The Logic of Interrogation
Classical Version*

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1 Logic and Conversation

On the standard view, logic is concerned with reasoning, more in particular with fixing criteria for the soundness and validity of arguments. If we apply standard logic in natural language semantics, we inherit this basic trait, and can only expect our logical semantics to have descriptive and explanatory value for the kind of linguistic phenomena that are related closely enough to what the logic is about.

Reasoning is just one particular language game. And if we think of our daily conversations, it does not have the same central position it has in logic. Cooperative information exchange seems a more prevailing linguistic activity. It is reasonable to assume that such a predominant function has a distinctive influence on the structure of natural language, which forms the subject matter of linguistics. For example, it is a widespread (and age-old) idea, that the organization of discourse is largely determined by a mostly implicit process of raising and resolving issues, and that even sentential structure, including the intonational contour of utterances, can only be properly understood, if we take that to heart. If there is some truth in this, then linguistic semanticists should be worried by the fact that by and large they base themselves on a logical paradigm that is biased to such an extent towards reasoning rather than exchange of information.

As a response to this fear, one might point out that Gricean pragmatics is as much part of an overall theory of meaning, as logical semantics is. And the Cooperation Principle, which is at the heart of it, precisely is a principle of rationality which governs information exchange. Grice proposed to use it in the explanation of linguistic phenomena that lie beyond the reach of logical semantics as such. Among other things, he employed the principle in a defense of standard

logic—in particular the truth functional analysis of the logical connectives—
against the allegation that it leaves important aspects of meaning unaccounted
for. He argued that standard logic together with the general assumption that
we follow the Cooperation Principle does provide us with the means to account
for such additional features of meaning. Hence, we are in no need to replace the
standard logical analysis by some other type of interpretation, we only have to
combine logical semantics with general pragmatic strategies to cover the relevant
facts.

One way to look at the logical investigations carried out in the present
paper, is to view them as an attempt to turn the Cooperation Principle as such
into the key notion of logical semantics. Instead of centering the logic around the
explication of what makes a piece of reasoning into a sound and valid argumenta-
tion, we intend the logic to judge whether a conversation proceeds in accordance
with the principles of cooperative information exchange.

2 The Game of Logic

In logic we use a simple picture of an argument. An argument is conceived of as
a sequence of sentences, of which all but the last one are called the premises, and
the last sentence is called the conclusion of the argument. One can look upon an
argument as the proceedings of a language game. If the game is played according
to the rules, then the truth of the premises guarantees the truth of the conclusion.
If such is the game of logic, then the logical notion of validity arbitrates whether
the game was played according to the rules.

Argumentation is just one particular language game. For one thing, al-
though there may be spectators, it is a solitary game, whence we can leave the
player out of the logical picture. The more typical case, at least from a linguistic
perspective, are dialogue games, which involve exchange of information among
two or more participants. If we generalize the picture of the game of argumenta-
tion sketched above, then we arrive at the following.

A discourse is a sequence of utterances, the proceedings of a particular
language game. The task of a logical analysis consists in providing us with logical
notions which enable us to arbitrate the game, to characterize an utterance as a
pertinent or impertinent move in the game.

In this paper, we study a simple dialogue game from this perspective:

**Definition 1 (The Game of Interrogation)** Interrogation is a game for two
players: the *interrogator* and the *witness*. The rules of the game are as follows:

A. The interrogator may only raise issues by asking the witness non-superfluous
questions.

B. The witness may only make credible non-redundant statements which ex-
clusively address the issues raised by the interrogator.
The game of interrogation is a logical idealization of the process of cooperative information exchange, which makes stiff demands on the witness. The elements of the rules can be linked to elements of the Gricean Cooperation Principle: The requirement that the witness makes credible statements is related to the Maxim of Quality; that the statements of the witness should be non-redundant, and the questions of the interrogator non-superfluous, relates to the Maxim of Quantity; and that the witness should exclusively address the issues raised by the interrogator is a formulation of the Maxim of Relation.

From a linguistic perspective, our interest does not lie in the game as such. The empirical success of the logic of interrogation depends on whether it can be used in the explication of structural linguistic facts. We will give an illustration of that in Section 11 of the paper.

3 The Tools of Interrogation

Relative to a suitable language, and a semantic interpretation for that language, the logic of interrogation has to provide us with logical notions by means of which we can arbitrate the game. As a language for the game of interrogation, we use a simple query-language, a language of first order predicate logic enriched with simplex interrogatives:¹

**Definition 2 (Query-Language)** Let PL be a language of predicate logic. The Query Language QL is the smallest set such that:

i. If $\phi \in PL$, then $\phi \in QL$;

ii. If $\phi \in PL$, $\bar{x}$ a sequence of $n$ variables ($0 \leq n$), then $?x\phi \in QL$.

In case the query-operator binds no variables, prefixing it to an indicative results in a yes/no-question. E.g., $?\exists xPx$ asks whether there is an object which has the property $P$. If the query-operator binds a single variable, a single who-question results. E.g., $?xPx$ asks which objects have the property $P$. When two variables are bound, as in $?xyRx$, we get a question asking for the denotation of a two-place relation, it asks for a specification of which pairs of objects stand in the relation $R$. So, in general, we interpret an interrogative $?x_1 \ldots x_n \phi$ as asking for a specification of the actual denotation of an $n$-place relation.

