Elements of Formal Semantics

An Introduction to the Mathematical Theory of Meaning in Natural Language

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Open Access Materials: Chapter 2

Elements of Formal Semantics introduces some of the foundational concepts, principles and techniques in formal semantics of natural language. It is intended for mathematically-inclined readers who have some elementary background in set theory and linguistics. However, no expertise in logic, math, or theoretical linguistics is presupposed. By way of analyzing concrete English examples, the book brings central concepts and tools to the forefront, drawing attention to the beauty and value of the mathematical principles underlying linguistic meaning.

See webpage below for further materials and information:

http://www.phil.uu.nl/~yoad/efs/main.html

CHAPTER 2

MEANING AND FORM

This chapter introduces some of the key notions about the analysis of meaning in formal semantics. We focus on entailments*: relations between premises and valid conclusions expressed as natural language sentences. Robust intuitions about entailment are distinguished from weaker types of reasoning with language. Speaker judgments on entailments are described using* models*: abstract mathematical structures, which emanate from semantic analyses of artificial logical languages. Model-theoretical objects are directly connected to syntactic structures by applying a general principle of* compositionality. *We see how this principle helps to analyze cases of structural ambiguity and to distinguish them from other cases of under-specification.*

What do dictionaries mean when they tell us that semantics is "the study of meaning"? The concept that people intuitively refer to as "meaning" is an abstraction inspired by observing how we use language in everyday situations. However, we use language for many different purposes, and those various usages may inspire conceptions of meaning that are radically different from one another. We cannot reasonably expect a theory of meaning to cover everything that people do with their languages. A more tractable way of studying meaning is by discerning specific properties of language use that are amenable to scientific investigation. These aspects of language use, if stable across speakers and situations, will ultimately guide us toward a theory of language "meaning".

ENTAILMENT

One of the most important usages of natural language is for everyday reasoning. For example, let us consider sentence (2.1):

(2.1) Tina is tall and thin.

From this sentence, any English speaker is able to draw the conclusion in (2.2) below:

(2.2) Tina is thin.

Thus, any speaker who considers sentence (2.1) to be true, will consider sentence (2.2) to be true as well. We say that sentence (2.1) *entails* (2.2), and denote it (2.1)$\Rightarrow$(2.2). Sentence (2.1) is called the *premise*, or *antecedent*, of the entailment. Sentence (2.2) is called the *conclusion*, or *consequent*.

The entailment from (2.1) to (2.2) exemplifies a relation that all English speakers will agree on. This consistency is remarkable, and all the more so since words like *Tina*, *tall* and *thin* are notoriously flexible in the way that they are used. For instance, you and I may have different criteria for characterizing people as being *thin*, and therefore disagree on whether Tina is thin or not. We may also disagree on the identity of Tina. You may think that Tina in sentences (2.1) and (2.2) is Tina Turner, while I may think that these sentences describe Tina Charles. However, we are unlikely to disagree on whether sentence (2.2) is a sound conclusion from (2.1).

We noted that when sentence (2.1) is judged to be true, so is sentence (2.2). However, the converse does not hold: (2.2) may be true while (2.1) is not – this is the case if Tina happens to be thin but not tall. Because of such situations, we conclude that sentence (2.2) does *not* entail (2.1). This is denoted (2.2)$\not\Rightarrow$(2.1). Just as with positive judgments on entailment, rejections of entailment are also often uniform across speakers and circumstances of use. Therefore, we consider both positive and negative judgments on entailment as important empirical evidence for semantic theory.

When studying simple entailments, we often pretend that our language vocabulary is very small. Still, as soon as our vocabulary has some simple adjectives and proper names, we can easily find entailments and non-entailments by looking at their different combinations with words like *and*, *or*, *is* and *not*. For instance:

(2.3) a. Tina is tall, and Ms. Turner is not tall $\Rightarrow$ Tina is not Ms. Turner.

b. Tina is tall, and Tina is not Ms. Turner $\not\Rightarrow$ Ms. Turner is not tall.

(2.4) a. Ms. Turner is tall, and Tina is Ms. Turner or Ms. Charles $\Rightarrow$ Tina is tall or Tina is Ms. Charles.

b. Ms. Turner is tall, and Tina is Ms. Turner or Ms. Charles $\not\Rightarrow$ Tina is tall.

The examples above may look unsurprising for anyone who is familiar with philosophical or mathematical logic. Indeed, similar entailments in natural language have inspired well-known logical formalisms like Propositional Logic and Predicate Logic. Readers may therefore wonder: don't the entailments above demonstrate puzzles that were solved long ago by logicians? The answer is "yes and no". Sure enough, these entailments can be translated to well-understood logical questions. However, logic does not traditionally focus on the details of the translation procedure from ordinary language to logical languages. This translation step is not "pure logic": it also involves intricate questions about the sounds and the forms of human languages, and about the nature of semantic concepts in the human mind. Consequently, in modern cognitive science, the study of entailment in natural language is not the sanctuary of professional logicians. Entailment judgments bring to the forefront a variety of questions about language that are also of primary concern for linguists, computer scientists, psychologists and philosophers. For instance, let us consider the following entailments:

(2.5) a. Sue only drank half a glass of wine $\Rightarrow$ Sue drank less than one glass of wine.

b. A dog entered the room $\Rightarrow$ An animal entered the room.

c. John picked a blue card from the pack $\Rightarrow$ John picked a card from the pack.

The entailments in (2.5) illustrate different aspects of language: measures and quantity in (2.5a); word meaning relations in (2.5b); adjective modification in (2.5c). These kinds of entailment are very common in natural language, but they were not systematically treated in classical logic. By studying the whole spectrum of entailments in ordinary language, formal semantics addresses various aspects of linguistic phenomena and their connections with human reasoning. Ideas from logic are borrowed insofar as they are useful for analyzing natural language semantics. More specifically, later in this book we adopt concepts from type theory and higher-order logics that have proved especially well suited for studying entailment in natural language. As we shall see, incorporating these concepts allows formal semantics to develop important connections with theories about sentence structure and word meaning.

Among the phenomena of reasoning in language, entailment is especially central because of its remarkable stability. In other instances of reasoning with natural language, conclusions are not fully stable, since they may rely on implicit assumptions that emanate from context, world knowledge or probabilistic principles. These lead to meaning relations between sentences that are often fuzzier and less regular than entailments. Consider for instance the following two sentences:

(2.6) Tina is a bird.

(2.7) Tina can fly.

Sentence (2.7) is a likely conclusion from (2.6), and most speakers will not hesitate too much before drawing it. However, upon some reflection we can come up with many situations in which sentence (2.6) truthfully holds without supporting the conclusion in (2.7). Think of young birds, penguins, ostriches, or birds whose wings are broken. Thus, in many natural discourses speakers may accept (2.6) while explicitly denying (2.7):

(2.8) Tina is a bird, but she cannot fly, because ... (she is too young to fly, a penguin, an ostrich, etc.).

We classify the inferential relation between sentences like (2.6) and (2.7) as *defeasible*, or cancelable, reasoning. By contrast, entailments are classified as *indefeasible reasoning*: all of the assumptions that are needed in order to reach the conclusion of an entailment are explicitly stated in the premise. For instance, the entailment (2.1)⇒(2.2) cannot be easily canceled by adding further information to the premise.

A discourse like (2.9) below that tries to contradict sentence (2.2) after asserting (2.1) will normally be rejected as *incoherent*.

(2.9) #Tina is tall and thin, but she is not thin.

A speaker who wishes to support this incoherent line of reasoning would need to resort to self-contradictory or otherwise counter-communicative arguments like "because I am lying when saying that Tina is tall and thin", or "because I am not using English in the ordinary way". The incoherence of (2.9) is marked by '#'. Sentence (2.9) is intelligible but nonsensical: its communicative value is dubious. This sort of incoherent, contradictory sentence should be distinguished from *ungrammaticality*. The latter notion is reserved for strings of words that clearly do not belong to natural language, e.g. *is and tall Tina thin*. Such ungrammatical strings are standardly marked by '*'.

