Plurals

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1 Introduction

In English and many other languages, noun phrases are subcategorized for number: they are either singular or plural. Though strictly speaking a morpho-syntactic phenomenon, this subcategorization has important semantic correlates. Whereas singular noun phrases typically refer to atomic individuals or to quantifiers over such individuals, plural noun phrases typically involve reference to (or quantification over) *collections* of individuals. For instance, the sentence "*the trees surround the pond*" describes a relation between a collection "*the trees*" and an individual "*the pond*". Despite their importance in many languages, collectivity phenomena were largely ignored in the proposals that laid the foundations of formal semantics of natural language. Accommodating plurals and collectivity in formal semantics has turned out to be a major challenge.

The aim of this chapter is to give an overview of different approaches to these challenges and to summarize some of their achievements. We concentrate on plurals in English, but many principles in their treatment carry over to several other languages. After introducing in section 2 some central facts and terms, we move on in sections 3-5 to three problems that have propelled much of the research on plurals. One problem concerns the basic ('ontological') properties of the collections denoted by plural nominals. In section 3 we discuss mereological and set-theoretical approaches to collective reference, and concentrate on one central differences between different proposals: whether they treat collections as 'flat' sets of primitive entities, or as possibly 'nested' sets that recursively admit collections as their members. A second major problem is the nature of *distributive interpretations* of plurals: interpretations that involve quantification over parts of collections. In section 4 we distinguish two approaches for deriving distributive interpretations: the lexical approach, based on the meaning of predicates, and a variety of operational approaches, based on introducing phonologically covert operators in the semantic analysis. Finally, in section 5 we discuss the problem of collectivity with *quantificational plurals*. Again we will consider two central approaches: one approach that analyses quantificational expressions as modifiers of predicates, and another approach that analyses them as determiners, i.e. relations between predicates in generalized quantifier theory (chapter **[GQ]**).

Given the complexity of the semantic questions involving plurality, their relations with more general theoretical paradigms, and the variety of existing approaches, we will not aim at promoting specific solutions to any of the major problems we discuss. Rather, we wish to point out merits and limitations of different semantic approaches to plurals, and to hint at possible ways in which they may be profitably used in future research.

2 Basic facts and terminology

When English noun phrases occur as sentence subjects, plural forms are readily distinguished from singular forms by their agreement with number marking on the inflected verb:

(1) a. The girl was/*were smiling.
 b. No girl was/*were smiling.
 c. John was/*were smiling.

(2) a. The girls were/*was smiling.
 b. No girls were/*was smiling.
 c. John and Mary were/*was smiling.

Accordingly, we refer to the subjects in (1) and (2) as 'singular NPs' and 'plural NPs', or in short singulars and plurals.[1] Sentences with singular or plural subjects are referred to as 'singular sentences' or 'plural sentences', respectively. In examples like (1) and (2) we may consider proper names, definite NPs ("*the girl*") and their conjunctions as referential NPs. Other NPs, whose analysis necessarily involves quantification ("*no girls*", "*many girls and two boys*") are referred to as quantificational NPs. This distinction is not critical for all theories, but it is often convenient for the sake of presentation.[2]

As demonstrated by the examples above, two morpho-syntactic processes in English that give rise to plural NPs are plural marking on nouns and conjunction between nominal expressions. Plural marking occurs in many (though not all) languages of the world (Dryer, 2005; Haspelmath, 2005). English plural nominals can appear bare (as in "*dogs bark*", "*dogs bit me*") or with articles and determiners like *the*, *some*, *all*, *no*, *many*, *most*, simple numerals (*zero*, *two*, *three*), and modified numerals such as *at most five*, *less/more than five*, *almost five*, *exactly five*, *approximately five*, *between five and ten*. In addition, plural nouns appear in comparative constructions such as "*more...than...*" and "*twice as many...as...*", as well as in partitives like "*many of the...*", "*at least half of all...*", and "*three out of five...*" that allow determiners to be nested.

NP conjunction is also a cross-linguistically common phenomenon that may trigger plurality (Haspelmath, 2004, 2007). In English, conjunctive NPs may be formed by all singulars and plurals. Consider, for instance, the following examples.

(3)	*Mary and the boys*	*Mary and [John and Bill]*
	the girls and the boys	*the girls and [John and Bill]*
	an actor and an artist	*all actors and two artists*
	more than one actor and two artists	*all actors and Bill*

The conjunctive NPs in (3) are themselves plural, hence their conjunction can reiterate and derive more complex NPs.

Having observed two major morpho-syntactic processes of plurals, we are interested in their semantics. What do plurals refer to? More generally, what sort of quantification do they involve? In many cases plural NPs quantify over simple entities, as it is the case in "*more than six girls smiled*" or "*one boy and one girl smiled*". This is elegantly described in generalized quantifier theory (chapter **[GQ]**). But plurals can also quantify over collections,

[1] We use the term 'noun phrase' ('NP') as a traditional designation, ignoring questions on $\hat{\mathrm{X}}$-structure (Abney, 1987). chapter **[GQ]** follows Abney and others and assumes the determiner to be the head of the phrase, thus talking about 'determiner phrases' ('DPs').

[2] Montague (1973) treats referential NPs as denoting quantifiers: "*John*" intuitively corresponds to an entity **j** but denotes a quantifier in the technical sense. This is advantageous when treating plurals like "*John and every girl*", which conjoin a referential NP and a quantificational NP (Winter, 2001).

as in “*more than six girls gathered*” or “*one boy and one girl met*”. Problems of collectivity were ignored in many classical works on Montague Grammar and generalized quantifiers theory. However, since the 1980s problems of collectivity have given rise to a lively research area in formal semantics, known as the “theory of plurals”.

For some simple cases of collective reference, consider the following sentences.

(4) a. Mary and Sue met.
b. The girls met.
c. John shuffled the cards.

A speaker who utters a sentence as in (4a-c) conveys a statement about collections of individuals. Sentences (4a-b) attribute the property “*meet*” to the relevant collection of people. Similarly, (4c) expresses a relation between John and a collection of cards. Such interpretations are referred to as collective interpretations.[3] As sentences (4a-c) demonstrate, collective interpretations may pertain to subject or non-subject positions of verbal predicates.

As mentioned above, plural sentences may also make statements about individual entities. Some examples are given below.

(5) a. The girls were smiling.
$\approx$ Each girl was smiling.
b. John killed the snakes.
$\approx$ John killed each snake.
c. Mary and Sue were sleeping.
= Each of Mary and Sue was sleeping.
= Mary was smiling and Sue was smiling.
d. Mary read the *Tractatus* and *Das Kapital*.
= Mary read each of the *Tractatus* and *Das Kapital*.
= Mary read the *Tractatus* and Mary read *Das Kapital*.

A speaker who utters a sentence as in (5a-d) conveys a statement about individual entities. In (5a), the sentence is interpreted as claiming that each, or at least many, of the individual girls smiled. Similarly, in (5b) John must have killed many of the individual snakes for the sentence to be true. We say that sentences of this sort have a distributive interpretation, and that the predicate *distributes* over the collection referred to by its plural argument.[4]

When the number of individuals is small, as in (5c-d), the distributive interpretation often requires strictly universal quantification: in (5c) both girls are sleeping; in (5d) both books were read. However, it has often been pointed out that universal quantification is not a generally valid way to articulate how predicate distribute over plural descriptions. The question of how to model distributive interpretations is the focus of section 4.

In many sentences, plurals admit both a distributive interpretation and a collective interpretation. Consider the following sentences.

(6) a. Mary and Sue weigh 50kg.
b. The girls weigh 50kg.

[3] Here and henceforth we use the term ‘interpretation’ to informally designate one type of situations in which a sentence may be used truthfully. The question of which interpretations should correspond to separate *analyses* of plural sentences will surface as one of our main themes.

[4] Historically, the term *distributive* refers to the intuition that predication in sentences like (5c-d) ‘distributes over’ the conjunction, as in the distribution of multiplication over addition in the equation $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$. This terminology is extended to other plurals as in (5a-b).