We call the formulae of PL the *indicatives*, and the other formulae in QL the *interrogatives* of the language. We use $\phi, \psi$, etc., as meta-variables which range over all sentences. Adding an exclamation point, as in $\phi!$, restricts the range to the indicatives, and adding an interrogation point, as in $\phi?$, to the interrogatives of the language. We refer to a sequence of sentences $\phi_1; \ldots; \phi_n$ as (the proceedings of) an *interrogation*, and use $\tau$ to range over such (possibly empty) sequences.

¹ For more discussion about the language and its interpretation, see Groenendijk & Stokhof (1996), in particular Section 4.
It is a convenient feature of the game of interrogation, that given the strict casting, we do not have to indicate who said what: interrogatives are uttered by the interrogator, indicatives by the witness. If the players were allowed to change roles, the proceedings of the game should include an indication of the source of each utterance.

4 Partitioning Logical Space

We state the semantics of the language in two steps. As our point of departure, we take a standard denotational semantics, and on top of that we define a notion of interpretation in terms of context change potentials.

As for the indicative part of the language, we assume a standard truth definition: \( \| \phi \|_{w,g} \in \{0,1\} \), where \( w \) is an element of the set of possible worlds \( W \) (first order models), and \( g \) an assignment of an element of the domain \( D \) to the individual variables. We assume a single domain for all worlds. Furthermore, we assume that the individual constants (names) of the language are interpreted as rigid designators.\(^2\)

For the interrogatives in the language, we employ a partition-semantics. We take the denotation of an interrogative in a world to be the set of worlds where the answers to the question are the same:\(^3\)

**Definition 3 (Semantics of Questions)**

\[ \| ? \bar{\varepsilon} \phi \|_{w,g} = \{ v \in W \mid \forall \bar{\varepsilon} \in D^n: \| \phi \|_{v,g[\bar{\varepsilon}/\bar{\varepsilon}]} = \| \phi \|_{w,g[\bar{\varepsilon}/\bar{\varepsilon}]} \} \]

Whereas an indicative \( \phi! \) selects a subset of the set of worlds: the worlds where \( \phi! \) is true, an interrogative \( \phi? \) divides the set of worlds into a number of (mutually exclusive) alternatives. For example, the question \( ? \exists x P x \) divides the set of worlds into two alternatives: the alternative consisting of the worlds where some object has the property \( P \), and the alternative consisting of the worlds where there is no such object in the domain. The question \( ?x P x \) divides the set of worlds in as many alternatives as there are possible denotations of the property \( P \). And the question \( ?x y R x y \) divides the set of worlds in as many alternatives as there are possible denotations of the relation \( R \).

The meaning of an interrogative corresponds to a partition of the set of possible worlds \( W \). Hence, it also corresponds to an equivalence relation on \( W \). It is the latter way of modeling a question that we will employ in formulating the context change potential of interrogatives.

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\(^2\) These are not very natural assumptions to make in an epistemic setting. See Aloni (1999) for a discussion of the issue, and an analysis which makes it possible to lift these assumptions.

\(^3\) See Groenendijk & Stokhof (1984,1996) for extensive discussion of the partition semantics for interrogatives.
5 Structuring the Context

In general, a semantics for a language in terms of context change potentials states the interpretation of a sentence as an operation on contexts. Hence, in order to formulate such a semantics for a particular language, we have to decide on a suitable notion of context.

Our query-language consists of two different types of sentences, with different functions, and different effects on the context. The function of indicatives is to provide data, the function of interrogatives is to raise issues. So, we could look upon a context as consisting of two elements: data and issues.4

We can model contextual data as a set of worlds, those worlds which are compatible with the data provided by the preceding discourse. Then, in general, the context change potential of an indicative will be to eliminate possible worlds.

We can model contextual issues as an equivalence relation on the set of possible worlds. If two worlds are non-related, i.e., if they belong to different contextual alternatives, then it is a contextual issue whether the actual world is like the one or like the other. The differences between related worlds, i.e., worlds which belong to the same alternative, is not a contextual issue.

Since interrogatives raise issues, their context change potential is to disconnect certain worlds, creating new or more fine-grained contextual alternatives. The context change potential of an interrogative consists in eliminating pairs of worlds — without eliminating the worlds themselves from the data: interrogatives do not provide data, they only raise issues.

Instead of splitting the context into two separate elements, a subset of the set of worlds representing the data, and an equivalence relation on the set of worlds representing the issues, we combine the two in modeling a context as an equivalence relation on a subset of the set of possible worlds. Or, equivalently.5

Definition 4 (Structured Contexts)

A context $C$ is a symmetric and transitive relation on the set of possible worlds $W$.

Two worlds are contextually related iff they both belong to the divided subset and to the same alternative. A world $w$ belongs to the divided subset iff $\langle w, w \rangle \in C$, which by abuse of notation, we write as $w \in C$. The set of contexts is partially ordered by $\subseteq$. The minimal context is $W^2$, the initial context of ignorance and indifference, where no data have been provided, and no issue has been raised. The

4. The terminology is taken from Hulstijn (1997), who defines an update semantics for questions in a similar way.

5. What is used here as the notion of context, a symmetric and transitive relation on the set of possible worlds, could also be taken as a notion of semantic content, replacing the usual notion of a proposition as a set (property) of possible worlds. The content of any sentence can then be taken to consist of a (possibly empty) assertive part, and a (possibly empty) interrogative part. The content of a sentence can be a mix of asserting/presupposing data and raising/supposing issues.
absurd context, $\emptyset$, results if the contextual data are inconsistent. An indifferent context is a context such that $\forall w, v \in C: \langle w, v \rangle \in C$, a context where all worlds in the data are related, i.e., a context where there are no (unresolved) issues.