Taking stock, we adopt the following notion of entailment:

> *Given an indefeasible relation between two natural language sentences S_1 and S_2, where speakers intuitively judge S_2 to be true whenever S_1 is true, we say that S_1* **entails** *S_2, and denote it $S_1 \Rightarrow S_2$.*

Just as intuitive judgments about sentence grammaticality have become a cornerstone in syntactic theory, intuitions about entailments between sentences are central for natural language semantics. As in other linguistic domains, we aim to build our semantic theory on judgments that do not rely on training in linguistics, logic or other scholarly disciplines. Entailments that robustly appear in ordinary reasoning give us a handle on common semantic judgments about language.

Entailments between sentences allow us to define the related notion of *equivalence*. For instance, the sentence (2.1)=*Tina is tall and thin* and the sentence S=*Tina is tall and Tina is thin* are classified as equivalent, because they entail each other. We denote this equivalence (2.1)$\Leftrightarrow S$. For more examples of equivalent sentences see Exercise 4. Another classical semantic notion is *contradiction*, which was lightly touched upon in our discussion of sentence (2.9) above. See Exercise 7 for some elaboration on contradictions and their relations to entailment.

MODELS AND THE TRUTH-CONDITIONALITY CRITERION

With this background on entailments in natural language, let us now see how formal semantics accounts for them. As mentioned above, formal semantics relies on some central principles from traditional philosophical and mathematical logic. Most versions of formal semantics account for entailments using theoretical structures that are called *models*. Models are mathematical abstractions that we construct and use as descriptions of hypothetical situations. We call these situations "hypothetical" because they do not necessarily correspond to actual situations in the world. Some of our models may agree with how we look at the world, but some of them will also describe situations that are purely imaginary. For instance, the models that we use for analyzing the sentence *Tina is thin* will describe situations in which Tina is thin, as well as situations where she is not thin. If you know a woman called Tina and you think that she is thin, you will consider the first models as closer to reality than the others. However, this is irrelevant for our purposes. For the sake of our semantic analysis we consider all the models that we construct as hypothetical. As such, they are all equal.

In order to encode hypothetical situations in models, we let models link words to abstract mathematical objects. For instance, since we want our models to describe situations in relation to the word *Tina*, we let each model contain some or other abstract entity that is associated with this word. Similarly, when we analyze the words *tall* and *thin*, we also let our models associate these adjectives with abstract objects. In this chapter we let models link adjectives to *sets* of entities. Thus, in each model we include a set of entities that is associated with the word *tall*. These are the abstract entities in that model that are considered tall in the hypothetical situation that the model describes. Similarly, each model associates the adjective *thin* with the set of entities that are considered thin in the situation.

In addition to dealing with words, models also treat *complex expressions*: phrases and sentences that are made up of multiple words. For example, let us consider the complex phrase *tall and thin*. Just like we did in the case of simple adjective words, we let each model associate this phrase with a set of entities. These are the entities that are considered to be tall and thin in the hypothetical situation that the model describes. Other words, phrases and sentences are associated with all sorts of abstract mathematical objects. The words, phrases and sentences that we treat are collectively referred to as *expressions*. In each of our models, we associate abstract objects with all the expressions that we treat.

Summarizing, we state our general conception of models as follows:

> *A* **model** *is an abstract mathematical structure that we construct for describing hypothetical situations. Models are used for analyzing natural language expressions (words, phrases and sentences) by associating them with abstract objects.*

Associating language expressions with abstract objects is part and parcel of a model definition. For instance, one of the models that we use, call it M, may associate the word *Tina* with some abstract entity a. In technical terms, we say that in the model M, the word *Tina* *denotes* the entity a. In all the models that we study in this chapter, the name *Tina* denotes some or other entity, and the adjective *tall* denotes some or other set of entities. Given a particular model M, we refer to those denotations as $[\![Tina]\!]^M$ and $[\![tall]\!]^M$, respectively. Similarly, $[\![tall\ and\ thin]\!]^M$ is the denotation of the phrase *tall and thin* in the model M. In general, we adopt the following notational convention:

> *Let* exp *be a language expression, and let M be a model. We write* $[\![\mathrm{exp}]\!]^M$ *when referring to the* **denotation** *of* exp *in the model M.*

To have a more concrete view on models and denotations, let us consider Figure 2.1. This figure describes two models, each of them containing three entities: a, b and c. In model M_1, the name *Tina* denotes the entity a, and the adjective *thin* denotes the set $\{\mathsf{a}, \mathsf{b}\}$. In model M_2, the denotation of *Tina* is again the entity a, but this time, the set denotation of *thin* is the set $\{\mathsf{b}, \mathsf{c}\}$. We formally write it as follows:

$$[\![Tina]\!]^{M_1} = \mathsf{a} \qquad [\![thin]\!]^{M_1} = \{\mathsf{a}, \mathsf{b}\}$$

$$[\![Tina]\!]^{M_2} = \mathsf{a} \qquad [\![thin]\!]^{M_2} = \{\mathsf{b}, \mathsf{c}\}$$

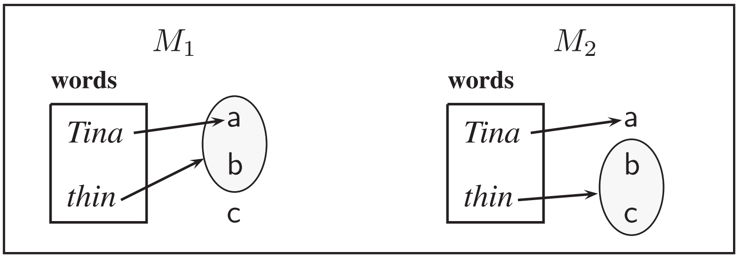

Figure 2.1 Models map words and other expressions to abstract mathematical objects. M_1 and M_2 are models with an entity denotation of Tina *and a set denotation of* thin. *The arrows designate the mappings from the words to their denotations, which are part of the model definition.*

Figure 2.1 only illustrates the assignment of denotations to simple words. However, as mentioned above, models are used in order to assign denotations to all expressions that we analyze, including complex expressions that are made of multiple words. In particular, models specify denotations for *sentences*. There are various ideas about what kinds of abstract objects sentences should denote. In most of this book, we follow the traditional assumption that sentences denote the two abstract objects known as *truth-values*, which are referred to as 'true' and 'false'. In more technical notation, we sometimes write '$\top$' for 'true' and '$\bot$' for 'false'. Yet another convention, which is most convenient for our purposes, is to use the number 1 for 'true' and the number 0 for 'false'.

Models assign truth-value denotations to sentences on the basis of the denotations they assign to words. For instance, the way we use models such as M_1 and M_2 in Figure 2.1 respects the intuition that the sentence *Tina is thin* is true in M_1 but false in M_2. Thus, we will make sure that M_1 and M_2 satisfy:

$$[\![\textit{Tina is thin}]\!]^{M_1} = 1 \qquad\qquad [\![\textit{Tina is thin}]\!]^{M_2} = 0$$

As we move on further in this chapter, we see how this analysis is formally obtained.

The truth-value denotations that models assign to sentences are the basis for our account of entailment relations. Let us return to the entailment between the sentence *Tina is tall and thin* (=(2.1)) and the sentence *Tina is thin* (=(2.2)). When discussing this entailment, we informally described our semantic judgment by saying that whenever sentence (2.1) is true, sentence (2.2) must be true as well. By contrast, we observed that the intuition does not hold in the other direction: when (2.2) is true, sentence (2.1) may be false. For this reason we intuitively concluded that sentence (2.2) does *not* entail (2.1). When analyzing (non-)entailment relations, we take into account these pre-theoretical intuitions about 'truth' and 'falsity'. We analyze an entailment $S_1 \Rightarrow S_2$ by introducing the following requirement: if a model lets S_1 denote *true*, it also lets S_2 denote *true*. When truth-values are represented numerically, this requirement means that if a model lets S_1 denote the number 1, it also lets S_2 denote 1.

Specifically, in relation to the entailment (2.1)⇒(2.2), we require that for every model where sentence (2.1) denotes the value 1, sentence (2.2) denotes 1 as well. Another way to state this requirement is to say that in every model, the truth-value denotation of (2.1) is *less than or equal to* the denotation of (2.2). Let us see why this is indeed an equivalent requirement. First, consider models where the denotation of (2.1) is 1. In such models, we also want the denotation of (2.2) to be 1. Indeed, requiring $[\![(2.1)]\!] \leq [\![(2.2)]\!]$ boils down to requiring that $[\![(2.2)]\!]$ is 1: this is the only truth-value for (2.2) that satisfies $1 \leq [\![(2.2)]\!]$. Further, when we consider models where the denotation of (2.1) is 0, we see that such models trivially satisfy the requirement $0 \leq [\![(2.2)]\!]$, independently of the denotation of (2.2).