A speaker may utter sentence (6a) to convey that Mary and Sue together weigh 50kg. In this case we say that the sentence has a collective interpretation. However, the same sentence can also be used to convey that each of the two girls weighs 50kg. In this case we say that its intended interpretation is distributive. The two interpretations also appear with the plural definite description in sentence (6b). More complex cases of this sort may often have interpretations that cannot be classified as purely distributive or purely collective. For instance, the sentence "*Mary and her dogs weigh 50kg*" admits an interpretation where Mary weighs 50kg as a single individual, and her dogs have the same weight as a group.

Predicates like "*weigh 50kg*" are often singled out as 'mixed predicates', but in general, most predicates are 'mixed' in one way or another. A predicate like "*smile*", which often invites distributive interpretations, can also be used so to invite a collective interpretation. Consider "*Arthur's lips smiled*", or, similarly, "*each of Patrick's facial muscles seemed motionless, but together they smiled*". Further, predicates like "*meet*", which typically give rise to collective interpretations, can also felicitously apply to singular NPs, as in "*the committee has met*". Nouns like "*committee*", "*class*" or "*group*" that show this phenomenon are singled out as group nouns (or 'collective nouns'). In British English, some of these nouns can agree with plural verbs also in their singular form (e.g. "*the committee are happy*"). When plural group nouns are used, also predicates like "*meet*" can give rise to distributive interpretations. For instance, the sentence "*the committees met*" has both a collective and a distributive interpretation, as does the sentence "*John shuffled the decks*".

The examples above illustrate collective and distributive interpretations with referential plurals. However, as mentioned, the distinction between distributivity and collectivity is directly relevant for quantificational NPs as well. Consider for instance:

(7) $$\left\{\begin{array}{l}\text{No girls}\\ \text{All of the girls}\\ \text{Most of the girls}\\ \text{Five girls}\end{array}\right\}\ \text{smiled / met / weigh 50kg.}$$

Depending on the verb, these sentences show a variety of distributive and collective interpretations, like the other sentences discussed above. In such cases the predicate "*smiled*" predominantly ranges over singular individuals and does not support a collective interpretation. However, to analyze "*all of the girls met*" or "*five girls met*", we need quantification over collections of girls rather than individual girls. In sentences like "*all girls weigh 50kg.*" or "*five girls weigh 50kg.*", many speakers accept both a distributive and a collective interpretation.

The facts surveyed above have evoked many questions about the semantics of plural NPs. When we start from the intuitive idea that plurals refer to or quantify over collections, the first question is what kinds of objects should be employed to model such "collections". This is the subject of section 3. Once we have decided how to model collections, the collective interpretation of referential plural NPs follows fairly directly. But if we take the collective interpretations to be the primary meanings of plurals, it is not immediately obvious how to account for distributive interpretations, which seem to make assertions about the individual elements of the collections. Starting out with referential plural NPs, we address this problem in section 4. Section 5 discusses quantificational plural NPs. These and other questions about plurals overlap with some other major topics in semantic theory: part-whole structure, mass terms, events, lexical semantics of predicates, cross-categorial semantics of coordination, implicature, tense and aspect, anaphoric dependency, bare plurals and genericity. Some of these issues will be touched upon as we go along.

3 The denotation of referential plurals

Sentences (4a-b) above are intuitively interpreted in terms of predication over collections. To say that "*Mary and Sue*" or "*the girls*" met is to ascribe a property "*meet*" to the relevant collection of people. This raises a question about the 'ontology' assumed by the semantic theory. If we take denotations of referential plurals to be collections of some sort, how are these collections to be formally defined? This section reviews some different answers that have been given to this question. In section 3.1 we discuss a family of largely equivalent treatments that represent collections as sets or mereological sums of entities. Section 3.2 addresses the question whether in addition to collections of structureless entities, the theory should also allow plurals to denote collections of collections.

3.1 The algebra of subsets and its mereological counterpart

History

Perhaps the oldest idea about referential plurals is that they may be modeled as denoting *sets*. The idea can be traced back to Bolzano (1851, pp. 2-4), who illustrated the intuitive idea of a set using sentences like "*the sun, the earth and the moon mutually influence each other*" and "*the rose and the concept of a rose are two very different things*". The earliest works on plurals in the Montague tradition also assumed that collective predicates apply to sets (Bennett, 1972; Bartsch, 1973; Hausser, 1974). Even earlier, McCawley (1968, p. 142) had noted that "a plural noun phrase usually refers not to an individual but to a set of individuals". Further, McCawley maintains [p. 146] that English does not distinguish between an individual x and the collection $\{x\}$ consisting of that individual. To model this property, he suggests to use a non-standard set theory; Massey (1976) and Schwarzschild (1996) suggest to use Quine's set theory (Quine, 1937, 1951, 1969). In standard set theory the same purpose may be achieved by lifting all singular denotations to range over sets, so that, for instance, proper nouns denote singleton sets rather than individuals (Verkuyl, 1981; Scha, 1981).

To cover the basic cases in all these set-based approaches, collections are represented by sets whose members are simple atomic entities, or 'individuals'. For instance, the denotation of plurals like "*John, Paul, and Charles*" or "*the boys*" may be the set of the relevant entities, $\{\mathbf{j}, \mathbf{p}, \mathbf{c}\}$. Collections whose elements are collections are not employed. We say that this approach assumes a domain of *flat* collections, which is contrasted with the *nested* collections of section 3.2 below. Domains of flat collections can be characterized as *boolean algebras* or, alternatively, as *complete atomic join semilattices* (Tarski, 1935, 1956; Link, 1983, 1998a). The latter is essentially the same structure as the 'mereological' part-whole structures proposed by Leśniewski (1928, 1930) and Leonard & Goodman (1940). Boolean algebras are special cases of such structures, which Hovda (2009) summarizes as follows: "[E]very complete Boolean algebra is a classical mereology, except for the presence of a single extra element called 0, an element that is a part of everything; and every classical mereology is a complete Boolean algebra, except for the presence of the 0 element" (this goes back to Tarski (1935, pp. 190-191, n. 5)). For the denotation of referential plurals, the decision between a boolean algebra and an atomistic mereology depends on a subtle issue: the status of "empty collections". If no one likes Amsterdam, what does the phrase "*the tourists who like Amsterdam*" denote? And if no one likes Brussels either, is "*the tourists who like Amsterdam are the tourists who like Brussels*" true or is it undefined? And along similar lines, is "*the tourists who like Amsterdam are numerous*" false or is it undefined in such situations?

Other considerations come into play, if we cast our net wider than the plural count nouns. English mass terms are nouns with denotations that are intuitively not atomic. Quantities of mud, gasoline, progress, or love are not measured by integer counts of their minimal constituent parts; in fact, the "naieve physics" assumed by English speakers does not seem to acknowledge such minimal parts. One may therefore analyze mass term denotations as having a non-atomic structure, and accordingly adopt mereological structures without atomic elements. Given this decision about mass terms, it becomes attractive to also treat plural count terms as denoting mereological sums – the only difference being that these sums are atomic. This approach was proposed by Link (1983, 1998a). An alternative way to create a common denominator between count terms and mass terms is to let mass terms denote structures that do have atoms, but avoid assumptions about the number or the character of these elements. This approach was proposed by Chierchia (1998a). Be it as it may, the distinctive properties of mereology are not very relevant for the study of plural count nouns. Similar points are made by Landman (1989) and Champollion (2010, pp.19-21). Also Link (1998a, Ch.3,13,14), who stresses the philosophical distinction between the approaches, accepts (p.64) that "for practical reasons (for instance, because people are 'used to it') we could stick to the power set model as long as we don't forget it is only a model".

Looking beyond the count-mass dichotomy in English, Grimm (2012) discusses several languages (in particular Welsh, Maltese and Dagaare) which make more fine-grained grammatical distinctions, for instance acknowledging separate categories for granular substances (*sand*) or distributed entities that habitually come together (*ants*). Grimm argues that nouns that occur in such a language can be ordered along a scale of individuation: *substances* $\prec$ *granular aggregates* $\prec$ *collectives* $\prec$ *individual entities*. To model the "aggregate nouns" and their number systems, Grimm augments standard mereology with relations that describe connectivity, thereby constructing a more expressive framework of "*mereotopology*".