6 Changing the Context

In defining the context change potentials of the formulae of our query-language, we restrict ourselves to the sentences, the closed formulae of $QL$. The definition uniformly interprets indicatives and interrogatives as functions from contexts to contexts, but they have a different kind of effect on the context:

**Definition 5 (Context Change Potentials)**

i. $C[\phi!] = \{ \langle w, v \rangle \in C \mid \|\phi!\|_w = \|\phi!\|_v = 1 \}$;

ii. $C[\phi?] = \{ \langle w, v \rangle \in C \mid \|\phi?\|_w = \|\phi?\|_v \}$;

iii. For $\tau = \phi_1; \ldots; \phi_n$, $C[\tau] = C[\phi_1] \ldots [\phi_n]$.

An indicative $\phi!$ eliminates a pair of worlds from the context as soon as $\phi!$ is false in one of the worlds of the pair. In effect, this means eliminating worlds from the contextual data. An interrogative $\phi?$ eliminates a pair of worlds (disconnects two worlds) if they belong to different alternatives, i.e., if the two worlds differ in such a way that the question would receive a different answer in them. Interpreting an interrogation, a sequence of a mix of interrogatives and indicatives, is just interpreting the sentences in the sequence one by one.

It can easily be checked that all context change potentials in the language have the classical update property: $\forall C, \phi : C[\phi] \subseteq C$. Further we note:

**Fact 1 (Indicatives and Interrogatives)**

a. $\forall C, w, v: \langle w, v \rangle \in C \& w, v \in C[\phi!] \Rightarrow \langle w, v \rangle \in C[\phi!]$.

b. $\forall C, w: w \in C \Rightarrow w \in C[\phi?]$.

Fact 1b says that interrogatives cannot eliminate worlds from the data, they can only eliminate pairs of worlds, i.e. disconnect worlds, leaving both of them in the data as such. Fact 1a says that indicatives cannot disconnect worlds; if two worlds are connected in the data, then if both remain in the data, they remain connected.

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6. This is why in the title of the paper it says: Classical Version. Originally, the logic of interrogations presented here was designed in a non-classical, dynamic setting, which lacks the classical update property. The richer system, also allowing for anaphoric relations across utterances, will be discussed in another paper. See also Groenendijk (1998).

7. This fact about the complete division of labor between indicatives and interrogatives is specific for the language at hand, and not a necessary feature. Mixed cases of sentences which both provide/presuppose data and issues can be accommodated without difficulty.
Now that we have specified the logical language and its semantics, we turn to a specification of the logical notions by means of which we can arbitrate whether an interrogation is played according to the rules of the game.

7 Consistency and Entailment

One of the elements of the rules of the game of interrogation, the Maxim of Quality, is that the witness may only make credible statements. From a minimal, purely logical perspective, giving the witness every benefit of the doubt, her statements can be judged credible as long as she does not contradict herself. This requirement is covered by the logical notion of contextual consistency:

Definition 6 (Consistency) \( \phi \) is consistent with \( \tau \) iff \( \exists C : C[\tau][\phi] \neq \emptyset \).

A sentence \( \phi \) is consistent with a preceding sequence \( \tau \), if there is at least some context \( C \) such that after an update of \( C \) with \( \tau \), a further update with \( \phi \) does not lead to absurdity.

Since interrogatives do not eliminate worlds from the data, but can at most disconnect worlds in the data (Fact 1b), as long as the context is not absurd, an interrogative will always be consistent with it. Hence, the Quality Maxim cannot fail to be obeyed by the interrogator, only the witness may fail to do so.

Two other elements of the rules, both instances of the Maxim of Quantity, are that the witness may only make non-redundant statements, and that the interrogator may only ask non-superfluous questions. From a minimal, purely logical perspective, a statement is redundant, and a question superfluous, in case it is already entailed by the preceding context:

Definition 7 (Entailment) \( \tau \models \phi \) iff \( \forall C : C[\tau] = C[\tau][\phi] \).

A sentence \( \phi \) is entailed by a preceding sequence \( \tau \), if after an update of a context \( C \) with \( \tau \), a further update with \( \phi \) will never make a difference.

Contrary to what is the case in the game of reasoning, entailment is a vice rather than a virtue in the game of interrogation. Although defined in a uniform way, non-entailment means something different for indicatives and interrogatives:

Fact 2 (Informativeness and Inquisitiveness)

a. \( \tau \not\models \phi \) iff \( \exists C, w : w \in C[\tau] \land w \notin C[\tau][\phi] \).

b. \( \tau \not\models \phi? \) iff \( \exists C, w, v : \langle w, v \rangle \in C[\tau] \land w, v \in C[\tau][\phi?] \land \langle w, v \rangle \notin C[\tau][\phi?] \).

Indicatives, and only indicatives, can be informative, which means that at least in some context, some world is eliminated. Interrogatives, and only interrogatives,
can be *inquisitive*, which means that at least in some context, some pair of worlds is disconnected.

The notions of consistency and entailment are standard logical notions. New is at most that they indiscriminately apply to statements and questions, and that we focus on the use of these notions in the formulation of Quality and Quantity requirements for the cooperative exchange of information, instead of as criteria for the soundness and validity of reasoning.