To conclude: saying that $[\![(2.2)]\!]$ is 1 in every model where $[\![(2.1)]\!]$ is 1 amounts to saying that $[\![(2.1)]\!] \leq [\![(2.2)]\!]$ holds in every model. Accordingly, we translate our intuitive analysis of the entailment (2.1)⇒(2.2) to the formal requirement that $[\![(2.1)]\!] \leq [\![(2.2)]\!]$ holds in every model. More generally, our aim is to account for entailments using the $\leq$ relation between truth-values in models. This leads to a central requirement from formal semantic theory, which we call the *truth-conditionality criterion* (TCC):

> *A semantic theory T satisfies the* **truth-conditionality criterion** (TCC) *for sentences S_1 and S_2 if the following two conditions are equivalent:*
>
> (I) *Sentence S_1 intuitively entails sentence S_2.*
>
> (II) *For all models M in T: $[\![S_1]\!]^M \leq [\![S_2]\!]^M$.*

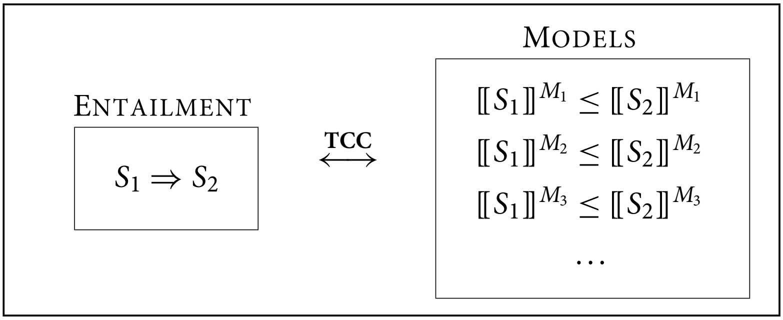

Figure 2.2 The TCC matches judgments on entailment with the $\leq$ relation. When the entailment $S_1 \Rightarrow S_2$ holds, the $\leq$ relation is required to hold between the truth-value denotations of S_1 and S_2 in all the models of the theory.

Table 2.1: Does $x \leq y$ hold?

	y=0	y=1
x=0	**yes**	**yes**
x=1	**no**	**yes**

Clause (I) of the TCC postulates an entailment between sentences S_1 and S_2. This is an *empirical* statement about the semantic intuitions of native speakers. By contrast, clause (II) is a statement about our *theory*'s treatment of sentences S_1 and S_2: the formal models we use indeed rely on intuitive notions of 'truth' and 'falsity', but they are purely theoretical. By imposing a connection between the empirical clause (I) and the theoretical clause (II), the TCC constitutes an *adequacy criterion* for semantic theory.

By way of recapitulation, Figure 2.2 illustrates how the TCC emulates the intuitive relation of entailment ('$\Rightarrow$') between sentences, by imposing the $\leq$ relation on sentence denotations in the models that our theory postulates. Table 2.1 summarizes how the requirement $x \leq y$ boils down to requiring that if x is 1, then y is 1 as well. Readers who are familiar with classical logic may observe the similarity between Table 2.1 and the truth table for *implication*. We will get back to this point in Chapter 5.

ARBITRARY AND CONSTANT DENOTATIONS

We have introduced the TCC as a general criterion for the empirical adequacy of formal semantics. Obviously, we want our theory to respect the TCC for as many intuitive (non-)entailments as possible. This will be our main goal throughout this book. Let us start our investigations by using the TCC to explain our favorite simple entailment: (2.1)⇒(2.2). We want to make sure that the models that we construct respect the TCC for this entailment. Thus, we need to define which models we have in our theory, and then check what truth-values each model derives for sentences (2.1) and (2.2). The models that we define fix the denotations of words like *Tina*, *tall* and *thin*. In technical jargon, words are also called *lexical items*. Accordingly, we will refer to the denotations of words as *lexical denotations*. Based on the lexical denotations that models assign to words, we will define the truth-values assigned to sentences containing them. To do that, let us explicitly state the assumptions that we have so far made about our models:

1. In every model M, in addition to the two truth-values 0 and 1, we have an arbitrary non-empty set E^M of the entities in M. We refer to this set as the *domain* of entities in M. For instance, in models M_1 and M_2 of Figure 2.1, the entity domains E^{M_1} and E^{M_2} are the same: in both cases they are the set {a, b, c}.
2. In any model M, the proper name *Tina* denotes an arbitrary entity in the domain E^M (cf. Figure 2.1).
3. In any model M, the adjectives *tall* and *thin* denote arbitrary sets of entities in E^M (cf. Figure 2.1).

When the model is understood from the context, we often write E for the domain of entities, suppressing the subscript M. We say that the domains of entities in the models we define are 'arbitrary' because we do not make any special assumptions about them: *any* non-empty set may qualify as a possible domain of entities in some model. Accordingly, we also treat the entity denotation of *Tina* as an arbitrary element of E. Whether this entity corresponds to a real-life entity like Tina Turner or Tina Charles is not our business here. We are not even insisting that the entity for *Tina* has 'feminine' properties, as might be suitable for a feminine English name. All we require is that in every model, the name *Tina* denotes *some* entity. In a similar fashion, we let the adjectives *tall* and *thin* denote arbitrary sets in our models. This arbitrariness is of course an over-simplification, but it will do for our purposes in this book. Here we study the meanings of words only to the extent that they are relevant for the study of entailment. Of course, much more should be said on word meanings. This is the focus of research in *lexical semantics*, which deals with many other important aspects of word meaning besides their contribution to entailments. For some readings on this rich domain, see some recommendations at the end of this chapter.

From now on we will often use words in **boldface** when referring to arbitrary denotations. For instance, by '**tina**' we refer to the element $[\![Tina]\!]$ of E that is denoted by the word *Tina* in a given model. Similarly, '**tall**' and '**thin**' are shorthand for $[\![tall]\!]$ and $[\![thin]\!]$: the *sets* of entities in E denoted by the words *tall* and *thin*. Putting words in boldface in this way is a convenience that spares us the use of the double brackets $[\![\,]\!]$. When we want to be more specific about the model M, we write $\mathbf{tina}^M$ or $[\![Tina]\!]^M$.

In our discussion of Figure 2.1, we noted that the sentence *Tina is thin* is intuitively true in M_1 but false in M_2. We can now see how this intuition is respected by our precise definition of models. To achieve that, we make sure that the sentences *Tina is thin* reflects a *membership assertion*. We only allow the sentence to be true in models where the entity denoted by *Tina* is a member of the set denoted by the adjective. Therefore, we analyze the word *is* as denoting a *membership function*. This is the function sending every entity x and set of entities A to the truth-value 1 if x is an element of A. If x is not an element of A, the membership function sends x and A to the truth-value 0. When referring to the membership function that the word *is* denotes, we use the notation '$\textsc{is}$'. Formally, we define $\textsc{is}$ as the function that satisfies the following, for every entity x in E and every subset A of E:

$$\textsc{is}(x, A) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}$$

For example, let us reconsider the models M_1 and M_2 that we saw in Figure 2.1. With our new assumption on the denotation of the word *is*, we now get:

In M_1: $[\![Tina\ is\ thin]\!] = \textsc{is}(\mathbf{tina}, \mathbf{thin}) = \textsc{is}(\mathsf{a}, \{\mathsf{a}, \mathsf{b}\}) = 1$ *since* $\mathsf{a} \in \{\mathsf{a}, \mathsf{b}\}$

In M_2: $[\![Tina\ is\ thin]\!] = \textsc{is}(\mathbf{tina}, \mathbf{thin}) = \textsc{is}(\mathsf{a}, \{\mathsf{b}, \mathsf{c}\}) = 0$ *since* $\mathsf{a} \notin \{\mathsf{b}, \mathsf{c}\}$

Thus, in M_1 the sentence *Tina is thin* denotes 1, and in M_2 it denotes 0, as intuitively required. More generally, in (2.10) below we summarize the denotation that the sentence *Tina is thin* is assigned in every model:

$$\text{(2.10)} \quad [\![\textit{Tina is thin}]\!]^M = \textsc{is}(\textbf{tina}, \textbf{thin}) = \begin{cases} 1 \text{ if } \textbf{tina} \in \textbf{thin} \\ 0 \text{ if } \textbf{tina} \notin \textbf{thin} \end{cases}$$

When referring to denotations, we have made a difference between the font for the denotations **tina**, **tall** and **thin**, and the font for the denotation IS. There is a reason for this notational difference. As mentioned, the denotations of the words *Tina*, *tall* and *thin* are *arbitrarily* chosen by our models. We have presented no semantic 'definition' for the meaning of these words. Models are free to let the name *Tina* denote any of their entities. Similarly, the adjectives *tall* and *thin* may denote any of set of entities. By contrast, the denotation of the word *is* has a *constant* definition across models: in all models we define this denotation as the membership function. We will have more to say about this distinction between denotations in Chapter 3. In the meantime, let us summarize our notational conventions:

> *Let* blik *be a word in a language. When the denotation* $[\![\text{blik}]\!]^M$ *of* blik *is arbitrary, we mark it* **blik**. *When it has a constant definition across models we mark it* BLIK.