Domains of individuals

In order to articulate the different formal approaches, we now introduce some further details and notation. We assume that natural language expressions directly denote objects in a model-theoretic framework (Montague, 1970).[5] The entities in the model are described by two distinct domains, consisting of singular individuals and plural individuals, and designated as 'D_{SG}' and 'D_{PL}', respectively. Natural language predicates range over elements of D_{SG} and D_{PL}. As we saw in section 2, mixed predicates like "*weigh 50kg.*" apply to elements of D_{SG} as well as D_{PL}. Thus, we introduce a domain D embracing both singular and plural individuals:

$$D = D_{SG} \cup D_{PL}.$$

Postulating this unified domain is the first step in specifying a domain of individuals. To complete it, we must specify D_{SG} and D_{PL}. We first do this for the "flat" set-based approach discussed above. In section 3.2 below we treat the alternative "nested" approach.

The individuals of the domain D_{SG} function as atoms of the domain D. We might allow D_{SG} to be any non-empty set modeling the basic entities in the model. However, as mentioned above, in order to allow simple operations on plural individuals it is technically more convenient to define the atoms in D_{SG} as the *singleton sets* constructed from elements of such an arbitrary non-empty set. Thus, any model M is defined in terms of a non-empty set E of entities. The elements of D_{SG} in M are the singletons over E, and the elements of D_{PL} are the subsets of E with at least two members. Summarizing:

[5] For more on frameworks that use direct interpretation see Keenan & Faltz (1978); Janssen (1983); Barker & Jacobson (2007) and the textbook Winter (2014).

Definition 1. *Let E be a non-empty set of entities. A* **flat domain over** *E is defined by:*

$$D_{SG} = \{\{x\} : x \in E\}$$
$$D_{PL} = \{A \subseteq E : |A| \geq 2\}$$
$$D \quad = D_{SG} \cup D_{PL} = \{A \subseteq E : A \neq \emptyset\}$$

The domain D in definition 1 is the set of all singular and plural individuals, which equals the powerset of E minus the empty set. This domain, endowed with the subset and union operations over it, has the structure of a *join semilattice*. This means that the structure $\langle D, \subseteq, \cup \rangle$ is a partially ordered set with the union operator $\cup$ as a least upper bound operator (Koppelberg, 1989). The set D is closed under unions, but since the empty set is not in D, it is not closed under complements and intersections. Because D has the structure of a join semilattice it is a notational variant of a mereological system, as we discussed above. Thus, flat domains as in Definition 1 can be translated into the ontology of Link (1983). This is done as follows.

Translation to Link's notation. Apply the following rules:

i. Instead of '$\{x\}$' for elements of D_{SG}, write 'x'.

ii. Instead of '$A \cup B$' for the union of sets $A, B \in D$, write '$A \oplus B$'.

iii. Instead of '$\bigcup \mathcal{A}$' for the union of the sets in a set $\mathcal{A} \subseteq D$, write '$\oplus \mathcal{A}$'.

Avoiding braces for singletons in D_{SG} as in (i) is innocuous since the sets D_{SG} and E are isomorphic. The '*i-sum*' notation '$\oplus$' in (ii) and (iii) (e.g. '$x \oplus A$' instead '$\{x\} \cup A$') reminds us of convention (i).

Let us consider the analysis of the coordination "*Mary and Sue*" using flat domains.[6] As in standard theories of anaphoric expressions (Büring, 2005), we assume that "*Mary*" has a different denotation than "*Sue*". This assumption is not part of the theory of plurals and not much will hinge on it, but we use it for the sake of exposition. Analyzing the coordination as set union, we get the following denotation for "*Mary and Sue*".

(8) $[\![\textit{Mary and Sue}]\!] \ = \ \{\mathbf{m}\} \cup \{\mathbf{s}\} \ = \ \mathbf{m} \oplus \mathbf{s} \ = \ \{\mathbf{m}, \mathbf{s}\}$

In words: the two singletons $\{\mathbf{m}\}$ and $\{\mathbf{s}\}$ in D_{SG} are merged by the denotation of "*and*". The resulting denotation for the plural NP is the set $\{\mathbf{m}, \mathbf{s}\}$ in D_{PL}. This leads to the following analysis of one of our basic examples, the sentence "*Mary and Sue met*" (=(4a)).

(9) $\mathbf{meet}(\{\mathbf{m}, \mathbf{s}\})$

Consider now the plural definite description "*the girls*" in (4b). Intuitively, we let this plural denote a set of plural individuals, and denote this set 'G'.[7] Accordingly, sentence (4b) (="*the girls met*") is analyzed as having the truth-value $\mathbf{meet}(G)$.

When simple definite plurals appear in conjunctive coordinations, we get examples like the following.

(10) The girls and the boys met.

We assume that the plural noun phrases "*the girls*" and "*the boys*" denote the respective sets G and B in D_{PL}. The union set $G \cup B$ is an element of the domain D_{PL} of plural individuals. Therefore, treating the conjunction "*and*" as set union leads to the following analysis.

[6] Analysis (8), like many theories of plurals, departs from analyses of "*and*" that use the boolean *meet* operator (Montague, 1973; Keenan & Faltz, 1978; Partee & Rooth, 1983). For theories of plurals that strive to adhere to the traditional boolean analysis of "*and*", see Winter (2001); Champollion (2013).

[7] For more details on plural nouns and the treatment of definite plurals, see section 5.1.

(11) $[\![\textit{the girls and the boys}]\!] \ = \ G \cup B$

This derives the following analysis of sentence (10).

(12) $\mathbf{meet}(G \cup B)$

In words: the plural individual consisting of the singular girls and singular boys satisfies the predicate "*meet*".

3.2 Hierarchical structures

The formula in (12) analyses sentence (10) as asserting that the group of children who met consists of the girls and the boys together. In this way, a semantics based on flat domains can use the union operator to support a basic analysis of collectivity with plural NPs and their conjunctive coordinations. In this account all referential plurals uniformly denote collections of singular individuals. This is an intuitively appealing analysis, but it is not complete. Interpretations of plurals may explicitly evoke parts of collections that are collections in their own right. For instance, consider the following sentences.

(13) The sun, the planets and the satellites of the planets influence each other

(14) The integers and the rationals are equinumerous.

Sentence (13) has a prominent interpretation where mutual influence is asserted about three specific objects. Two of these three objects – the planets and their satellites – involve collections and are denoted by plural NPs. Similarly, the subject of sentence (14) has two plural conjuncts, and the sentence as a whole expresses a relation between their denotations. In view of such examples it has often been argued that denotations of plurals may need to have more internal structure than what sets of singular entities allow (Hoeksema, 1983; Link, 1984; Scha & Stallard, 1988; Landman, 1989). In this approach, the sets denoted by plurals may have plural individuals as their elements, not only singular individuals. For instance, sentence (14) may then be analyzed as expressing the statement $\mathbf{equinumerous}(\{I, R\})$. In this analysis, $\{I, R\}$ is a plural individual containing elements that are collections in their own right. We refer to such plural individuals as nested collections.

Interpretations of complex plurals

Before introducing technical details about nesting of plural individuals, let us more systematically discuss some of the interpretations of plurals that motivate such a move. We can appreciate many of the relevant empirical questions by considering plurals like "*the girls and the boys*" as in sentences (10), (13) and (14). These coordinations have syntactic sub-parts that are themselves plural. We refer to such plural NPs as 'complex plurals'.

The interpretations of sentences with complex plurals will inform the decision between flat domains and nested domains. We classify three different kinds of such interpretations.

The 'union' interpretation. As we saw, a prominent interpretation of the sentence "*the girls and the boys met*" (10) involves only one meeting, of the girls and the boys together. This interpretation is directly modeled by letting the predicate **meet** apply to the union $G \cup B$. If the boys and the girls together constitute the children, then under the union interpretation, sentence (10) is semantically indistinguishable from the sentence "*the children met*". Salient union interpretations appear with many verbs, as illustrated by the following sentences.

