Geschiedenis van de logica

Modal Logic, Temporal Logic and the Logic of Agency

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Outline

Modal Logic
* From predicate logic to modal logic

Temporal Logic
* Arthur Prior and the development of (tense) logic after 1950
* Tensed vs tenseless talk
* Semantics for the future tense

Logic of Agency
* Review of branching time
* Agents and choices
* “Seeing to it that”
Arthur Prior
**Arthur Prior**

1914 born in Masterton, New Zealand  
1946 Lecturer, Canterbury University College, NZ  
1956 John Locke Lectures, Oxford; initiated British Logic Colloquium  
1958 Professor in Manchester  
1960 Editor, *The Journal of Symbolic Logic*  
1966 Fellow and Tutor, Balliol College, Oxford  
1969 died in Trondheim, Norway

Main works:

1957 *Time and Modality*  
1967 *Past, Present and Future*  
1971 *Objects of Thought* (ed. P.T. Geach and A.J.P. Kenny)  
1977 *Worlds, Times and Selves* (ed. K. Fine)
Arthur Prior and the development of (tense) logic

Technical developments in logic:

* among the first explicitly semantic approaches to modal logic

* among the earliest expressiveness results (Hans Kamp)

* earliest developments towards “hybrid logic”

Other fields:

* Philosophy of language: phenomenology of “essential indexicality”

* Metaphysics: logical analysis of the problem of futura contingentia
Prior on logic and natural language

* Foundational problem: How do we know what the logical connectives mean?

* Prior’s argument (The runabout inference-ticket): Giving introduction- and elimination-rules alone cannot give the meaning of a connective

* Logic as a certain (formal) way of studying natural language / the world:

* Logic is about the real world;
* No fixed boundary between logic and other sciences
Modal logic: motivation I (Prior, “What is logic?”)

A (1) If Polly is an animal then Polly is an animal.

(2) If Polly is a feathered animal then Polly is an animal.

(3) If all feathered animals breathe air then what does not breathe air is not a feathered animal.

B (1) If Polly is a feathered animal she breathes air.

(2) If Polly has feelings, I am obliged to treat her kindly.

(3) If Polly isn’t my parrot, she’s Peter’s.
C (1) If there are parrots, there will always have been parrots.

(2) If I know that Polly breathes air, she does breathe air.

(3) If I am obliged to treat Polly kindly, then I am not obliged not to treat Polly kindly.

(4) If it is not possible for Polly not to be a parrot, then it is possible for her to be a parrot.
Modal logic: Kripke semantics

* Propositions are true or false locally, at an index (a “world”)

* Adverbial modification of proposition represented as quantification over indexes

* Main example: alethic modalities

  – Possibly $p$ ($\Diamond p$) ⇔ At some (accessible) index, $p$

  – Necessarily $p$ ($\Box p$) ⇔ At all (accessible) indexes, $p$

  – Slogan: “necessary truth is truth in all possible worlds”
Time and tense in natural language

(1) Socrates is sitting.

* English (and other Indo-European languages): *tensed* language

* natural language sentences are complete without dates

* ancient and medieval discussion: propositions are complete without dates

* 20th century (Frege, Russell): explicit dates needed, or token-reflexive analysis:

(2) Socrates is* sitting at \(t\). ("is*" a tenseless copula)

(3) Socrates is* sitting while this sentence is being uttered.
Essential indexicality

* Many uses of indexicals like “I”, “now”, and (maybe) “here” cannot be eliminated

* Famous example (John Perry, 1979): The sugar trail in the supermarket

* Indexicals are vital for explaining actions and emotions

* Names can be mis-applied, “I” cannot

* Prior (1959): Tense is essentially indexical
Prior’s “thank goodness” argument: The essential indexicality of tense

[...] half the time I personally have forgotten what the date is, and have to look it up or ask somebody when I need it for writing cheques, etc.; yet even in this perpetual dateless haze one somehow communicates, one makes oneself understood, and with time-references too. One says, e.g. ‘Thank goodness that’s over!’, and not only is this, when said, quite clear without any date appended, but it says something which it is impossible that any use of a tenseless copula with a date should convey. It certainly doesn’t mean the same as, e.g. ‘Thank goodness the date of the conclusion of that thing is Friday, June 15, 1954’, even if it be said then. (Nor, for that matter, does it mean ‘Thank goodness the conclusion of that thing is contemporaneous with this utterance’. Why should anyone thank goodness for that?)
Formalising the tenses

* Tense is essential \Rightarrow take atomic sentences to be tensed

* Introduce (modal) operators $F$ (future) and $P$ (past)

* Iterability argument for use of operators

* $P$ and $F$ are weak operators;
  * duals $G$ (always going to be) and $H$ (has always been)