ANALYZING AN ENTAILMENT

In order to analyze the entailment (2.1)⇒(2.2), let us now also look at the denotation of the sentence *Tina is tall and thin*. Since we let the sentence *Tina is thin* denote the truth-value of a membership assertion, it is only natural to analyze the sentence *Tina is tall and thin* in a similar way. Thus, we want this sentence to be true if the entity **tina** is a member of a set denoted by the conjunction *tall and thin*. But what should this set be? The same semantic intuitions that supported the entailment (2.1)⇒(2.2) can guide us to the answer. Obviously, for Tina to be tall and thin, she has to be tall, and she also has to be thin. And vice versa: if Tina is tall, and if in addition she is also thin, there is no way to avoid the conclusion that she is tall and thin. Elementary as they are, these considerations suggest that if we are going to let the conjunction *tall and thin* denote a set, it had better be the *intersection* of the two sets for the adjectives *tall* and *thin*. Formally, we write:

$$[\![\textit{tall and thin}]\!]^M = [\![\textit{tall}]\!]^M \cap [\![\textit{thin}]\!]^M = \textbf{tall} \cap \textbf{thin}.$$

Thus, we define the denotation of the word *and* to be the *intersection function* over E. This is the function AND that satisfies the following, for all subsets A and B of E:

$$\begin{aligned}\textsc{and}(A, B) &= A \cap B \\ &= \text{the set of all members of } E \text{ that are both in } A \text{ and in } B\end{aligned}$$

Now there is also no doubt about the denotation of the sentence *Tina is tall and thin*. Using the same kind of membership assertion that we used for the sentence *Tina is thin*, we reach the following denotation for this sentence:

(2.11) $$[\![\textit{Tina is tall and thin}]\!]^M = \textsc{is}(\,\textbf{tina}, \textsc{and}(\textbf{tall},\textbf{thin})\,) = \begin{cases} 1 \text{ if } \textbf{tina} \in \textbf{tall} \cap \textbf{thin} \\ 0 \text{ if } \textbf{tina} \notin \textbf{tall} \cap \textbf{thin} \end{cases}$$

In words: in every given model M, the sentence *Tina is tall and thin* denotes the truth-value 1 if the entity **tina** is in the intersection of the sets **tall** and **thin**; otherwise the sentence denotes 0.

We have now defined the truth-value denotations that the sentences *Tina is tall and thin* and *Tina is thin* have in every model. These are the truth-values specified in (2.11) and (2.10), respectively. Therefore, we can use the TCC in order to verify that our theory adequately describes the entailment between the two sentences. As a matter of set theory, the truth-value (2.10) must be 1 if the truth-value in (2.11) is 1: if the entity **tina** is in the intersection **tall** ∩ **thin**, then, by definition of intersection and set membership, it is also in the set **thin**. This set-theoretical consideration holds for all possible denotations **tina**, **tall** and **thin**. Thus, it holds for all models. This means that our assignment of denotations to sentences (2.1) and (2.2) has been successful in meeting the TCC when accounting for the entailment between them.

At this point you may feel that the games we have been playing with entities, sets, functions and truth-values are just restating obvious intuitions. This is perfectly true. Indeed, there is reason to feel satisfied about it. Semantic models provide us with a general and mathematically rigorous way of capturing common intuitions about entailment. A model is a small but precise description of a particular situation in which different sentences may be true or false. By specifying the denotations of the words *Tina*, *tall* and *thin*, a model describes, in an abstract way, who Tina is, and what the tall entities and thin entities are. As we have seen, and as we shall see in more detail throughout this book, models also take care of more "logical" denotations for words like *and* and *is*. This assignment of denotations to lexical items enables us to systematically assign denotations to complex expressions, including conjunctive phrases like *tall and thin* and sentences like *Tina is tall and thin*. If we are successful in assigning denotations to such complex expressions, we may be reasonably hopeful that our strategies will also be useful for much higher levels of hierarchical embedding (e.g. Dylan's description of the sad-eyed lady on page 1). In fact, by defining truth and falsity in models for two simple sentences, we have been forced to dive rather deep into the meanings of conjunction, predication, adjectives and proper names, and the ways in which they combine with each other. As we shall see in the following chapters, much of our elementary set-theoretical maneuvering so far is valuable when tackling more advanced questions in formal semantics.

When looking at a class of models that is heterogenous enough, we can "see", so to speak, whether one sentence must denote 1 when another sentence does. Let us get a feel of what is going on in the simple example we have been treating, by playing a little with some concrete models. Let us consider Table 2.2, which summarizes our assumptions so far and illustrates them concretely in the three models described in the rightmost columns. Each of these models has the set $E = \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{d}\}$ as its domain of entities. In model M_1, the word *Tina* denotes the entity a, and the word *thin* denotes the set of three entities $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$. Model M_2 assigns different denotations to these words: *Tina* denotes the entity b, and *thin* denotes the set $\{\mathsf{b}, \mathsf{c}\}$. In model M_3, the denotation of *Tina* remains the entity b, as in model M_2, while the adjective *thin* denotes a set of three entities: $\{\mathsf{a}, \mathsf{c}, \mathsf{d}\}$. Accordingly, the truth-values in the three models for the sentence *Tina is thin* are 1, 1 and 0, respectively. Similarly, using the assumed denotations for *tall*, we can also verify that the truth-values in these three models for the sentence *Tina is tall and thin* are 0, 1 and 0, respectively. Satisfying

Table 2.2: Denotations for expressions in the entailment (2.1)⇒(2.2).

Expression	**Cat.**	**Type**	**Abstract denotation**	**Denotations in example models with** $E = \{a, b, c, d\}$		
				M_1	M_2	M_3
Tina	PN	entity	**tina**	a	b	b
tall	A	set of entities	**tall**	{b, c}	{b, d}	{a, b, d}
thin	A	set of entities	**thin**	{a, b, c}	{b, c}	{a, c, d}
tall and thin	AP	set of entities	AND(**tall**, **thin**)	{b, c}	{b}	{a, d}
Tina is thin	S	truth-value	IS(**tina**, **thin**)	1	1	0
Tina is tall and thin	S	truth-value	IS(**tina**, AND(**tall**, **thin**))	0	1	0

Categories: PN = proper name; A = adjective; AP = adjective phrase; S = sentence.

the TCC means that the latter value must be less than or equal to the former value, which is indeed the case.

Model M_1 in Table 2.2 shows that the TCC is also met for the non-entailment (2.2)$\not\Rightarrow$(2.1). This model makes sentence (2.2) true while making (2.1) false. This means that our theory respects the requirement in the TCC that, if an entailment is missing, then at least one model does not satisfy the $\leq$ relation between the truth-values of the two sentences in question. In formula, model M_1 satisfies $[\![(2.2)]\!]^{M_1} \not\leq [\![(2.1)]\!]^{M_1}$. Furthermore, model M_1 also respects our intuition of *why* an entailment is absent in this case. As pointed out above, if somebody tried to convince you that Tina must be tall and thin just because she happens to be thin, you might reasonably object by pointing out the possibility that Tina may *not* be tall. Model M_1 highlights this possibility.