(15) a. The students and the teachers performed together.
b. The managers and the bureaucrats outnumber the workers.
c. The soldiers and the policemen surrounded the factory.

In (15a) the union interpretation involves a performance by the union set of students and teachers; in (15b) the non-productive employees outnumber the productive ones; in (15c) the agents of organized violence surround the factory together.

The 'semi-distributive' interpretation. Under this interpretation the sentence makes a separate statement about each of the sets denoted by the parts of the complex plural. For instance, a speaker may use sentence (10) to describe a situation where the girls met and the boys met, but there was no meeting of all the children together. Thus, under this interpretation the predicate "distributes over" the denotations of the NP conjuncts, though not down to the atoms of their denotations as in (2). This 'semi-distributive' interpretation is often as coherent as the union interpretation. Which of the two interpretations is more salient depends on lexical meaning, world knowledge and contextual information. For instance, sentence (15a) above can be true in case there were two different performances, one by the students and one by the teachers. Similarly, (15b) may be employed to assert that both the managers and the bureaucrats outnumber the workers. Sentence (15c) is perhaps less likely to report two events in which the factory was surrounded, but this possibility cannot be ruled out, e.g. if in two different events, two different groups, of soldiers and of policemen respectively, were called in to surround the same factory.

The 'symmetric' interpretation. Under this interpretation the sentence makes a statement about a relation holding symmetrically between the given individuals. For instance, in sentence (13) above, the relation *influence* holds between the sun, the planets and the satellites. In sentence (14), the relation *equinumerous* holds between the two sets of numbers. Similarly, sentences (16a-b) below are both equivalent to sentence (17), expressing a relation between the two sets of children.

(16) a. The girls and the boys were separated.
b. The girls and the boys were separated from each other.

(17) The girls were separated from the boys (and vice versa).

In both (16a) and (16b) a collective sentence is interpreted as expressing a symmetric relation between sets as in (17). Below we give two more examples for sentences with such a prominent *symmetric interpretation* (Lakoff & Peters, 1969).

(18) a. The girls and the boys disagree (with each other).
b. The Frenchmen and the Germans were fighting (with each other).

In sentence (18a) a prominent interpretation is that the girls disagree with the boys (and vice versa). Similarly, sentence (18b) prominently describes a situation where the group of Frenchmen fought the group of Germans (and vice versa).

In their ability to derive the first two kinds of interpretations we have surveyed, the flat approach and the nested approach have no fierce competition with each other. Union interpretations are immediately derived in flat domains, and, with some care, also in nested domains (Landman, 1989). Semi-distributive interpretations are easily handled by using standard boolean conjunction, which are routinely assumed in both approaches (Winter, 2001). It is the symmetric interpretations that are critical for deciding between the two lines. We return to this central point shortly, but before we do that, let us spell out some more formal details about nested domains for plural individuals.

Nested domains

Like flat domains, also nested domains use the set D_{SG} of singular individuals for constructing the set D_{PL} of plural individuals. However, the set D_{PL} is now inductively extended by lumping together sets that are already in D_{PL} into new members of D_{PL}. For instance, when D_{PL} already has the sets $\{a, b\}$ and $\{c, d\}$, a new element is added to D_{PL}, which contains these two sets as members. This new element is the set $\{\{a, b\}, \{c, d\}\}$. Intuitively, a nested domain D contains all the sets that are derived from E in set theory, save those that involve the empty set or singletons. Formally, nested domains are defined as follows.

Definition 2. *Let E be a non-empty set of entities. We define $D_0 = E$, and for every $i \geq 1$ we define:*

$$D_i = D_{i-1} \cup \{A \subseteq D_{i-1} : |A| \geq 2\}$$

A **nested domain over** *E is now defined by:*

$$D = \bigcup_{i \geq 0} D_i$$

In words: on the basis of the set $D_0 = E$, each indexed domain D_i with $i \geq 1$ is inductively defined by adding to D_{i-1} the powerset of D_{i-1} minus the empty set and singletons. The infinite union of all the indexed domains is used as the domain D of singular and plural individuals.

Within the domain D, the domains for singular and plural individuals are naturally given by:

$$D_{SG} = D_0$$
$$D_{PL} = D - D_{SG}$$

Note that here, unlike our definition of flat domains, we use the set E itself as the domain D_{SG} of singular individuals.

Let us consider two simple examples. For the set $E = \{a, b\}$ we have the following indexed domains up to D_2:

$$D_0 = \{a, b\}$$
$$D_1 = D_0 \cup \{\, \{a, b\} \,\}$$
$$D_2 = D_1 \cup \{\, \{a, \{a, b\}\},\ \{b, \{a, b\}\},\ \{a, b, \{a, b\}\} \,\}$$

Consequently we have:

$$D_{SG} = \{a, b\}$$
$$D_{PL} = \{\, \{a, b\},\ \{a, \{a, b\}\},\ \{b, \{a, b\}\},\ \{a, b, \{a, b\}\},\ \ldots,\ \}$$

For the set $E = \{a, b, c\}$ we have:

$$\begin{aligned} D_0 &= \{a, b, c\} \\ D_1 &= D_0 \cup \{\ \ \{a, b\},\ \{a, c\},\ \{b, c\},\ \{a, b, c\}\ \} \\ D_2 &= D_1 \cup \{\ \ \{a, \{a, b\}\}, \{a, \{a, c\}\}, \{a, \{b, c\}\}, \{a, \{a, b, c\}\}, \\ &\qquad\qquad \{a, b, \{a, b\}\}, \{a, b, \{a, c\}\}, \{a, b, \{b, c\}\}, \{a, b, \{a, b, c\}\}, \\ &\qquad\qquad \{a, \{a, b\}, \{a, c\}\}, \{a, \{a, b\}, \{b, c\}\}, \{a, \{a, b\}, \{a, b, c\}\}, \\ &\qquad\qquad \ldots, \{a, b, c, \{a, b\}, \{a, c\}, \{b, c\}\}\ \} \end{aligned}$$

In the examples above, the set E of entities is very small, but the domain D_3 has already many plural individuals, and D has infinitely many of them. This infinity of the nested domain D is in contrast with the definition of flat domains.[8]

Nested domains invites a different treatment of conjunction. Instead of the union analysis in (8) and (12) above, we can now treat the coordinator "*and*" as denoting a *set formation operator* (sf). The sf operator takes two entities X and Y (possibly sets), and returns a set $\{X, Y\}$ that consists of X and Y as members. Formally:[9]

[8] Whether or not this infinity should be restricted is a complex matter, which also depends on what you say about expressions like "*Mary, and Mary and John, and Mary and [Mary and John], and Mary and [Mary and John] and [Mary and [Mary and John]]*" etc.

[9] Unlike (8), here the assumption that X and Y are different becomes technically crucial, otherwise $\{X, Y\}$ would become a singleton, which is not allowed given our definition of nested domains.

(19) $X \,\mathsf{sf}\, Y = \{X, Y\}$

By the definition of the nested domain D, whenever X and Y are different members of D, the set $X \,\mathsf{sf}\, Y$ is in D as well. Reconsider now the coordinations "*Mary and Sue*" and "*the girls and the boys*", and their analysis using the sf operator over nested domains:

(20) $[\![\textit{Mary and Sue}]\!] \;=\; \mathbf{m} \,\mathsf{sf}\, \mathbf{s} \;=\; \{\mathbf{m}, \mathbf{s}\}$

(21) $[\![\textit{the girls and the boys}]\!] \;=\; G \,\mathsf{sf}\, B \;=\; \{G, B\}$

The atoms $\mathbf{m}$ and $\mathbf{s}$ are in $D_0 = D_{SG}$. By definition of D_1 and our standard assumption $\mathbf{m} \neq \mathbf{s}$, we have $\{\mathbf{m}, \mathbf{s}\}$ as a member of D_1, hence of D_{PL}. Similarly, with our routine assumptions that the sets $G \subseteq D_1$ and $B \subseteq D_1$ are different and that each of them consists of at least two atoms, we have $\{G, B\}$ as a member of D_2, hence of D_{PL}.