* Prior considers propositional and quantified languages

* Problems of contingently existing individuals; modal system $Q$
Prior’s syntax: Polish notation

The intuitive connection between each of these theses and non-branching in the future is straightforward; we can see how each of them could have exceptions if there were no such thing as 'the' future but only a number of alternative possible futures. As a specimen, we prove T5, as follows:

\[
\begin{align*}
(1) & CnscpCnFPnPnPp \\
(2) & CnscpCnFPp \\
(3) & CnscpCnFPnPpApPpFPp \\
(4) & CnscpCnFPnPpszNPnsnp \\
(6) & CnscpCnFPnPpPpPpPp \\
(7) & CnscpCnFPnPpPpPpPp \\
(8) & CnscpCnFPnPpPpPpPp \\
(9) & CnscpCnFPnPpPpPpPp \\
(10) & CnscpCnFPnPpPpPpPp \\
(11) & CnscpCnFPnPpPpPpPp \\
(12) & CnscpCnFPnPpPpPpPp \\
(13) & CnscpCnFPnPpPpPpPp \\
(14) & CnscpCnFPnPpPpPpPp \\
(15) & CnscpCnFPnPpPpPpPp \\
(16) & CnscpCnFPnPpPpPpPp \\
(17) & CnscpCnFPnPpPpPpPp \\
Semantics for modal logics

* use a modal object language, what about the semantics?

* models: time-flow as a binary relation (earlier than/later than)

* language of the earlier-later-relation: “$U$-calculus” ($m < m'$ etc.)

* tension: if the tenses are basic, the formalism should reflect this
  * the models cannot be more fundamental than the tense operators

* Prior on the status of models: “handy diagrams”
  * no metalanguage

* aim: interpreting the $U$-calculus within tense logic

* expressiveness: irreflexivity (easy in $U$-calculus, no tense-logical analogue)
Hybrid logic: Prior on “world-states”

* $Wp$ (“$p$ is the world state”): $Wp \rightarrow p$ and $(Wp \land q) \rightarrow \Box(p \rightarrow q)$
* (“The world is everything that is the case”, Wittgenstein, *TLP* 1)

* sorted language: ordinary propositional variables ($p, q, r, \ldots$) and world-variables ($a, b, c, \ldots$); for world-variables, have $\Diamond a$ and $\Box(a \rightarrow p) \lor \Box(a \rightarrow \neg p)$

* “$p$ holds at $a$” as $\Box(a \rightarrow p)$, “$a$ is earlier than $b$” as $\Box(b \rightarrow Pa)$

* need for a modality $\Diamond$ (“somewhere in the model”) and $\Box$ (“everywhere”)
* linear models: $p \lor Pp \lor Fp$; branching time: $p \lor Pp \lor Fp \lor PPp$
* generally, not definable (generated submodels!)
Semantics for the future tense I: Paying one’s gambling debts

* We assert future-tensed statements in the face of indeterminism

* Betting as a prime example: “The coin will show heads”

* If the sentence was true (or false) at the time of utterance, then the world must be deterministic, contrary to assumption (⇒ “logical determinism”)

* If the sentence was neither true nor false, then why should I pay my gambling debts? After all, neither I nor my opponent said something true.
Semantics for the future tense II: Branching time

* Metaphysical question about the nature of time

* *Descriptive metaphysics* (Strawson): Focus on *actual* conceptual scheme;
  * use natural language and the way we act as guidelines

⇒ Overwhelming support for clear distinction between open future/fixed past

* *(Revisionary metaphysics* might urge to revise our attitude (Spinoza, Russell))

* Formally, tree-like structure of time:

  * no backward branching: \( \forall x, y, z ((x \leq z \land y \leq z) \rightarrow (x \leq y \lor y < x)) \)

  * historical connection: \( \forall x, y \exists z (z \leq x \land z \leq y) \)
Semantics for the future tense III: Occam vs Peirce

* $F\phi$ for “it will be the case that $\phi$”, evaluate at moment $m$

* semantics for $F$ analogous to alethic modal logic: basic tense logic $K_t$
  * $m \models F\phi$ iff there is $m' > m$ s.t. $m' \models \phi$

* this semantic definition does not reflect our use of “it will be that”

* histories: maximal chains; $H(m)$ the set of histories through $m$
  * moving along histories backward and forward is unproblematic (linear order)

* Peircean: $m \models F\phi$ iff in every $h \in H(m)$ there is $m' > m$ s.t. $m' \models \phi$

* Occamist: relative to $h$: $m, h \models F\phi$ iff there is $m' > m, m' \in h$, s.t. $m', h \models \phi$

