Skolem Functions in Linguistics

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Technion/Utrecht University

Utrecht, 5 July 2008

Syntax, Semantics, and Discourse: the Theory of the Interface

Workshop in memory of Tanya Reinhart

Indefinites: existential quantifiers?

“The great majority of logicians who have dealt with this question were misled by grammar.”
(Russell 1919)

My understanding: “indefinite descriptions may behave as if they were ‘referential’ like proper names, but let syntax not confuse us gentlemen – their meaning is that of existential quantifiers”.

What’s wrong about existential quantification?

The Epsilon Calculus
(Hilbert 1920)

\[ \exists x \, [A(x)] \iff A(\varepsilon x. A(x)) \]

\[ \forall x \, [A(x)] \iff A(\varepsilon x. \neg A(x)) \]

Motivation: provide witness for every existential claim.
(Meyer-Viol 1995)

David Hilbert
(1862-1943)

Modern Natural Language Semantics

1970s-1980s: Quantifiers Everywhere

Syntax as a guide for theories of meaning:

All noun phrases denote generalized quantifiers

Montague (1973)

Richard Montague
(1930-1971)

Russell’s distinctions – left for philosophy of language
Hilbert’s concerns – left for proof theory
Modern Natural Language Semantics
1980s-1990s: A Dynamic Turn

Empirical problems for Montagovian uniformity:

Every farmer who owns a donkey beats it.
(Kamp 1981, Heim 1982)

If a friend of mine from Texas had died in the fire, I would have inherited a fortune.
(Fodor and Sag 1982, Farkas 1981)

Hilbert strikes back – perhaps indefinites are (discourse) “referential” after all?

What are Skolem Functions?

In the logical tradition:
Functions from (tuples of) $n$ entities to entities.

For example:

$f : \langle a, a \rangle \mapsto b \quad \langle a, b \rangle \mapsto a \quad \langle b, a \rangle \mapsto a \quad \langle b, b \rangle \mapsto b$

SF from pairs (2-tuples) over a simple domain with elements $a$ and $b$.

Early signs of SFs – branching
(historical observation by Schlenker 2006)

Henkin (1961): non-linear quantifier scope?

Branching quantifiers:
\[
\forall x \exists z \quad \forall y \exists u \quad \Phi(x,y,z,u)
\]
Henkin’s Semantics involves Skolem Functions (next slide).

Hintikka (1973): branching in natural language –

Some book by every author is referred to in some essay by every critic.

\[
[\forall x: author(x)] [\exists z: book-by(z,x)] [\forall y: critic(y)] [\exists u: essay-by(u,y)] \quad \text{referred-to-in}(z,u)
\]

Skolemization (higher-order Hilbertization)

Removing existential quantifiers from formulas in Predicate Calculus.

Example:

(1) Everyone gave everyone something.

$\Rightarrow$ For every two people $x$ and $y$ we can find a thing $f(x,y)$ that $x$ gave $y$.

The function $f$ is an Skolem Function of arity 2 that witnesses (1).
Skolemization (cont.)

Everyone gave everyone something.

\[(1) \quad \forall x \forall y \exists z[R(x, y, z)] \implies \forall x \forall y[R(x, y, f(x, y))]
\]

Suppose that \( R \) satisfies:

\[R(a, a, b) \land R(a, b, a) \land R(b, a, a) \land R(b, b, b)\]

Such an \( R \) satisfies (1) and with \( f \) they satisfy (2):

\[f : \langle a, a \rangle \mapsto b \quad \langle a, b \rangle \mapsto a \quad \langle b, a \rangle \mapsto a \quad \langle b, b \rangle \mapsto b\]

In linguistics: restricted quantifiers

Everyone gave everyone some present.

\[
\forall x \forall y [\exists z : A(z)][R(x, y, z)] \implies \forall x \forall y[R(x, y, f(x, y, A))]
\]

In the linguistic practice:

**Skolem Functions** are functions from \( n \)-tuples of entities and non-empty sets \( A \) to entities in \( A \).

When \( n=0 \) (no entity arguments) the function is a **choice function**: it chooses a fixed element from \( A \).