The set $\{\mathbf{m}, \mathbf{s}\}$ that we get in the sf-based analysis (20) is the same as in the union-based analysis in (8). By contrast, with the nested analysis (21), the subject "*the girls and the boys*" of sentence (10) denotes the plural individual $\{G, B\}$, which is outside the flat domain. Therefore, in (10) and other sentences containing complex plurals, there is a potential descriptive difference between the two approaches. This puts us at an important crossroad for the theory of plurals.

Symmetric interpretations using nested collections

We now get back to the problem of symmetric interpretations of complex plurals, and see how nested domains allow us to address it. For a start, consider sentence (22a) below. This plural intransitive sentence is equivalent to the singular transitive sentence (22b).

(22) a. Mary and John were separated (from each other).
b. Mary was separated from John (and vice versa).

How are such equivalences to be accounted for? Because sentence (22a) contains a simple plural subject, its symmetric interpretation can be easily derived in both approaches. In a simplistic manner we can do that using the following rule, which establishes a semantic relation between the denotations of the intransitive predicate in (22a) and the transitive predicate in (22b).[10]

(23) For every plural individual $\{x, y\} \in D_{PL}$:
$\textbf{were\_separated}(\{x, y\})$
$\Leftrightarrow \textbf{were\_separated\_from}(x, y) \wedge \textbf{were\_separated\_from}(y, x)$

In both approaches, the analysis of sentence (22a) is $\textbf{were\_separated}(\{\mathbf{m}, \mathbf{j}\})$, and rule (23) renders this analysis equivalent to the analysis of sentence (22b), as intuitively required.

Under the nested approach, the same analysis immediately applies to complex plurals. For instance, sentence (24a) below (=(16)) has the nested analysis in (25a). By rule (23), this analysis is equivalent to the analysis of (24b) in (25b).

(24) a. The girls and the boys were separated (from each other).

If co-reference between conjuncts is needed for the analysis, e.g. of "*Mary and Sue are the same person*", one solution would be to admit singletons, but other solutions have also been proposed (Landman, 1989).

[10] As we see below, we need more complex formulations of rule (23) to account for sentences like "*Mary, John and Sue were separated*". Furthermore, rule (23) also does not hold between all collective predicates and their transitive correlates: if Mary kissed John on the cheek and he ignored her, and later John kissed Mary on the cheek and she ignored him, it does not follow that "*Mary and John kissed*", even though we can conclude that "*Mary and John kissed each other*". See Siloni (2001); Dimitriadis (2008b) and section 5.5.

b. The girls were separated from the boys (and vice versa).

(25) a. $\textbf{were\_separated}(\{G, B\})$

b. $\textbf{were\_separated\_from}(G, B) \wedge \textbf{were\_separated\_from}(B, G)$

Thus, the nested approach directly accounts for the symmetric interpretation of sentence (24a). On the basis of similar principles, nesting of plural individuals can account for the symmetric interpretation of sentences with complex plurals like the ones we have surveyed above.

Symmetric interpretations using flat collections

When using flat domains the situation is quite different. With flat domains, rule (23) is not applicable for analyzing complex plurals like "*the girls and the boys*". Specifically, under the union analysis the denotation of this complex plural is the set $G \cup B$, which is not a doubleton as required by rule (23). More generally, because of the absence of nested collections, flat domains do not allow us to derive the symmetric interpretation as directly following from the structure of the complex plurals. Should semantic theory adopt nested collections in order to analyze such plurals? Schwarzschild (1990, 1996) doubts it, maintaining that flat collections are sufficient for the semantic analysis, and that the symmetric interpretation should be derived by other means rather than nesting in the domain of plural individuals. Schwarzschild observes that all predicates expressed by English verb phrases can be applied to simple plurals: no predicate selects for nested collections. For instance, the complex plural sentence (24a) ("*the girls and the boys were separated (from each other)*") has the parallel sentence (26) below, with a simple plural replacing the complex plural subject.

(26) The children are were separated (from each other).

Assuming that the boys and the girls are just the children, Schwarzschild observes that utterances of (24a) and (26) may differ in their salient interpretations, but there is no difference in the *range* of interpretations that they support. When sentence (24a) is uttered out of the blue its salient interpretation is the symmetric one, according to which the children are separated by gender. But it is not the only interpretation of (24a): as Schwarzschild shows, different contexts may promote other separation criteria. For instance, a context may specify two distinct groups of children, determined according to the children's ages. In this case, "*the girls and the boys*" may be used as if it were synonymous with "*the children*", and separation by age becomes easier. Even more dramatically, we may add an adverbial modifier as in "*the girls and the boys were separated by age*", which only allows the age-based separation.

Based on this and similar observations, Schwarzschild proposes that all plurals denote individuals in a flat domain, i.e. sets of singular individuals. For instance, the complex plural "*the girls and the boys*" in (24a) is assumed to denote the union $G \cup B$, which in the intended models is the same as the denotation C of the plural "*the children*". To distinguish between sentences like (24a) and (26), Schwarzschild introduces a context-dependent parameter that defines a *cover* of the plural's denotation. This pragmatically induced cover specifies subsets of the set denotation of a referential plural. For both plurals "*the children*" and "*the girls and the boys*", the context may trigger any cover with sets C_1 and C_2 whose union equals the set of children C. By determining the cover, the context determines the criterion according to which a predicate applies to the set denotation of plurals (sec. 4.4). In particular, it is the pragmatically induced cover, not the NP denotation, that determines the criterion for separation in sentences (24a) and (26).

Suppose now that sentence (24a) is uttered out of the blue. Schwarzschild assumes that in this case, the salient cover consists of the subsets G and B, hence it specifies a gender criterion for separation between the children. This cover is selected because, in the lack of other knowledge about the structure of the group of children, the main factor affecting the pragmatics is the structure of the complex plural in (24a): "*the girls and the boys*". By contrast, within the subject "*the children*" of sentence (26) there is no information that favors one cover over another. Accordingly, there is no single cover that is salient for this sentence when it is uttered out of the blue. We see that in Schwarzschild's approach, as in the nested approach, the difference between sentences (24a) and (26) follows from their different syntactic structure. In the nested approach it follows directly. In Schwarzschild's approach the NP conjunction in (24a) only indirectly affects the choice of the cover, due to pragmatic mechanisms that are not fully spelled out.

Importantly, also a semantics based on nested domains cannot work correctly without some pragmatic principles. As Schwarzschild pointed out, in a context where there are two age groups of children, the prominent interpretation of both (24a) and (26) may involve separation by age. How does the nested approach deal with such cases? One way, proposed in Winter (2000), is to use the peculiar anaphoric and metonymic properties of definites. Here the relevant fact is that noun phrases like "*the girls and the boys*" and "*the children*" may used as 'proxies' for "*the relevant groups of children*" (see sec. 4.4).[11]

Schwarzschild's work makes it clear that the theoretical decision between the different approaches to the structure of collections should hinge on these pragmatic considerations, or else on other phenomena besides complex plurals. One such phenomenon that is most relevant for the theoretical decision is the treatment of singular and plural *group terms*, as in "*these people are the committee(s)*". We believe that in treating such cases, some versions of the nested approach have descriptive advantages over the flat approach. However, the details of these analyses go beyond the scope of this review. For details, see Scha & Stallard (1988); Landman (1989); Barker (1992); Landman (1996); Pearson (2011); de Vries (2013).

We have surveyed some key questions about the decision between flat domains and nested domains for theories of plurals. Despite the delicate theoretical and empirical debates that are involved in the decision between them, there is by now a rather wide agreement that both approaches are useful for treating many phenomena of plurality. Therefore, we now set aside the decision on flat vs. nested domains, and move on to other problems that are relevant for deciding on the denotation of referential plurals.

