, negation, relative clauses)
$\tau(\tau\tau)$	coordinators (conjunction, disjunction, restrictive relative pronouns)

Function Application and constituency



What would be the type of IS with the follow (infelicitous) structure?

[Tina is] tall


a.
S
FUNNY
tall
Tina
is
b.
tall'(**is**'(**tina**')): *t*
is'(**tina**'): *e*
tall': *et*
tina': *e*
is': *ee*

Exercise

give types to words the following sentences

Mary [walked [in Utrecht]]

[Walk -ing] [is fun]

[[Walk -ing] [in Utrecht]] [is fun]

[The man] smiled

[The [tall man]] smiled

[If [you smile]] [you win]

There [is [trouble [in Paradise]]]

I [[love it] [when [you smile]]]