DIRECT COMPOSITIONALITY

So far we have been paying little attention to sentence structure. However, as mentioned in the introduction, one of our main interests is how meanings of natural language expressions are related to the syntactic forms of these expressions. For instance, let us consider the following two sentences:

(2.12) a. All pianists are composers, and Tina is a pianist.
b. All composers are pianists, and Tina is a pianist.

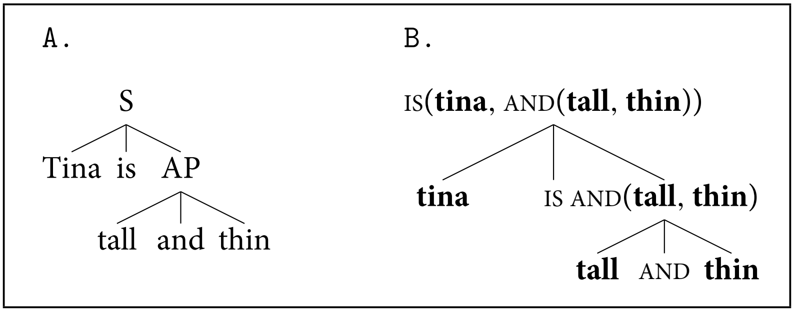

Figure 2.3 Syntactic structure and compositional derivation of denotations for Tina is tall and thin.

Sentences (2.12a) and (2.12b) contain the same words but in a different order. Consequently, the meanings of these two sentences are rather different. In particular, while (2.12a) entails the sentence *Tina is a composer*, sentence (2.12b) does not. The meaning of an expression is not a soup made by simply putting together the meanings of words. Rather, the order of the words in a complex expression and the hierarchical structures that they form affect its meaning in systematic ways. Since entailment relations between sentences reflect an aspect of their meanings, entailments are also sensitive to sentence structure. In the framework that we assume here, entailments are explained by appealing to model-theoretic denotations. Therefore, the question we are facing is: how are syntactic structures used when defining denotations of complex expressions? The general principle known as *compositionality* provides an answer to this question. According to this principle, the denotation of a complex expression is determined by the denotations of its immediate parts and the ways they combine with each other. For instance, in our analysis of the entailment (2.1)⇒(2.2), we treated the denotation of sentence (2.1) (*Tina is tall and thin*) as derived step by step from the denotations of its parts: the name *Tina*, the verb *is*, and the adjective phrase *tall and thin*. Figure 2.3 summarizes our compositional analysis.

Figure 2.3A shows the syntactic part-whole relations that we assume for the sentence. In this structure we group together the string of words *tall and thin* into one adjectival phrase, which we denote *AP*. More generally, tree diagrams as in Figure 2.3A represent the sentence's *constituents*: the parts of the sentence that function as grammatical units. In this case, besides the sentence itself and the words it contains, the only syntactic constituent assumed is the adjectival phrase *tall and thin*. Figure 2.3B describes how denotations of constituents are derived from the denotations of their immediate parts. The denotation of the adjectival phrase *tall and thin* is determined by combining the denotations of its immediate parts: **tall**, AND and **thin**. The denotation of the whole sentence is determined by the denotations of its parts: **tina**, IS, and AND(**tall**, **thin**). What we get as a result is the truth-value denotation in (2.11). The way in which this truth-value is derived is sanctioned by the compositionality principle on the basis of the structure in Figure 2.3A. Note that compositionality would not allow us to derive the truth-value in any other order. For instance, on the basis of the structure in Figure 2.3A, we would not be able to compositionally define the denotation of the whole sentence directly on the basis of the denotations of the adjectives *tall* and *thin*. These words are not among the sentence's immediate parts. Therefore, according to the compositionality principle, they can only indirectly affect its denotation.

Summarizing, we have adopted the following general principle, and seen how we follow it in our analysis of the entailment (2.1)⇒(2.2).

> **Compositionality**: *The denotation of a complex expression is determined by the denotations of its immediate parts and the ways they combine with each other.*

A word of clarification should be added here about the role of semantic formulas in our analysis. Consider for instance the formula IS(**tina**, AND(**tall**,**thin**)) that we derive in Figure 2.3B. This formula is not a representation of some abstract meaning, independent of the sentence structure. To the contrary, this formula is almost completely identical to the structure in Figure 2.3A, while adding only the necessary semantic details for describing how the denotation is derived for sentence (2.1). Most importantly, the formula specifies the function-argument relations between the denotations of the sentence constituents. Thus, the formula IS(**tina**, AND(**tall**, **thin**)) is simply the syntactic bracketing [*Tina is* [*tall and thin*]] imposed by the tree in Figure 2.3A, with two modifications: (i) symbols for words are replaced by symbols of their denotations; and (ii) symbols for denotations may be shuffled around in order to follow the convention of putting function symbols to the left of their arguments. This respects the highly restrictive nature of compositional analysis: the process only requires a syntactic structure, denotations of lexical items, and a way to glue the denotations together semantically. The real "semantic action" is within these three components of the theory, not within the semantic formulas we use. This version of compositionality is sometimes referred to as *direct compositionality*. In this paradigm, denotations in the model are directly derived from syntactic structures, with no intermediate level of semantic or logical representation.

STRUCTURAL AMBIGUITY

Direct compositionality helps to clarify an important issue in linguistic theory: the phenomenon of *structural ambiguity*. Consider the following sentence:

(2.13) Tina is not tall and thin.

Let us consult our intuitions with respect to the following question: does (2.13) entail sentence (2.2) (=*Tina is thin*) or not? This is much harder to judge than in the case of the entailment (2.1)⇒(2.2). However, there is a common intuition that (2.13) entails (2.2), but only *under particular usages*. A speaker who wishes to convey the entailment (2.13)⇒(2.2) can do so by stressing the prosodic boundary after the word *tall*:

(2.14) Tina is not tall, and thin.

Without such an intonational pattern, a speaker can also use (2.13) felicitously for describing a situation where Tina is not thin. For instance, if we tell Sue that Tina is tall and thin, she may use (2.13) for denying the assertion, by saying something like:

(2.15) Come on, that isn't true! *Tina is not tall and thin*: although she is indeed very tall, you couldn't possibly think of her as thin!

In this reaction, the way in which Sue uses sentence (2.13) clearly indicates that she does not consider it to entail sentence (2.2).

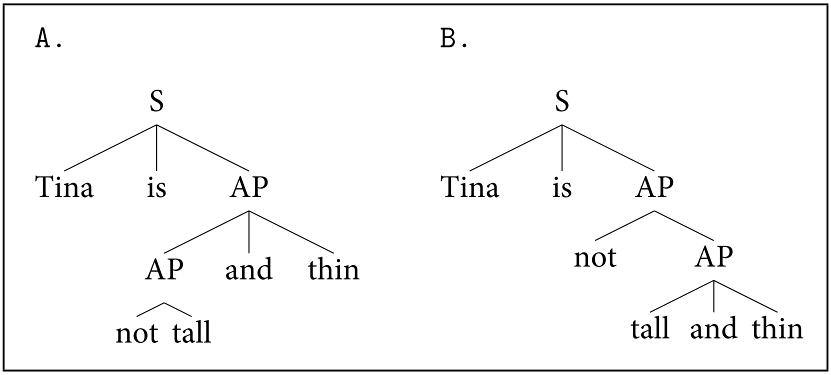

Figure 2.4 Structural ambiguity.

Because the two usages of sentence (2.13) show differences between its "entailment potential", we say that it is *ambiguous*. The "comma intonation" in (2.14) *disambiguates* the sentence. Another way of disambiguating sentence (2.13) is illustrated in (2.15), using the context of our conversation with Sue. In the former disambiguation, sentence (2.13) is used for entailing (2.2); in the latter disambiguation, the entailment from (2.13) to (2.2) is blocked. We refer to the two possible usages of sentence (2.13) as *readings* of the sentence. One reading entails (2.2), the other does not. Another way to describe the "two readings" intuition is to note that sentence (2.13) may be intuitively classified as both true and false in the same situation. Consider a situation where Tina is absolutely not thin. Context (2.15) highlights that sentence (2.13) may be used as true in this situation. By contrast, (2.14) highlights that the sentence also has the potential of being false in the same situation.