3.3 Events and 'anti-pluralist' approaches

A radical idea on the ontological status of collections comes from a philosophical tradition that wishes to avoid them altogether. In this tradition, launched by Boolos (1984, 1985), it is maintained that the model-theoretic interpretation of a logical language should only refer to individuals without internal structure.[12] Higginbotham & Schein (1989) embrace this line and attempt to avoid reference to collections by employing a neo-Davidsonian event semantics (Davidson 1967; Parsons 1990 and chapter [**TA**]). The meaning of a sentence is taken to involve quantification over 'eventualities', i.e. both states and events. Verbs are analyzed as a one-place predicates over eventualities, and the verb's arguments and adjuncts all specify properties of events (agent, patient, instrument, location, time etc.). Within event semantics, Higginbotham & Schein (henceforth *H&S*) analyze plurals as one-place predicates over singular individuals.[13] To illustrate H&S's analysis, consider the following sentence.

[11] In contrast to (26), this line expects sentences like "*Mary, John and Sue were separated from each other*" to only be interpreted as "*each of the children was separated from the other two*" (Winter, 2000; Sabato & Winter, 2005).

[12] Boolos argues that reference to collections gives rise to a variant of the Russell-paradox. Schein (1993) follows this line, but Scha (2013) argues that it is basically mistaken.

[13] H&S do analyze some collective sentences, e.g. "*the apostles are twelve*" as involving predication over plural individuals. This is in agreement with Kroch (1974, p.193) and Dowty (1986), and see also Winter (2002).

(27) The girls lifted the piano

To analyze the distributive/collective interpretation of (27), H&S suggest two neo-Davidsonian representations. In (28a-b) below we present their proposal using Davidsonian notation, which is simpler but preserves the import of H&S's analysis (cf. (18) and (23) in H&S, p.168).

(28) a. $\forall x.\mathbf{G}(x) \rightarrow \exists e.\forall y.y = x \leftrightarrow \mathbf{lift\_p}(y, e)$
In words: every girl is the unique agent of some "lifting eventuality" e.

b. $\exists e.\forall x.\mathbf{G}(x) \leftrightarrow \mathbf{lift\_p}(x, e)$
In words: for some "lifting eventuality" e, the set of girls is the set of e's agents.

Following H&S's avoidance of plural individuals, we use the symbol $\mathbf{G}$ in formulas (28a-b) as a one-place predicate. In both formulas, $\mathbf{G}$ does not serve as an argument of other predicates, unlike plural individuals in other approaches to plurals, whose primary use is to serve as arguments of collective predicates. At an intuitive level, the analyses in (28) capture two possible interpretations of sentence (27). Analysis (28a) requires that every girl lift the piano in a different event ("distributivity"). Analysis (28b) requires that the piano was lifted in one event, and each of the girls contributed to this event as one of its agents ("collectivity"). H&S go further than that and claim that analyses as in (28a-b) capture the difference between distributive and collective interpretations as a "matter of scope" (H&S,p.169). This is an important remark about the motivation of H&S's approach, but it is unclear. Formulas (28a-b) differ in more than just the relative scope of the universal quantifier $\forall x$ over girls and the existential quantifier $\exists e$ over events. Furthermore, the connection between the formulas in (28) and the structure of sentence (27) is not made explicit in H&S's proposal. As chapter **[SCO]** explains, the concept of "scope" in semantic theory is strongly tied with syntactic structure and compositionality. H&S's account contains no explanation of how the purported ambiguity of (27) is related to other cases of scope ambiguity. Because of that, it is not obvious which mechanisms are responsible for generating the two analyses in H&S's approach.

Schein (1993) further extends and elaborates on H&S's analysis, but does not address the issue of compositionality in more detail. In this sense H&S's and Schein's subsequent work are distinguished from most other theories of plurals, which involve compositional principles underlying the semantic analysis. Event semantics has considerable virtues, as in the analysis of optional verbal arguments, adverbial modifiers, or tense and aspect. However, as Landman (2000) points out, the optimal version of event-based approaches to plurality may be one that does allow plural individuals. Event-based approaches to plurality have been pursued by many other authors (Lasersohn, 1995, 1990b; Kratzer, 2000, 2007; Schwarzschild, 2011; Champollion, 2010), usually independently of Boolos' 'anti-pluralist' agenda. Furthermore, some of these works even treat events as having a structure similar to that of plural individuals, possibly nested ones.

4 Distributivity

As we noted above, the sentence "*the girls were smiling*" has a prominent distributive interpretation, i.e. it seems more or less synonymous with "*each girl was smiling*". Many plural sentences share this property. Accordingly, it has often been proposed that plural definites should have a distributive analysis equivalent with the meaning of "*each*" (Woods, 1967; Bartsch, 1973; Hausser, 1974; Bennett, 1974; Kroch, 1974; Cushing, 1977; VanLehn, 1978; Barwise & Cooper, 1981). However, it has been commonly observed that distributive interpretations of plurals exhibit a certain vagueness. If "*the girls*" denotes a large enough collection, exceptions are usually tolerated. Furthermore, the variety of distributivity effects shows that paraphrases such as "*most girls*" are also inadequate. For instance, Yoon (1996) and Malamud (2012) consider the distinct interpretations of the sentence "*the windows are open*". A context of an impending storm may lead to an 'existential' interpretation ("*some windows are open*"). By contrast, if house-painters come to paint the window frames, this promotes a 'universal' interpretation ("*all the windows are open*"). Malamud gives an extensive discussion of pragmatic factors that play a role here. See also Schwarz (2013) and Poortman (2014) for recent experimental studies.

When we consider verbs with two or more definite plural arguments, the possibilities multiply (Kroch 1974:202, Scha 1981). Hearing that "*the boys were dancing with the girls*", one will not necessarily imagine that each boy was dancing with each girl. In many settings the sentence is true if enough boys were dancing with some girl or other, and and enough girls were dancing with some boy or other. But hearing the sentence "*the books are on the shelves*", one may conclude that every book is on some shelf, while many shelves may be loaded with other things or be empty. Further, in "*the squares overlap with the circles*" it suffices that *some* square overlaps with *some* circle.

In face of this diversity, Scha (1981) proposes that plural definites should only be analyzed by predication over plural individuals.[14]. According to this view, an utterance that uses plural definite descriptions forces the hearer to think at the level of collective predications, and then to decide, on the basis of pragmatic reasoning, how to project such an abstract meaning representation onto an actual or imagined real-world situation. We believe that this view is to a large extent correct. To test it we first explore models that *reinterpret* collective predications as quantificational statements, and show that those reinterpretations follow from plausible assumptions about lexical semantics and pragmatic processes. We then move on to limitations of lexical reinterpretation processes and discuss some quantificational mechanisms for distributivity that have been proposed on top of them. We show that there is considerable evidence for a *distributivity operator* that quantifies over singularities within collections. Then we discuss some proposed complications of this mechanism that are still under debate.

4.1 Lexical reinterpretation

According to the lexical reinterpretation approach, distributive interpretations of plural NPs emerge through the elasticity of predicate concepts, without any structural semantic ambiguity. Referential plural NPs are uniformly treated as denoting plural individuals that act as predicate arguments, also in sentences that have distributive interpretations. This approach was explicitly proposed by Kroch (1974) and Scha (1981), and was adopted with some variations by Dowty (1986); Winter (1997, 2000, 2001); Champollion (2010) and de Vries (2012), among others. The starting point for the lexical reinterpretation approach

[14] A similar idea was independently adopted in studies of generic sentences. Following Carlson (1977), many works on generic interpretations of bare plurals treat them as a pseudo-quantificational epiphenomenon of predication over 'kind' individuals.

is a simple observation about the behavior of natural language predicates with respect to part-whole structure. For instance, considering the following pairs of sentences, we note that the sentences in each pair are very close in their meaning.

(29) a. This surface is white – *Every* part of this surface is white.
  b. This surface is dented – *Some* part of this surface is dented.
  c. Mary's car touches the tree – *Some* part of the Mary's car touches *some* part of the tree.
  d. Mary's car is in Dodoma – *Every* part of Mary's car is in *some* part of Dodoma.

We refer to such pairs of sentences as *pseudo-equivalent*. As most semanticists assume, such pseudo-equivalences result from the lexical semantics of the predicates (Cruse, 1979; Dowty, 1986; Winston *et al.*, 1987; Casati & Varzi, 1999). For instance, the connection between the two sentences in (29a) can be analyzed as a property of the predicate "*white*", which is semi-formally stated below.