In fact, the latter would only make sense for the indicative part of the language. Which is not to say that, e.g., \( \phi \models \psi \), or \( \phi! \models \psi \), makes no sense. The latter means that \( \psi \) is a superfluous question to ask after having been told that \( \phi! \), i.e., that \( \phi! \) has already completely resolved the issue raised by \( \psi \). It is not unusual to read this as: \( \phi! \) *gives a complete answer to* \( \psi \), which is only a bit unnatural given that in \( \phi! \models \psi \), the answer precedes the question. However, when read in the other direction, \( \psi \models \phi! \), the entailment only holds in case \( \models \phi! \), which is only logical, given that questions provide no data. What \( \phi! \models \psi \) means is that the question \( \psi \) is superfluous after \( \phi! \) has already been asked, which is the case if whenever the issue raised by \( \phi! \) is resolved, the issue raised by \( \psi \) cannot fail to have been resolved as well.

Although the familiar notions of contextual consistency and entailment have a minor role to play in the logic of interrogation as minimal requirements on the sensibility of utterances as moves in a game of information exchange, we have not yet touched upon the more central aspect, which is that information provided by the witness should be relevant to the issues that have been raised by the interrogator. We turn to that heart of the matter now.

### 8 Licensing and Pertinence

The last element of the rules, the *Maxim of Relation*, is that the statements of the witness should *exclusively address the issues* raised by the interrogator. This requirement is covered by the new logical notion of licensing:

**Definition 8 (Licensing)**

\( \tau \) licenses \( \phi \) iff \( \forall C, w, v : \langle w, v \rangle \in C[\tau] \ \& \ w \notin C[\tau][\phi] \Rightarrow v \notin C[\tau][\phi] \).

A sentence is contextually licensed if whenever a world is eliminated from the data, all worlds related to it are eliminated as well, i.e., the whole alternative to which the world belongs is eliminated. Licensing forbids the elimination of some world in some alternative, leaving some other world from the same alternative in the data. In eliminating some world, a sentence would be informative, but if it does not eliminate a whole alternative at the same time, the information provided does not exclusively address the contextual issues. The sentence would provide
irrelevant information, information not directly related to the contextual issues.\textsuperscript{8} Note that since interrogatives never eliminate any world from the data, they are trivially licensed. As was the case with consistency, licensing only puts constraints on the statements of the witness, but reckons any question from the interrogator to be relevant.\textsuperscript{9} Note also that if an indicative $\phi$ is inconsistent with $\tau$ or is entailed by $\tau$, then $\phi$ is trivially licensed by $\tau$.

Consistency and non-entailment are added to the requirement of licensing in the over-all notion of pertinence, the logical notion which arbitrates whether an interrogation is played according to the rules:

\textbf{Definition 9 (Pertinence)} \ $\phi$ is pertinent after $\tau$ iff

\begin{itemize}
  \item[i.] $\phi$ is consistent with $\tau$ (\textit{Quality})
  \item[ii.] $\phi$ is not entailed by $\tau$ (\textit{Quantity})
  \item[iii.] $\phi$ is licensed after $\tau$ (\textit{Relation})
\end{itemize}

As indicated, the three elements of logical pertinence can be related to the Gricean Conversational Maxims (leaving \textit{Manner} out of consideration) which constitute the Cooperation Principle. But whereas the Gricean notions are usually thought of as belonging to a level of pragmatics which comes on top of logical semantics, here they make up the logic as such. In the logic of interrogation the notion of pertinence plays the same methodological role as the notion of entailment normally does. Whereas the latter arbitrates the game of argumentation, the former arbitrates the game of interrogation.

\section{Putting Licensing to the Test}

Intuitively, a good criterion for logical relatedness of a sentence $\phi$ to the contextual issues is the following: If $\phi$ gives any information in the context at all, then $\phi$ at least partially resolves the contextual issues. The latter is the case if at least one of the contextual alternatives is eliminated.\textsuperscript{10} The notion of licensing meets this criterion:

\begin{itemize}
  \item[8.] In Jäger (1996), a similar relevance notion can be found, but baked right into the semantics as such, and not as a logical notion which comes on top of the semantics to arbitrate appropriateness.
  \item[9.] This is a feature particular to the present set-up. One could add requirements of relatedness for the questions of the interrogator as well.
  \item[10.] The notion of resolution, defined as eliminating at least one alternative, is the usual notion of a giving a partial answer in a partition semantics for questions, next to the notion of a giving a complete answer, defined as $\phi \models \psi$?.
\end{itemize}
Fact 3 (Adequacy Test) \( \tau \) licenses \( \phi \) iff for all contexts \( C \):
if \( \exists w: w \in C[\tau] \& w \notin C[\tau][\phi] \)
(\emph{if \( \phi \) is informative in \( C[\tau] \)},
then \( \exists w \in C[\tau]: \forall v: \langle w, v \rangle \in C[\tau] \Rightarrow v \notin C[\tau][\phi] \)
(\emph{then \( \phi \) is resolvent in \( C[\tau] \)}).

This says that \( \tau \) licenses \( \phi \) is materially the same as: for any context \( C \), if \( \phi \) is informative in \( C \) after \( \tau \), then \( \phi \) is resolvent in \( C \) after \( \tau \). I.e., as soon as \( \phi \) eliminates a world from the data, \( \phi \) cannot fail to eliminate a contextual alternative.