The striking thing about the ambiguity of sentence (2.13) is the ease with which it can be described when we assume the compositionality principle. Virtually all syntactic theories analyze sentence (2.13) as having two different syntactic structures, as illustrated in Figure 2.4. A simple phrase structure grammar that generates the structural ambiguity in Figure 2.4 is given in (2.16) below:

(2.16) AP $\longrightarrow$ *tall*, *thin*, ...

AP $\longrightarrow$ AP *and* AP

AP $\longrightarrow$ *not* AP

In words: an adjective phrase (AP) can be a simple adjective, or a conjunction of two other APs, or a negation of another AP. These rules derive both structures in Figure 2.4. When a grammar generates more than one structure for an expression in this way, we say that it treats the expression as *structurally ambiguous*. You may think that the syntactic ambiguity in Figure 2.4 is by itself already an elegant account of the semantic ambiguity in (2.13). However, there is a gap in this account: why does it follow that the structural ambiguity of sentence (2.13) also makes it *semantically* ambiguous? Compositionality provides the missing link. When the two structures in Figure 2.4 are compositionally analyzed, we immediately see that the same model may assign them two different truth-values. Concretely, let us assume that the denotation of the negation word *not* in (2.13) is the *complement function*, i.e. the function NOT that maps any subset A of E to its complement set:

$$\text{NOT}(A) = \overline{A} = E - A = \text{the set of all the members of } E \text{ that are not in } A$$

Figure 2.5 uses the denotation NOT for illustrating the compositional analysis of the two structures in Figure 2.4. As Figure 2.5 shows, the compositional process works differently for each of the two structural analyses of sentence (2.13). For each of the denotations in Figures 2.5A and 2.5B to be 1, different requirements have to be satisfied. This is specified in (2.17a) and (2.17b) below:

(2.17) a. $\text{IS}(\textbf{tina}, \text{AND}(\text{NOT}(\textbf{tall}), \textbf{thin})) = 1$
This holds if and only if (iff) $\textbf{tina} \in \overline{\textbf{tall}} \cap \textbf{thin}$.

b. $\text{IS}(\textbf{tina}, \text{NOT}(\text{AND}(\textbf{tall}, \textbf{thin}))) = 1$
This holds if and only if $\textbf{tina} \in \overline{\textbf{tall} \cap \textbf{thin}}$.

For the denotation in Figure 2.5A to be 1, the requirement in (2.17a) must hold. When this is the case, the entity **tina** is in the set **thin**, hence the truth-value assigned to the sentence *Tina is thin* (=(2.2)) is also 1. Thus, our compositional analysis of the structure in Figure 2.4A captures the reading of sentence (2.13) that entails sentence (2.2). By contrast, the denotation (2.17b) that is derived in Figure 2.5B does not guarantee that the entity **tina** is in the set **thin**. This is because the entity **tina** may be in the complement set $\overline{\textbf{tall} \cap \textbf{thin}}$ while being in the set **thin**, as long as it is not in the set **tall**. Specifically, consider a model

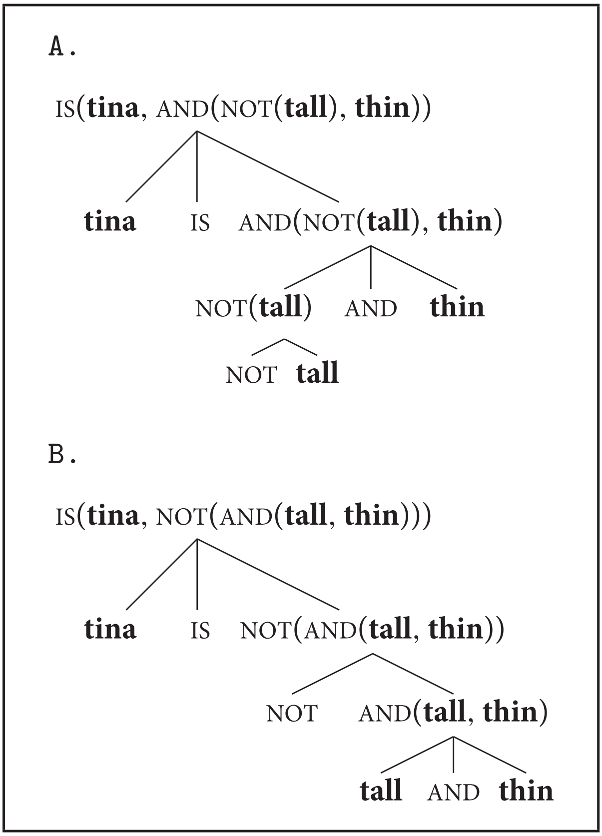

Figure 2.5 Compositionality and ambiguity.

M where the entities **tina**, **mary** and **john** are t, m and j, respectively. Suppose further that the model *M* assigns the following denotations to the adjectives *thin* and *tall*:

$$\textbf{thin} = \{\mathsf{t}, \mathsf{j}\} \quad \textbf{tall} = \{\mathsf{m}, \mathsf{j}\}$$

The model *M* represents a situation where Tina is thin but not tall, Mary is tall but not thin, and John is both thin and tall. In this model, the denotation in Figure 2.5B is the truth-value 1, but sentence (2.2) denotes the truth-value 0. This means that our compositional analysis of the structure in Figure 2.4B captures the reading of sentence (2.13) that does not entail sentence (2.2).

In compositional systems, the structure that we assume for a sentence strongly affects the entailment relations that our theory expects for it. When a sentence is assumed to be structurally ambiguous, a compositional theory may assign different truth-values to its different structures. As a result, the theory may expect different entailment relations to hold for the different structures. Accordingly, when speakers are confronted with such a sentence, they are expected to experience what we informally call "semantic ambiguity", i.e. some systematic hesitations regarding some of the sentence's entailments. Structural ambiguity is used as the basis of our account of semantic ambiguity. Once we have acknowledged the possibility of ambiguity, we prefer to talk about the entailments that sentence *structures* show, and of the truth-values that are assigned to these structures. However, for the sake of convenience, we often say that sentences themselves have entailments and truth-values. This convention is harmless when the sentences in question are unambiguous, or when it is clear that we are talking about a particular reading.

Semanticists often distinguish the syntactic-semantic ambiguity of sentences like (2.13) from another type of under-specification, which is called *vagueness*. For instance, as we noted above, the sentence *Tina is tall* says little about Tina and her exact height. In some contexts, e.g. if Tina is known to be a Western female fashion model, the sentence may be used for indicating that Tina is above 1.70 meters. In other contexts, e.g. if Tina is known to be member of some community of relatively short people, the sentence may indicate that Tina is above 1.50 meters. However, we do not consider these two usages as evidence that the sentence must be assigned different structures with potentially different denotations. Rather, we say that the sentence *Tina is tall* is vague with respect to Tina's height. Further specification of relevant heights is dealt with by augmenting our semantic treatment with a *pragmatic theory*. Pragmatic theories also consider the way in which sentences are used, and the effects of context on their use. Pragmatic theories aim as well to account for the way speakers resolve (or partly resolve) vagueness in their actual use of language. Classical versions of formal semantics did not aim to resolve vagueness, but current semantic theories often interact with pragmatics and describe the way context helps in resolving vagueness in actual language use.

Vagueness is very prominent in the way natural languages are used, and most sentences may be vague in one way or another. For instance, the sentence *Sue is talking* tells us nothing about Sue's voice (loud or quiet, high or low, etc.), what Sue is talking about, who the addressee is, etc. However, upon hearing the sentence, we may often use the context to infer such information. For instance, suppose that we are at a conference and know that Sue is one of the speakers. In such a context, we may draw some additional conclusions about the subject of Sue's talk and the addressees. Hearers often use context in this way to extract more information from linguistic expressions, and speakers often rely on their hearers to do that. In distinction from entailment, such inferential processes which are based on contextual knowledge are defeasible. For instance, even when the context specifies a particular conference where Sue is a speaker, we may use the sentence *Sue is talking* to indicate that Sue is talking to a friend over the phone. What we saw in sentence (2.13) is quite different from the defeasible reasoning that helps speakers and hearers in their attempts to resolve vagueness of language utterances. The comma intonation in (2.14) illustrated that one phonological expression of sentence (2.13) *indefeasibly* entails sentence (2.2). This convinced us that both structures that we assigned to sentence (2.13) are semantically useful. The theoretical consideration was the key for our treatment of the sentence as semantically ambiguous, more than any "pure" linguistic intuition. Most decisions between ambiguity and vagueness involve similar theoretical considerations, rather than the direct judgments of a speaker's linguistic intuitions.