(30) For every individual x: $\mathbf{white}(x) \leftrightsquigarrow \forall x'.\mathbf{part\_of}(x', x) \to \mathbf{white}(x')$.

Similar rules can be used for the other semantic paraphrases in (29). By using the '$\leftrightsquigarrow$' arrow, we stress that rules such as (30) are less stable than standard logical rules. The situations in which (30) applies depend on the concepts of *white* and *part of* that speakers have. Such concepts are notoriously context-sensitive. Thus, as in theories of non-monotonic reasoning and mental concepts, pseudo-equivalences as in (29) should be understood as reflecting weaker reasoning than logical equivalence (Laurence & Margolis, 1999).

The lexical reinterpretation approach to distributivity adopts a similar approach to distributive interpretations of plurals. Consider the following examples.

(31) a. The books are white – *Every* book is white.
  b. The books are damaged – *Some* book(s) is/are damaged.
  c. The books touch the boxes – *Some* book touches *some* box.
  d. The books are in the boxes – *Every* book is in *some* box.

In sentences (31a-d) we observe pseudo-equivalences which run parallel to those in (29a-d). This cannot be considered a coincidence. Instead of singular individuals and their parts, sentences (31a-d) refer to plural individuals and their parts, i.e. the singular individuals that constitute them. In the same way as rule (30) describes the pseudo-universal interpretation of sentence (29a), the following rule describes the distributive interpretation of (31a).

(32) For every plural individual A: $\mathbf{white}(A) \leftrightsquigarrow \forall x' \in A.\mathbf{white}(x')$.

Similarly, we may describe the other equivalences in (31) by the following postulates on the relations **damaged**, **touch** and **in**.

(33) For all plural individual A and B:
  a. $\mathbf{damaged}(A) \leftrightsquigarrow \exists x' \in A.\mathbf{damaged}(x')$
  b. $\mathbf{touch}(A, B) \leftrightsquigarrow \exists x' \in A.\exists y' \in B.\mathbf{touch}(x', y')$
  c. $\mathbf{in}(A, B) \leftrightsquigarrow \forall x' \in A.\exists y' \in B.\mathbf{in}(x', y')$

These schemes represent knowledge about predicates that should be embedded in any lexical theory about part-whole structures that includes plural individuals.

Part-whole structure is not the only kind of world-knowledge that affects distributive interpretations. Our default example "*the girls are smiling*" illustrates another case. Smiling is done by individual persons, and is not intuitively applicable to groups. However, note that sentences like "*the group is smiling*" are acceptable. Reasonably, conceptual processes of metonymy allow the transfer of properties from individual members to the group as a whole (Bartsch, 1973; Kroch, 1974; de Vries, 2012). Another case is "*the boys were dancing with the girls*", where in a ballroom context there is an assumption that dancing is done in pairs. Kroch (1974, 204-6) discusses "*to be married to*", which has similar properties, and proposes a lexical reinterpretation rule to get the desired distributive interpretation.

4.2 Quantificational distributivity

Arguably, lexical reinterpretation is the null hypothesis about the origins of distributivity with referential plurals. However, it is doubtful that this hypothesis alone can account for all distributive interpretations. Consider for instance the following sentence.

(34) The girls are wearing a blue dress.

Many speakers judge sentence (34) to be acceptable, and infer from it that different girls are wearing different blue dresses. Intuitively, this interpretation requires that the subject "*the girls*" behaves like a quantifier taking scope over the existential quantifier denoted by the object.

This kind of 'quantificational distributivity' is a problem that lexical reinterpretation alone cannot easily handle. Let us see why. Suppose that we keep assuming that subject "*the girls*" denotes a set G, which serves as the argument of the complex predicate *wear a dress*. When the object "*a blue dress*" is standardly analyzed as denoting an existential quantifier, this leads to the following analysis of sentence (34) (chapter **[GQ]**, cf. (44b) below).

(35) $[\![\textit{wear a blue dress}]\!](G)$
$\Leftrightarrow\ (\lambda x.\exists y.\mathbf{blue\_dress}(y) \wedge \mathbf{wear}(x,y))(G)$
$\Leftrightarrow\ \exists y.\mathbf{blue\_dress}(y) \wedge \mathbf{wear}(G,y)$
In words: "There exists a blue dress y such that the girls are wearing y".

Lexical information may allow us to derive from (35) further information about individual girls. Similarly to the additional information in (32)-(33), we may assume that when a group wears a dress, every member of that group wears it. Thus, we may assume the following about the predicate "*wear*".

(36) For every plural individual A and singular individual y:
$\mathbf{wear}(A,y) \ \leftrightsquigarrow\ \forall x' \in A.\mathbf{wear}(x',y)$

The information in (36) still does not allow the analysis (35) to capture the acceptable interpretation of sentence (34). According to (36), we can only derive from (35) a pragmatically unlikely conclusion: that there is some blue dress that *every* girl is wearing. Formally, from (35) we can only conclude by (36):

(37) $\exists y.\mathbf{blue\_dress}(x) \wedge \mathbf{wear}(G,y) \qquad (=(35))$
$\leftrightsquigarrow\ \exists y.\mathbf{blue\_dress}(x) \wedge \forall x' \in G.\mathbf{wear}(x',y)$

This is still not the acceptable information that speakers infer from sentence (34). Intuitively, the acceptable interpretation of sentence (34) requires distribution over individual girls to behave like a quantifier in the compositional analysis of the sentence. This quantifier must take scope over the existential quantifier within the complex predicate "*wear a dress*", and it cannot just be confined to the lexical analysis of the predicate "*wear*". The formula in (38) below models this behavior by assigning sentential scope to a universal quantifier over girls.

(38) $\forall x' \in G.\exists y.\mathbf{blue\_dress}(y) \wedge \mathbf{wear}(x',y)$

Quantification over girls in (38) is introduced as part of the compositional analysis of the sentence, not as part of the lexical interpretation of words. When a plural sentence shows a distributive interpretation that requires such a quantificational analysis, we refer to it as a case of quantificational distributivity.

Various effects of quantificational distributivity have been identified with referential plurals. Consider for instance the following sentences, with paraphrases of the relevant interpretations (Heim *et al.*, 1991; de Vries, 2012).

(39) The boys think they will win.
"Each boy thinks that he will win."

(40) The children are hiding somewhere.
"Each child is hiding in some place or other."

(41) The semanticists are walking or cycling.
"Each semanticist is walking or cycling."

(42) The boys have fewer coins than Mary.
"Each boy has fewer coins than Mary."

As in (34), these distributive interpretations cannot be generated by predication over plural individuals and lexical reinterpretation. The conclusion is that in some cases it is necessary to include a quantifier in the formal analysis of referential plurals. In the rest of this section we discuss some semantic mechanisms that were proposed for deriving such cases of quantificational distributivity.

4.3 Link's distributivity operator

Kroch (1974, 194-6) and Link (1983, 1987) analyze distributive interpretations by introducing a universal quantifier into the formal analysis of plural sentences. In Link's analysis, this operator has an effect similar to the effect of the floating quantifier *each* in "*the girls are each wearing a blue dress*" and "*the boys each think they will win*". As we saw, quantificational distributivity may also appear in sentences when there is no overt phonological indication like "*each*". Accordingly, in such cases Link adds a distributivity operator to the analysis. Link's distributivity operator is implemented as a function that maps unary predicates onto unary predicates, as defined in (43) below.[15]

(43) For every predicate P over D, $\mathcal{D}(P) \;=\; \lambda A.\forall y \in A.P(y)$.

For simplicity we assume here a flat approach, where E is the set of atoms and $D = \{A \subseteq E : A \neq \emptyset\}$ is the domain of singular and plural individuals. In words, the predicate $\mathcal{D}(P)$ holds of a (plural) individual A if and only if the predicate P holds of any (singular) individual y that is an atomic part of A. The $\mathcal{D}$ operator makes it possible to analyze sentences like (34) as formally ambiguous. Under the distributive analysis, the $\mathcal{D}$ operator applies to the VP denotation as in (44a) below. Under the non-distributive analysis, the $\mathcal{D}$ operator does not apply, and the VP denotation applies directly to the subject denotation as in (44b).