At first sight, this property may seem weaker than licensing. Relative to a particular context, a sentence \( \phi \) can be informative and resolvent, in case next to eliminating some whole alternative, \( \phi \) also eliminates some world in some other alternative without eliminating that alternative as a whole. However, if that were the case, then there would also be some other context where \( \phi \) is informative, but not resolvent. It is by quantifying over all contexts, that being resolvent when informative, amounts to the same as licensing.\(^{11}\)

That logical relatedness requires addressing contextual issues, is most clearly indicated by the fact that an indicative \( \phi \) is licensed iff the corresponding \textit{yes/no}-question \( ?\phi \) is contextually non-inquisitive:

Fact 4 (Relatedness Test) Let \( \phi \) be an indicative. \( \tau \) licenses \( \phi \) iff \( \tau \models ?\phi \).

We refer to this fact as the Relatedness Test, because it gives a way of judging whether an indicative utterance is related to the contextual issues. If when \( \phi \) is uttered, the corresponding question whether \( ?\phi \) is inquisitive, this means that the question is new, and not already present. Hence, the utterance is not licensed by the issues that have already been raised (and are not yet resolved) in the context.

Pertinence is a notion of contextual appropriateness, where the latter is usually taken to relate to presuppositions. Pertinence is a presuppositional notion:

Fact 5 (Presupposition Test) \( \neg \phi \) is pertinent after \( \tau \) iff \( \phi \) is pertinent after \( \tau \).

Putting the last two facts together, we can say that under the notion of pertinence, an indicative sentence presupposes the corresponding \textit{yes/no}-question, in the sense that it should be non-inquisitive in the context, i.e., it should be a contextual issue.

\(^{11}\) There is no space to go into this here, but there is also an important difference between the notion of licensing and the notion of being resolvent when informative. Unlike the latter notion, licensing is \textit{grounded}. By this we mean that being licensed is the same as being licensed in the initial context of ignorance and indifference, updated with whatever went on in the discourse. The notions of consistency and non-entailment are grounded as well, which means that pertinence is also a grounded notion. So, in calculating pertinence one only has to reckon with one single minimal context. Whatever counts as appropriate there, is appropriate \textit{per se}.\)

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In Section 11 we shall see, that taking the intonation contour of sentences into account, indicative sentences may also presuppose stronger who-questions.

10 Pertinent Answers

The new notion of licensing also gives rise to a new logical notion of an answer. An answer can be characterized as the special case of an indicative being licensed in the context of a single interrogative:

**Definition 10 (Answers)** \( \phi! \) is an answer to \( \psi \) iff \( \phi! \) is licensed by \( \psi \).

In Section 7, we noted that inconsistency and entailment imply relatedness. Hence, tautologies and contradictions are borderline cases of trivial and absurd answers. Apart from absurd and trivial answers, which answer any question, there are two (non-equivalent) answers to yes/no-questions:

**Fact 6 (Yes/No)** \( \phi! \) is an answer to \( ?\psi \) iff \( \models \phi \) or \( \models \lnot \phi \) or \( \phi \iff \psi \) or \( \phi \iff \lnot \psi \).

Adding Quality and Quantity to the requirement of Relation, we arrive at the more informed notion of pertinent answers:

**Definition 11 (Pertinent Answers)**

\( \phi! \) is a pertinent answer to \( \psi \)? iff \( \phi! \) is pertinent after \( \psi \).

Being a pertinent answer just excludes absurd and trivial answers:

**Fact 7 (Pertinency and Contingency)**

\( \phi \) is a pertinent answer to \( \psi \)? iff \( \phi \) is an answer to \( \psi \)? & \( \models \lnot \phi \) & \( \models \lnot \lnot \phi \).

Only non-trivial questions (\( \models \lnot \psi \)) have pertinent answers, and only equivalents of yes and no, are pertinent answers to non-trivial yes/no-questions. As for single who-questions, such as \( ?x \; Px \), an atomic sentence like \( Pa \) is a (pertinent) answer.\(^{12}\)

**Fact 8 (Literal Answers)** \( [\vec{c}/\vec{x}] \phi \) is an answer to \( ?\vec{x} \; \phi \).

\(^{12}\) This feature makes it possible to link the logically elegant partition view of questions with a notion of answers that meets linguistic intuitions. In Groenendijk & Stokhof (1984) and elsewhere, we argued at length on logical grounds against Hamblin’s and Karttunen’s semantic analyses of questions. Nevertheless, almost without exception, linguistic semanticists fall back on these analyses, because they dislike the notion of exhaustive answers that seems to be baked into the partition view. Under the present notion of an answer, linguists can have their cake and eat it.
Given the presuppositional nature of licensing and pertinence, answerhood is preserved under negation:

**Fact 9 (Negative Answers)**

\( \phi \) is a (pertinent) answer to \( \psi \) iff \( \neg \phi \) is a (pertinent) answer to \( \psi \).

Sentences which only state something about the cardinality of the set of objects that have the property \( P \), are also answers to the question \( ?x \ Px \). For example, \( \exists x \ Px \) and \( \forall x \ Px \) are (pertinent) answers to \( ?x \ Px \).

The notion of an answer defined in terms of licensing differs from the standard notion of an answer in a partition theory of questions, which, as we mentioned in Section 7, is formulated as \( \phi ! \models \psi \). The standard notion is both less and more demanding than the one defined here in terms of licensing.