* Prior-Thomason semantics for $F$: Occamist
Semantics for the future tense IV: Stand-alone sentences

* Take Occamist approach seriously. Assertion problem:

* Context of utterance supplies moment of evaluation $m$
* Context of utterance does not supply history of evaluation $h$

$\Rightarrow$ Sentence $F\phi$ cannot be evaluated in given context, no truth value

* Solution: Later moment $m'$ singles out set of histories through $m$;
* at $m'$ the previous assertion will then be vindicated (or not)

* Assertions about the future share the pragmatics of betting
Recap: branching time

* Tree-like partial ordering of moments \( \langle T, < \rangle \)

* no backward branching: \( \forall x, y, z ((x \leq z \land y \leq z) \rightarrow (x \leq y \lor y < x)) \)

* historical connection: \( \forall x, y \exists z (z \leq x \land z \leq y) \)

* histories \( h \): maximal linear subsets of \( T \)

* historical modalities quantifying over \( h \): \textit{Poss} (possible) and \textit{Sett} (settled)

* undividedness at \( m \) (for \( h, h' \in H_m \), \( m \) not maximal):

\[
h \equiv_m h' \text{ iff there is } m' \in h \cap h' \text{ s.t. } m < m'
\]

* \( \equiv_m \) an equivalence relation; partition \( \Pi_m \): \textit{elementary possibilities} at \( m \)
Agents and choices in a branching framework

* partition $\Pi_m$ describes nature’s indeterminism

* $m$ is an indeterministic point iff $\Pi_m$ has more than one element

* descriptive metaphysics: sometimes we are in control of nature’s indeterminism

* formally: $Agents$ a set of labels for agents

* for $\alpha \in Agents$; partition $Choice^\alpha_m$ describes $\alpha$’s choices at $m$

* $Choice^\alpha_m(h) \subseteq H(m)$; “=” means that $\alpha$ has no choice

* no choice between undivided histories:

$$ (h' \in Choice^\alpha_m(h) \land h'' \equiv_m h') \rightarrow h'' \in Choice^\alpha_m(h) $$
Multiple agents and independent choices

* agents’ choices at $m$ are simultaneous, so should be independent

$\Rightarrow$ for any function $f_m$ that maps $Agents$ to elements of $Choice_m^\alpha$,

$$\bigcap_{\alpha \in Agent} f_m(\alpha) \neq \emptyset$$

* strong constraint on $Choice_m$

* implausible if, e.g., two agents can manipulate the same object

* spatial separation as a precondition for independence

* branching time not a theory of space

$\Rightarrow$ need to use branching space-times as a formal basis for agency
Seeing to it that I: Stit normal form

* many natural language expressions are *agentive* for some $\alpha$; contrast

(4) Ishmael sailed over the seven seas (agentive)

(5) Ishmael sailed over the side of the *Pequod* (not agentive)

* some operators need agentive complements, e.g., imperatives, deontic notions

* normal form for agentives: $\alpha$ sees to it that $\phi$ ([$\alpha$*stit* : $\phi$])

* thesis: $\phi$ is agentive for $\alpha$ iff it can be paraphrased as “$\alpha$ sees to it that $\phi$”

* *stit* as a family of agent-indexed modal operators; allow nesting
Seeing to it that II: Semantics

* various stit operators in the literature

* consider \( dstit \), the “deliberative stit”: current choice secures outcome

* two conditions: (i) positive: secure outcome, (ii) negative: non-trivial

\[
m, h \models \alpha dstit : \phi \text{ iff }
\]

(i) for all \( h' \in \text{Choice}_m(h) \), we have \( m, h' \models \phi \)

(ii) there is \( h'' \in H(m) \) for which \( m, h'' \not\models \phi \)

* nobody sees to it that \( 2 + 2 = 4 \)

* usually, \( \phi \) will be of the form \( F \psi \) for contingent \( \psi \)
Seeing to it that III: Refraining

* refraining both an action (refrainings are attributed to agents; one can be prai-sed or blamed for refrainings) and a non-action (after all, refraining means *not acting*)

* negated *stit* is inappropriate

* von Wright: refraining = ability plus negation of action

\[ [\alpha \text{ref} : \phi] \text{ as } \neg[\alpha \text{stit} : \phi] \land \text{Poss} : [\alpha \text{stit} : \phi] \]

* for *dstit*, equivalent to nested *stit*:

\[ [\alpha \text{ref} : \phi] \text{ as } [\alpha \text{stit} : \neg[\alpha \text{stit} : \phi]] \]

* refraining from refraining equivalent to acting
Literature


Per Hasle and Peter Øhrstrøm, *WWW-site for Prior studies*. URL= http://www.prior.aau.dk/