SF semantics for Hintikka’s examples?

(Henkin/Hintikka)

**Some book by every author is referred to in some essay by every critic.**

\[\exists f [\forall x:author(x) \land \forall y:critic(y)]
\]

\[referred-to-in(f(x, z, book-by(z, x)), g(y, u, essay-by(u, y)))\]

But the status of branching has remained undecided in the logical-linguistic literature:

- Doubts about evidence for branching (Fauconnier 1975, Beghelli et al. 1997)
- Intermediate positions (Schlenker 2006).

More signs of SFs – functional questions

(1) Which woman does every man love? His mother.

(2) Which woman does no man love? His mother-in-law.


\[\forall x [\text{man}(x) \rightarrow \text{love}(x, f(x, \text{woman}))]\]
Early 90s – the plot thickens
Choice functions derive the special scope properties of indefinites and \textit{wh}-in-situ:

“Quantification over choice functions is a crucial linguistic device and its precise formal properties should be studied in much greater depth than what I was able to do here.”

Reinhart (1992)

Hilbert strikes harder: CFs (SFs) as a general semantics for indefinites and \textit{wh}-elements.

Reinhart’s CF thesis
Exceptional scope of indefinites belongs in the semantics – neither (logical) syntax nor pragmatics (Fodor and Sag) are responsible.

If a friend of mine from Texas had died in the fire, I would have inherited a fortune.

Reinhart’s analysis, with DRT-style closure:
\[
\exists f \left[ CH(f) \land [\text{die}(f(\text{friend})) \rightarrow \text{fortune}] \right]
\]

Precursors semantic scope mechanisms:
Cooper (1975), Hendriks (1993)

Summary: short history of SFs in linguistics

– 1960s logico-philosophical foundations
1970s branching quantification
1980s functional questions
1990s – scope of indefinites, and more…

Caveat: more researchers have studied epsilon-terms and their possible relations to anaphora, predating current attempts – see Slater (1986), Egli (1991).

Mid 90s: new questions

- Formalizing CFs/SFs in linguistics
- CFs vs. general SFs
- Empirical consequences of attributing the scope of indefinites to semantics
- Functional pronouns
- General role of CFs/SFs within the DP: definites, numerals, anaphoric pronouns
Precise use of CFs/SFs

Empty set problem:
Some fortuneteller from Utrecht arrived.

\[ \exists f [CH(f) \land \text{arrive}(f) \text{(fortuneteller))] \]

Winter (1997):
\[ \exists f [CH_0(f) \land \text{arrive}(f) \text{(fortuneteller)]} \]

Montague-style

Do away with existential closure of CFs?
Kratzer (1998):
\[ \text{arrive}(f \text{(fortuneteller)}) \]

Hilbert / Fodor & Sag-style

CFs or general SFs?

The problem of “intermediate scope”:

(1) *Every professor will rejoice if a student of mine/his cheats on the exam.*

Is there a contrast in cases like (1)?
Fodor and Sag – Yes.
Wide agreement nowadays – No.
(Farkas, Abusch, Ruys, Reinhart, Chierchia)

Kratzer: Evidence for “referential” general SFs
Reinhart: Evidence for intermediate existential closure
Chierchia: Evidence for both

Advantages of “semantic scope”

Ruys’ problem of numeral indefinites:

(1) *If three workers in our staff have a baby soon we will have to face hard organizational problems.*

Winter (1997)

Double scope:
1- Existential scope – island insensitive
2- Distribution scope – island sensitive

Explained by CF semantic strategy.
On-going work on SFs in Linguistics

- **Indefinites/functional readings**
  (Winter 2004)

- **Branching and indefinites**
  (Schlenker 2006)

- **Donkey anaphora and SFs**
  Peregrin and von Heusinger 2004
  Elbourne 2005 → Brennan 2008

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### References