The standard notion of an answer is less demanding in that it allows for *over-informative answers*, whereas the notion of an answer in terms of licensing typically does not. Under the standard notion, if \( \phi \) counts as an answer to \( \psi \), then for arbitrary \( \chi \), also \( \phi \land \chi \) counts as an answer to \( \psi \). Under the present notion, it does so only if \( \chi \) as such, is also an answer to \( \psi \):

**Fact 10 (Conjoined Answers)**

If \( \phi \) is an answer to \( \psi \), and \( \chi \) is an answer to \( \psi \), then \( \phi \land \chi \) is an answer to \( \psi \).

Given that answerhood is also preserved under negation, other logical operations which can be defined in terms of negation and conjunction, like disjunction, also preserve answerhood.

The standard notion of an answer is more demanding in that it is a notion of *exhaustive* answering. E.g., whereas under the present notion \( Pa \land Pb \) counts as a (pertinent) answer to \( ?x \ Px \), under the standard notion it does not. Only an explicitly exhaustive answer, like \( Pa \land Pb \land \neg \exists x (Px \land x \neq a \land x \neq b) \), is an answer under the standard notion. Under the notion defined here, the explicitly exhaustive answer can be characterized as a better, a more informative answer.\(^{13}\)

**Definition 12 (Comparing Answers)** Let \( \phi, \chi \) be pertinent answers to \( \psi \).

\( \phi \) is a more informative answer to \( \psi \) than \( \chi \) iff \( \phi \models \chi \land \chi \not\models \phi \).

In fact, the explicitly exhaustive answer counts as an *optimal answer* to the question, in the sense that there are no pertinent answers to \( ?x \ Px \) which are more informative. Note that: If \( \phi \) is an optimal answer to \( \psi \), then \( \phi \models \psi \).

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13. Precisely because the notion of licensing forbids over-informativeness, we obtain this easy way of comparing answers in terms of informativeness. Compare this with the much more intricate notions of comparing answers in Groenendijk & Stokhof (1984, 1996).
The focus of the present paper is not so much on the relation of answering as such, but rather on the more general issue of the role of the logical notions of licensing and pertinence in arbitrating the appropriateness of utterances from the perspective of cooperative information exchange. The following section is devoted to the discussion of some examples.

11 An Illustration. And Nothing Else

The examples given below are only intended as an illustration of, and partly as further motivation for, the logical notions introduced above, in particular the new notion of licensing. We make no claims to the effect that we present linguistic analyses, or provide alternative explanations as compared to other approaches.

11.1 Resolving an Ambiguity with an Issue

Consider the following example. Out of context, and without intonational information, (1a) is ambiguous between (1b) and (1c):

(1) a. Alf rescued Bea. And no-one else.
   b. Rab; $\neg \exists x(Rxb \land x \neq a)$
   c. Rab; $\neg \exists x(Rax \land x \neq b)$

However, after the interrogative in (2a), or with the intonational information indicated by underlining in (2a), the ambiguity in (1a) is resolved:

(2) a. (Who rescued Bea?) Alf rescued Bea. And no-one else.
   b. ?x Rxb; Rab; $\neg \exists x(Rxb \land x \neq a)$
   c. ?x Rxb; Rab; $\neg \exists x(Rax \land x \neq b)$

Only (2b) is a plausible interpretation for (2a), and not (2c). Alternatively, after the interrogative in (3a), or with the intonational information indicated by underlining in (3a), (3a) can only be interpreted as (3c), and not as (3b):

(3) a. (Whom did Alf rescue?) Alf rescued Bea. And no-one else.
   b. ?x Rax; Rab; $\neg \exists x(Rxb \land x \neq a)$
   c. ?x Rax; Rab; $\neg \exists x(Rax \land x \neq b)$

Our logic of interrogation accords with the difference between (2a) and (3a). Both the interrogations (2b) and (3c) are pertinent. The interrogatives ?x Rax and ?x Rxb are both inquisitive. And both the sequence of indicatives in (3b) and in (3c) are contingent. More importantly, Rab is licensed by (is an answer to) both ?x Rxb and ?x Rax. And $\neg \exists x(Rxb \land x \neq a)$ is licensed by ?x Rxb; Rab, just as $\neg \exists x(Rax \land x \neq b)$ is licensed by ?x Rax; Rab.

Given that ?x Rxb asks for the specification of the (whole) denotation of the property $\lambda x Rxb$, the answer that $a$ has that property may leave the interrogator with the question whether anyone else does. And this is precisely the issue.

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14. English is not the perfect language for this type of example, because of the easy availability of do-support. Lacking do-support, Dutch would be better.
that \( \neg \exists x (RxB \Land x \neq a) \) addresses. We can also inspect this by performing the Relatedness Test: the yes/no-question \(?x (RxB \Land x \neq a)\) is non-inquisitive after \( ?x RxB; Rxa \). Hence, \( \neg \exists x (RxB \Land x \neq a) \) is contextually licensed, it is an issue the witness is entitled to address.

But not the other way around: the sequences in (2c) and (3b) are impertinent. The sentence \( \neg \exists x (Rax \Land x \neq b) \) is not licensed by \( ?x RxB; Rab \). And the sentence \( \neg \exists x (Rxb \Land x \neq a) \) is not licensed by \( ?x Rax; Rab \). That Alf rescued no-one else but Bea, can be informative in a state in which the question has been raised who rescued Bea, without being resolvent after the answer has been given that Alf rescued Bea, and hence, is not licensed by the context.