FURTHER READING

Introductory: For methodological aspects of logical semantics, including truth-values, entailment and compositionality, see Gamut (1982, vol. 1). For more examples and discussion of structural ambiguity, see Zimmermann and Sternefeld (2013, ch. 3), and, in relation to vagueness, Kennedy (2011). For further discussion of compositionality, see Partee (1984). On defeasible reasoning, see Koons (2014). Levinson (1983) is an introductory textbook on pragmatics. On lexical semantics, see Cruse (1986); Murphy (2010). Meaning relations between words and *concepts* they refer to are extensively studied in the literature on *categorization*. See Laurence and Margolis (1999); Smith (1988); Taylor (1989) for introductions of these topics.

Advanced: The idea that sentences denote truth-values, and more generally, *propositions* (cf. Chapter 6), was proposed as central for communication (Austin 1962; Searle 1969). The centrality of entailment and the model-theoretic TCC was also highlighted in semantic theories of non-indicative sentences, especially *interrogatives* (Groenendijk and Stokhof 1984, 2011). An alternative to the model-theoretic approach to entailment is *proof-theoretic semantics* (Schroeder-Heister, 2014). In its application to natural language, proof-theoretic approaches are sometimes referred to as *natural logic*. Some examples of work in this area are McAllester and Givan (1992); Sánchez (1991); Moss (2010). Defeasible reasoning in language is related to *common sense reasoning* in work in artificial intelligence (Brewka et al. 1997) and cognitive psychology (Stenning and van Lambalgen 2007; Adler and Rips 2008). For more on pragmatic theories, and specifically the notion of *implicature*, see Grice (1975); Geurts (2010); Chierchia et al. (2012). Much of this work pays close attention to the meaning and use of the word *or* (cf. the choice between 'inclusive' and 'exclusive' denotations in Exercise 6). Direct compositionality in contemporary semantics of natural language was first illustrated in Montague (1970*a*). For further work on compositionality, see Montague (1970*b*); Janssen (1983); Janssen with Partee (2011); Barker and Jacobson (2007); Pagin and Westerståhl (2010); Werning et al. (2012).

EXERCISES (ADVANCED: **4**, 5, **6**, 7, **8**, 9, 10)

1. In the following pairs of sentences, make a judgment on whether there is an entailment between them, and if so, in which of the two possible directions. For directions in which there is no entailment, describe informally a situation that makes one sentence true and the other sentence false. For example, in the pair of sentences (2.1) and (2.2), we gave the judgment (2.1)⇒(2.2), and supported the non-entailment (2.2)⇏(2.1) by describing a situation in which Tina is thin but not tall.
   (i) *a*. Tina got a B or a C. *b*. Tina got a B.
   (ii) *a*. Tina is neither tall nor thin. *b*. Tina is not thin.
   (iii) *a*. Mary arrived. *b*. Someone arrived.
   (iv) *a*. John saw fewer than four students. *b*. John saw no students.
   (v) *a*. The ball is in the room. *b*. The box is in the room and the ball is in the box.
   (vi) *a*. Hillary is not a blond girl. *b*. Hillary is not a girl.

(vii) *a*. Hillary is a blond girl. *b*. Hillary is a girl.
(viii) *a*. Tina is a Danish flutist and a physicist. *b*. Tina is a Danish physicist and a flutist.
(ix) *a*. Tina is not tall but taller than Mary. *b*. Mary is not tall.
(x) *a*. Mary ran. *b*. Mary ran quickly.
(xi) *a*. I saw fewer than five horses that ran. *b*. I saw fewer than five black horses that ran.
(xii) *a*. I saw fewer than five horses that ran. *b*. I saw fewer than five animals that ran.
(xiii) *a*. Exactly five pianists in this room are French composers. *b*. Exactly five composers in this room are French pianists.
(xiv) *a*. No tall politician is multilingual. *b*. Every politician is monolingual.
(xv) *a*. No politician is absent. *b*. Every politician is present.
(xvi) *a*. At most three pacifists are vegetarians. *b*. At most three vegetarians are pacifists.
(xvii) *a*. All but at most three pacifists are vegetarians. *b*. At most three non-vegetarians are pacifists.

2. Each of the following sentences is standardly considered to be structurally ambiguous. For each sentence suggest two structures, and show an entailment that one structure intuitively supports and the other structure does not:
(i) I read that Dan published an article in the newspaper.
(ii) Sue is blond or tall and thin.
(iii) The policeman saw the man with the telescope.
(iv) Rich Americans and Russians like to spend money.
(v) Sue told some man that Dan liked the story.
(vi) Dan ate the lettuce wet.
(vii) Sue didn't see a spot on the floor.
3. Table 2.2 shows different denotations for the expressions in sentences (2.1) and (2.2) in different models. We used these models and the truth-values they assign to the sentences to support our claim that the TCC explains the entailment (2.1)$\Rightarrow$(2.2), and the non-entailment (2.2)$\not\Rightarrow$(2.1).
The table on the right gives the expressions for the two analyses in Figure 2.5 of the sentence *Tina is not tall and thin*.

   a. Add to this table the missing denotations of these expressions within the three models M_1, M_2 and M_3.

b. Verify that the truth-values that you assigned to the two analyses in Figure 2.5 support the intuition that the analysis in Figure 2.4A entails sentence (2.2), whereas the analysis in Figure 2.4B does not entail (2.2).

Expression	**Denotations in example models with** $E = \{a, b, c, d\}$		
	M_1	M_2	M_3
Tina	a	b	b
tall	{b, c}	{b, d}	{a, b, d}
thin	{a, b, c}	{b, c}	{a, c, d}
not tall			
[*not tall*] *and thin*			
Tina is [[*not tall*] *and thin*]			
tall and thin			
not [*tall and thin*]			
Tina is [*not* [*tall and thin*]]			
Tina is thin			

4. a. Mark the pairs of sentences in Exercise 1 that you considered equivalent.
   b. Give three more examples for pairs of sentences that you consider intuitively equivalent.
   c. State the formal condition that a semantic theory that satisfies the TCC has to satisfy with respect to equivalent sentences.
5. Consider the ungrammaticality of the following strings of words.
   (i) **Tina is both tall* **Tina is both not tall* **Tina is both tall or thin*

   To account for this ungrammaticality, let us assume that the word *both* only appears in adjective phrases of the structure *both* AP_1 *and* AP_2. Thus, a constituent *both X* is only grammatical when *X* is an *and*-conjunction of two adjectives or adjective phrases; hence the grammaticality of the string *both tall and thin* as opposed to the ungrammaticality of the strings in (i), where *X* is *tall*, *not tall* and *tall or thin*, respectively. We assume further that the denotation of a constituent *both* AP_1 *and* AP_2 is the same as the denotation of the parallel constituent AP_1 *and* AP_2 as analyzed in this chapter.

a. With these syntactic and semantic assumptions, write down the denotations assigned to the following sentences in terms of the denotations **tina**, **tall**, **thin**, IS and AND.
(ii) Tina is both not tall and thin. (iii) Tina is not both tall and thin.

b. Explain why the denotations you suggested for (ii) and (iii) account for the (non-)entailments (ii)$\Rightarrow$(2.2) and (iii)$\not\Rightarrow$(2.2).

c. Consider the equivalence between the following sentences:
(iv) Tina is both not tall and not thin. (v) Tina is neither tall nor thin.
Suggest a proper denotation for the constituent *neither tall nor thin* in (v) in terms of the denotations **tall** and **thin** (standing for sets of entities). Explain how the denotation you suggest, together with our assumptions in items 5a and 5b above, explain the equivalence (iv)$\Leftrightarrow$(v).