A simple counterexample against licensing, is the situation where the interrogator already knows that one and only one person rescued Bea. She wants to know who it was. After her question to that effect, and having been told by the witness that it was Alf, the state of the interrogator is a state of indifference. Still, that Alf rescued no-one else, can very well be informative in her state. Only she did not ask for that. That Alf rescued no-one else does not resolve a contextual issue. Such a counterexample shows that the last sentence of (2c) is not licensed by the context, which makes it impertinent.

Again, we can also put the Relatedness Test to work: the yes/no-question \(?x (Rax \Land x \neq b)\) is inquisitive after \( ?x RxB; Rab \). This means that \( \neg \exists x (Rax \Land x \neq b) \) is not licensed by the context. In the context of \( ?x RxB; Rab \), it addresses an issue which was not raised by that context.

What the discussion of these examples suggests is the following. One way of accounting for the resolution of the ambiguity in (1a), in the contexts (2a) and (3a), is that we cooperatively interpret (2a) and (3a) in such a way that our interpretation gives rise to a pertinent discourse, where each sentence is licensed by the preceding context. That is how we arrive at (2b) and (3c), and not at (2c) and (3b), as appropriate interpretations for (2a) and (3a).

### 11.2 Presupposing an Issue

If we swap the interrogatives in (2a) and (3a), leaving the intonational contour of the utterances the same, the resulting interrogations are not appropriate, e.g., compare (2a) with (4a):


b. \( ?x Rax; \Leftrightarrow ?x RxB \Rightarrow Rab; \neg \exists x (Rxb \Land x \neq a) \)

The intuition is that with the intonation contour as indicated in (4a), the first indicaticve simply does not fit the interrogative. It fits the interrogative we originally had in (2a), not this one in (4a). A natural conclusion to draw is that the intonation contour as such has some semantic impact, because otherwise, we are (semantically) out of business in explaining what is wrong with (4a).

Along not unusual lines, we might account for the unacceptability of (4a), in a presuppositional setting, by assuming that the intonation contour of the first
indicatives in (2a) and (3a), presuppose the issue raised by the interrogatives in (2a) and (3a). We can look upon the sequences in (2b-c) and (3b-c) as the result of presupposition accommodation. In (4b), I indicated that by fronting the first utterance of the witness, with the corresponding presupposed question between double angled brackets.\textsuperscript{15}

Now we are back in business. If anything may be assumed, then it is that, leaving accommodation aside, if a question is presupposed, it is to be non-inquisitive in the context. Just as, leaving accommodation aside, a presupposed indicative should be non-informative in the context. Then we are quickly ready with explaining what is wrong with (4a): \(?x Rxb\) is inquisitive after \(?x Rax\), the unacceptability of (4a) is due to presupposition failure.

A general feature of presuppositions is that they are preserved under negation. As we noted above, contextual relatedness is of a presuppositional nature. An utterance of an indicative \(\phi\) always presupposes the corresponding \textit{yes/no}-question. Returning to the type of examples we are discussing here, where we take intonational contour into consideration, if we think along these presuppositional lines, then we can represent (3a) out of context, but with the intonation contour as indicated in (5a), as (5b):

\[(5) \begin{align*}
  \text{a. } & \text{Alf rescued Bea. And no-one else.} \\
  \text{b. } & \ll \lnot \alpha Rxb \gg \lnot Rab; \lnot \exists x (Rxb \land x \neq a)
\end{align*}\]

Just concentrating on the first sentence, we see that as compared to the general presupposition of indicatives we just noted, that \(\phi\) presupposes the \textit{yes/no}-question \(?\phi\), the effect of the intonation contour in the first sentence of (5a), according to the representation in (5b), leads to a \textit{stronger} presupposed \textit{who}-question. The stronger presupposition is also preserved under negation:

\[(6) \begin{align*}
  \text{a. } & \text{Alf did not rescue Bea. And, also, no-one else.} \\
  \text{b. } & \ll \lnot \alpha Rxb \gg \lnot Rab; \lnot \exists x (Rxb \land x \neq a)
\end{align*}\]

Observe that if we consider the first sentence in (6a) with a neutral intonation contour, we get back the same kind of ambiguity we found in (1a), where the second reading is the only one which (7a) has:

\[(7) \begin{align*}
  \text{a. } & \text{Alf did not rescue Bea. And, also, no-one else.} \\
  \text{b. } & \ll \lnot \alpha Rax \gg \lnot Rab; \lnot \exists x (Rax \land x \neq b)
\end{align*}\]

Next to preservation under negation, the possibility to be cancelled is another characteristic feature of presuppositional phenomena. Compare (2a) with (8a):

\[(8) \begin{align*}
  \text{a. } & (\text{Who rescued Bea?}) \text{ Alf rescued Bea. And, actually, no-one else.} \\
  \text{b. } & ??(\text{Who rescued Bea?}) \text{ Alf rescued Bea. And he rescued no-one else.}
\end{align*}\]

\textsuperscript{15} This is only a bit of suggestive notation. The semantics presented in Section 6 does not take presuppositions into account. If it did, it would declare \(C[\ll \phi \gg \psi] = C[\psi]\), if \(C[\phi] = C\), else undefined. Note that indicative and interrogative presuppositions are uniformly dealt with in this way.
Unlike in (2a), in (8a) the ambiguity of (1a) turns up again. Actually, I tend to believe that for (8a) the reading in (2c), which was excluded for (2a), is more salient than the reading in (2b), the only acceptable reading of (2a). The word *actually* crucially seems to give rise to the availability of both readings. Apparently, the conversational effect of *actually*, is an indication of the fact that the issue at hand is being overruled.