6. Consider the following sentence:
(i) Tina is [tall or thin].
The *inclusive* or *exclusive* analyses for the coordinator *or* involve denotations that employ the *union* and *symmetric difference* functions, respectively – the functions defined as follows for all $A, B \subseteq E$:

$$\text{OR}_{\text{in}}(A, B) = A \cup B$$
$$= \text{the set of members of } E \text{ that are in } A\text{, in } B \text{ or in both } A \text{ and } B$$

$$\text{OR}_{\text{ex}}(A, B) = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$
$$= \text{the set of members of } E \text{ that are in } A \text{ or } B\text{, but not both } A \text{ and } B$$

Consider the following sentential structures:
(ii) Tina is not [tall and thin].
(iii) Tina is not [tall or thin].
(iv) Tina is [not tall] and [not thin].
(v) Tina is [not tall] or [not thin].

a. Assuming that the word *or* denotes the function OR_{in}, write down all the entailments that the TCC expects in (ii)–(v). Answer the same question, but now assuming that *or* denotes OR_{ex}.

b. Which of the two entailment patterns in 6a better captures your linguistic intuitions about (ii)–(v)?
c. Under one of the two analyses of *or*, one of the structures (ii)–(v) is expected to be equivalent to (i). Which structure is it, and under which analysis? Support your answer by a set-theoretical equation.

7. A pair of sentences (or readings/structures) is said to be treated as *contradictory* if, whenever one of the sentences is taken to denote 1, the other denotes 0. For instance, under the analysis in this chapter, the sentences *Mary is tall* and *Mary is not tall* are contradictory.

a. Give more examples for contradictory pairs of sentences/structures under the assumptions of this chapter.
b. Consider the sentences *The bottle is empty* and *The bottle is full*. Suggest a theoretical assumption that would render these sentences contradictory.
c. Give an entailment that is accounted for by the same assumption.
d. Show that according to our account, the denotation of the sentence *Tina is tall and not tall* is 0 in any model. Such a sentence is classified as a *contradiction*. Show more examples for sentences that our account treats as contradictions.
e. Show that according to both our treatments of *or* in Exercise 6, the denotation of the sentence *Tina is tall or not tall* is 1 in any model. Such sentences are classified as *tautological*. Show more examples for sentences that our account treats as tautological.
f. Show that the TCC expects that any contradictory sentence entails any sentence in natural language, and that any tautology is entailed by any sentence in natural language. Does this expectation agree with your linguistic intuitions? If it does not, do you have ideas about how the problem can be solved?

8. We assume that entailments between sentences (or structures) have the following properties.

Reflexivity: Every sentence S entails itself.

Transitivity: For all sentences S_1, S_2, S_3: if S_1 entails S_2 and S_2 entails S_3, then S_1 entails S_3.

Reflexivity and transitivity characterize entailments as a *preorder* relation on sentences/structures.

Consider the following entailments:
(i) Tina is tall, and Ms. Turner is neither tall nor thin $\Rightarrow$ Tina is tall, and Ms. Turner is not tall.
(ii) Tina is tall, and Ms. Turner is neither tall nor thin $\Rightarrow$ Tina is not Ms. Turner.
Show an entailment that illustrates transitivity together with entailments (i) and (ii).

9. Consider the following structurally ambiguous sentence (=(ii) from Exercise 2).
(i) *Tina is blond or tall and thin.*
   a. For sentence (i), write down the denotations derived for the two structures using the inclusive denotation of *or* from Exercise 6, and the denotations **tina**, **blond**, **tall** and **thin**.
   b. Give specific set denotations for the words *blond*, *tall* and *thin* that make one of these denotations *true* (*1*), while making the other denotation *false* (*0*).
   c. Using the *both… and* construction from Exercise 4, find two unambiguous sentences, each of which is equivalent to one of the structural analyses you have given for sentence (i).
   d. Under an inclusive interpretation of *or*, which of the two sentences you found in 9c is expected to be equivalent to the following sentence?
   (ii) *Tina is both blond and thin or both tall and thin.*
   e. Write down the set-theoretical equation that supports this equivalence.
   f. Using our assumptions in this chapter, find a structurally ambiguous sentence whose two readings are analyzed as equivalent.

10. Consider the following entailment:
(i) Tina has much money in her bank account, and Bill has one cent less than Tina in his bank account $\Rightarrow$ Bill has much money in his bank account.
   a. We adopt the following assumption: *Tina has m cents in her bank account*, where *m* is some positive natural number. Further, we assume that entailment is transitive. Show that with these assumptions, you can use the entailment pattern in (i) to support an entailment with the following contradictory conclusion: *Ms. X has much money in her bank account, and Ms. X has no money in her bank account.*

b. The ability to rely on transitivity of entailments to support such absurd conclusions is known as the *Sorites Paradox*. Suggest a possible resolution of this paradox by modifying our assumptions in 10a and/or our assumption that entailment relations are transitive.

SOLUTIONS TO SELECTED EXERCISES

3.

Expression	**Denotations in example models with** $E = \{a, b, c, d\}$		
	M_1	M_2	M_3
Tina	a	b	b
tall	{b, c}	{b, d}	{a, b, d}
thin	{a, b, c}	{b, c}	{a, c, d}
not tall	{a, d}	{a, c}	{c}
[*not tall*] *and thin*	{a}	{c}	{c}
Tina is [[*not tall*] *and thin*]	1	0	0
tall and thin	{b, c}	{b}	{a, d}
not [*tall and thin*]	{a, d}	{a, c, d}	{b, c}
Tina is [*not* [*tall and thin*]]	1	0	1
Tina is thin	1	1	0

4.c. In any theory T that satisfies the TCC, sentences S_1 and S_2 are equivalent if and only if for all models M in T, $[\![S_1]\!]^M = [\![S_2]\!]^M$.

5.a–b. The truth-values and the accounts of the (non-)entailments are identical to the truth-values for the ambiguous sentence (2.13) and the corresponding (non-)entailment from (2.13) to (2.2).

c. $[\![\textit{neither tall nor thin}]\!] = \overline{\textbf{tall}} \cap \overline{\textbf{thin}} = \textsc{and}(\textsc{not}(\textbf{tall}), \textsc{not}(\textbf{thin})) = [\![\textit{both not tall and not thin}]\!]$

6.a. $\textsc{or}_{in}$: (iii)⇒(ii); (iv)⇒(ii); (ii)⇔(v); (iii)⇔(iv); (iii)⇒(v); (iv)⇒(v).

$\textsc{or}_{ex}$: (iv)⇒(ii); (v)⇒(ii); (iv)⇒(iii).

c. (v); the $\textsc{or}_{ex}$ analysis; $(\overline{A} - \overline{B}) \cup (\overline{B} - \overline{A}) = (B - A) \cup (A - B) = (A - B) \cup (B - A)$.

7\. a. Mary is neither tall nor thin, Mary is tall or thin; Mary is tall and not thin, Mary is thin; Mary is [not tall] or [not thin], Mary is tall and thin.

b. The adjectives *empty* and *full* denote *disjoint sets*: $\textbf{empty} \cap \textbf{full} = \emptyset$.

c. The bottle is empty $\Rightarrow$ The bottle is not full; The bottle is full $\Rightarrow$ The bottle is not empty.

8\. Tina is tall, and Ms. Turner is not tall $\Rightarrow$ Tina is not Ms. Turner (=(2.3a)).

9\. a. $\textsc{is}(\textbf{tina},\textsc{and}(\textsc{or}_{\text{in}}(\textbf{blond},\textbf{tall}),\textbf{thin}))$

$\textsc{is}(\textbf{tina},\textsc{or}_{\text{in}}(\textbf{blond},\textsc{and}(\textbf{tall},\textbf{thin})))$

b. $\textbf{blond} = \{\textbf{tina}\}$; $\textbf{tall} = \textbf{thin} = \emptyset$

c. Tina is both blond or tall and thin.
Tina is blond or both tall and thin.

d. Tina is both blond or tall and thin.

e. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

f. Tina is blond and tall and thin

10\. a. Consider the following general entailment scheme, based on (i):
(i') *Ms. n* has much money in *Ms. n*'s bank account and *Ms.* $n+1$ has one cent less than *Ms. n* in *Ms.* $n+1$'s bank account $\Rightarrow$ *Ms.* $n+1$ has much money in *Ms.* $n+1$'s bank account.
We can deduce from (i'), by induction on the transitivity of entailment, that the following (unacceptable) entailment is intuitively valid:
Ms. 1 has much money in her bank account, and *Ms.* 1 has *m* cents in her bank account $\Rightarrow$ *Ms.* $m+1$ has much money in her bank account, and *Ms.* $m+1$ has no cents in her bank account.

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