Unlike in the artificial language game of interrogation, in real discourse we may invent the issues we want to address ourselves. As (8a) shows, although we are not asked for that, we may provide the additional piece of information that rescuing Bea was Alf’s only heroic act. Does this get in the way of the role of our strict notion of relatedness in steering discourse, and determining its appropriateness? I don’t think so. The relevant observation is, that if one overrules relatedness to a contextually given issue, and addresses a new issue, as happens in (8a), then one explicitly marks one’s utterance for having this effect. If relatedness did not operate, there would be no need for that. So, my hypothesis is, that (8b) is not an appropriate sequence, that is, unless one way or the other, for example, by adding special intonation contour to the utterance (*Ahaand!...*), the utterance is marked for providing extra unsolicited information.

11.3 How Accommodating Can One Get?

The two sentence sequence in (9a) is just as alright as the three sentence sequence in (2a); and from the unavailability of the reading (2c) for (2a), we may expect that (10a) is hardly acceptable:

   b. $\exists x. R_{xb}; \ R_{ab} \land \neg \exists x (R_{xb} \land x \neq a)$

(10) a. ??Who rescued Bea? Alf rescued only Bea.
   b. $\exists x. R_{xb}; \ R_{ab} \land \neg \exists x (R_{ax} \land x \neq b)$

The following examples also give an illustration of that:

(11) Did Alf rescue Bea? Yes he did. And, in fact, he rescued only Bea.
(12) ??Did Alf rescue Bea? Alf rescued only Bea.

The last two sentences of (11), and the last sentence in (12), provide the same information. Still, the discourse in (11), where we first just resolve the issue raised by the interrogative, and then go on to provide some extra information that is not asked for as such, is alright. But if we make the answer as such over-informative, as in (12), by putting the extra information already in it, the acceptability of the resulting discourse is questionable.

Although the examples discussed above support the idea that the strict notion of contextual relatedness embodied in the notion of licensing is operative in a structural way, it is hard to believe that just being a bit over-informative is always punished so harshly. The following example is a case in point:
b.  ?∃x Rxb; Rab

The indicative in (13), is impertinent after the yes/no-question. Only ∃x Rxb and
¬∃x Rxb are pertinent in the context of the question ?∃x Rxb. The sentence Rab
properly entails ∃x Rxb, and hence counts as over-informative. However, intuitively,
the information that Rab is such a natural elaboration of ∃x Rxb, anticipat-
ing the further question: Who?, that it seems wrong to deem it impertinent
in the context. Rather than blaming her for being uncooperative, the witness
deserves praise for her accommodating attitude.

   Note, first of all, that the indicative in (13a) really needs the intonation
contour indicated in (14a):

b.  ?∃x Rxb; ≪?x Rxb ≫ Rab

In line with the observations made above, this means that the indicative presup-
poses the issue who rescued Bea, and should be represented as in (14b), and not
as in (13b). However, this does not yet explain why the sequence feels alright.
The issue ?x Rxb is not implied by ∃x Rxb, but rather the other way around:
?x Rxb ⊨ ∃x Rxb. The issue presupposed by the indicative in (14), is stronger
than the issue posed by the question, and hence is inquisitive in the context.

   Note, secondly, that although it is perhaps a more standard way to react
to the question, it seems not really obligatory to first say: Yes, as in (15a):

b.  ?∃x Rxb; ∃x Rxb; ≪?x Rxb ≫ Rab

If this were the case, we would arrive at (15b), and the present examples would fit
in with the observation made above, that providing extra information is allowed
only after the contextual issue has been resolved.

   However, if, as I assume, (14a) as such is fully appropriate, then, as it
stands, the logic of interrogation does not give us the means to account for this.
One way to approach the matter might be to add a notion of contextual rel-
latedness for questions, which explains why the issue presupposed by the last
utterance in (14a) is so closely related to the opening yes/no-question, that its
accommodation takes no effort.

   Another way to address this issue might be to interpret the effect of fo-
cussing in the indicative utterance in (14a) in such a way, that it involves exis-
tential quantification, and amounts to the same thing as we find in (15b). But
further investigations along these lines have to be left to another occasion.
12 Summary and Conclusion

In this paper, we investigated the prospects of basing logic on cooperative information exchange instead of valid reasoning. To this end, we introduced a simple dialogue game of interrogation. Relative to a minimal logical query-language suitable for the game, and a semantic interpretation for that language in terms of context change potentials, we defined a logical notion of pertinence, which enables us to arbitrate whether the game is played according to the rules. The elements of pertinence—contextual consistency, non-entailment, and licensing—were seen to correspond to elements of the Gricean Cooperation Principle. The main novelty is the notion of licensing, by which we can judge whether an utterance is logically related to the context. We illustrated the use of the logic of interrogation in natural language semantics by considering some linguistic examples, which exhibit phenomena which are inherently related to the communicative function of language.

We hope to have shown that a reorientation of logic towards raising and resolving issues is a feasible enterprise, which is interesting both from a logical and from a linguistic perspective. It leads to a new notion of meaning as cognitive content, which treats data and issues as equal citizens. In doing so, logical semantics invades the territory of pragmatics. Instead of viewing semantics and pragmatics as constituting two separate components within a theory of meaning, we make a move towards an integrated theory by shifting the logical perspective from valid argumentation to cooperative communication.

References