[15] We here focus on Link's popular analysis, which is quite similar to Kroch's earlier proposal. Dowty (1986); Roberts (1987); Lasersohn (1995) show motivation for defining $\mathcal{D}$ as a predicate modifier, discussing examples like "*the girls met in the bar and had a beer*", which require collectivity for the first VP conjunct but quantificational distributivity for the second VP conjunct.

(44) a. $(\mathcal{D}([\![\textit{wear a dress}]\!]))([\![\textit{the girls}]\!])$

$$= (\mathcal{D}(\lambda z.\exists u.\mathbf{wear}(z,u) \wedge \mathbf{dress}(u)))(G) \quad \text{VP and subject denotations}$$

$$= (\lambda A.\forall y \in A.(\lambda z.\exists u.\mathbf{wear}(z,u) \wedge \mathbf{dress}(u))(y))(G) \quad \text{definition of } \mathcal{D} \text{ operator}$$

$$= \forall y \in G.\exists u.\mathbf{wear}(z,u) \wedge \mathbf{dress}(u) \quad \text{simplification}$$

b. $([\![\textit{wear a dress}]\!])([\![\textit{the girls}]\!])$

$$= (\lambda z.\exists u.\mathbf{wear}(z,u) \wedge \mathbf{dress}(u))(G) \quad \text{VP and subject denotations}$$

$$= \exists u.\mathbf{wear}(G,u) \wedge \mathbf{dress}(u) \quad \text{simplification}$$

Analysis (44a) captures the prominent interpretation of (34), according to which each girl is wearing a (possibly different) dress. Analysis (44b) only describes pragmatically odd situations in which there is a dress that the girls are wearing jointly (cf. (35)). This pragmatic implausibility does not mean that analysis (44b) is semantically redundant. Consider the following sentence.

(45) The girls ate a pizza.

Without the $\mathcal{D}$ operator, the analysis derived for (45) amounts to "*there is a pizza that stands in the relation 'eat' to the collection of girls*". This statement is true if the girls shared a pizza (i.e. each girl had a slice), but also if one girl ate a pizza while acting as a proxy for the whole group. Thus, if we do not apply the $\mathcal{D}$ operator, the analysis of sentence (45) reflects an intuitively "collective" interpretation. With the $\mathcal{D}$ operator, sentence (45) gets the analysis involving quantificational distributivity, tantamount to "*every girl ate a (possibly different) pizza*".[16]

A point to keep in mind is that quantificational distributivity also appears with arguments of binary predicates and other non-unary predicates as in "*John gave the girls a present*". In the example, if we want a quantifier over singular girls to take scope over the existential argument "*a present*", we need to apply the distributivity operator to a complex *binary* predicate with the meaning "*give a present*". Formally, we need to derive the following analysis.

(46) $\forall x \in G.\exists y.\mathbf{present}(y) \wedge \mathbf{give}(\mathbf{j}, x, y)$

Link's $\mathcal{D}$ operator on unary predicates cannot directly apply in such cases, and a proper extensions of this operator is required (Lasersohn, 1998).

There is a wide consensus that the $\mathcal{D}$ operator or a variation thereof (see below) is needed for deriving interpretations involving quantificational distributivity as in sentences (34) and (45), and similarly in (39)-(42). However, there is no consensus on whether this is the only mechanism required for deriving distributive interpretations. While some authors following Link (1983) have tacitly assumed that this is the case, de Vries (2012) shows that this view is incompatible with the need to derive distributive interpretations for sentences like "*the class is asleep*". Similarly, in (31) we have seen interpretations involving singular individuals that are not universal, which is not explained by Link's operator. Based on these and similar observations, de Vries develops previous approaches from Kroch (1974); Dowty (1986); Roberts (1987) and Winter (1997, 2000), where both lexical reinterpretation and quantificational distributivity are explicitly postulated as a means for capturing the variety of distributive interpretations.

[16] Another use of $\mathcal{D}$ is for deriving the denotation of a plural noun from the corresponding singular noun, e.g. when defining the denotation of "*girls*" by $\mathcal{D}([\![\textit{girl}]\!])$ (section 5.1, Landman 1996). However, $\mathcal{D}$ is not a general *denotation* of plural morphology. With relational nouns like "*friends*" (Barker, 2011), the contribution of plural morphology is more complicated. For further work on the relevance of Link's distributivity operator for the compositional semantics of plural nouns, see Link (1983); Winter (2002); Zweig (2009).

4.4 Beyond Link's distributivity operator?

Link's $\mathcal{D}$ operator is a universal quantifier over singular individuals, and applies to one argument of a predicate at a time. We can therefore characterize it as *atomic* and *unary*. In the literature on plurals there are variations on this operator that allow more complex forms of quantification. Motivation for non-atomic distributivity was given based on examples like the ones below.

(47) a. The shoes cost \$75. (Lasersohn, 2006)
b. The potatoes weigh 100kg. (Schwarzschild, 1996)

When uttering sentence (47a) we may refer to different pairs of shoes, each pair costing \$75. Similarly, (47b) may be true when there are different baskets of potatoes, each of which weighing 100kg. These interpretations are favored by our world knowledge that shoes are normally sold in pairs, and that a single potato is unlikely to weigh 100kg.

A different motivation for non-atomic distributivity was suggested based on examples like the following.

(48) a. Rodgers, Hammerstein and Hart wrote musicals. (Gillon, 1987)
b. Mary, John, Sue and Bill built rafts.

These sentences may be easily accepted in any case where each of the agents wrote a musical (or built a raft), or did that in collaboration with one or more of the other agents, and the total number of musicals written (or rafts built) is two or more. For instance, based on our world knowledge, the prominent interpretation of sentence (48a) involves the two duos *Rodgers & Hammerstein* and *Hammerstein & Hart*. Each of these duos wrote musicals independently of the other duo. Thus, sentence (48a) may be judged true even if no one of the three individuals wrote any musical on his own, and they never collaborated as a trio. Similarly, sentence (48b) may be easily accepted if Mary and John built several rafts together, John and Sue built one raft together, and Bill built one raft alone. Thus, it is not required that the four people collaborate as one team, nor is it necessary that any of them built any raft single-handedly.

To account for such 'semi-distributive' interpretations, many works since Gillon (1987, 1990) have assumed a generalization of Link's $\mathcal{D}$ operator that quantifies over sub-collections. One popular mechanism is the *cover* approach mentioned in section 3.2. Suppose that the plural individual denotation of "*the shoes*" in (47a) is a set of four shoes $\{s_1, s_2, s_3, s_4\}$. In the cover approach, to get the prominent interpretation of sentence (47a), the context first specifies a cover with sub-collections of this set, e.g. the pairs $\{s_1, s_2\}$ and $\{s_3, s_4\}$. Link's $\mathcal{D}$ operator is extended and allowed to distribute over the sub-collections in the cover. This is illustrated in the following analysis of sentence (47a).

(49) $\forall x \in \{\{s_1, s_2\}, \{s_3, s_4\}\}.\mathbf{cost\_\$75}(x)$

In (49), each in the pairs in the cover is required to cost \$75.

For Rodgers, Hammerstein and Hart in (48a), the relevant cover contains the collections $\{\mathbf{r}, \mathbf{h}_1\}$ and $\{\mathbf{h}_1, \mathbf{h}_2\}$. Distribution over these collections leads to the following analysis.

(50) $\forall x \in \{\{\mathbf{r}, \mathbf{h}_1\}, \{\mathbf{h}_1, \mathbf{h}_2\}\}.\mathbf{wrote\_musicals}(x)$

We can reach the relevant cover here by either assuming that the context provides it, or by introducing various *cumulation* mechanisms that amount to existential quantification over covers (see below). Note that covers may also include singular individuals. For sentence (34) ("*the girls are wearing a blue dress*") we can use a cover that consists of each of the singular individuals for "*the girls*", which derives the same result as using Link's $\mathcal{D}$ operator.

Theories that rely on pragmatic specification have a lot of covers to choose from. For obtaining the interpretation of the sentence "*Mary, John and Sue ate pizzas*" where Mary ate pizzas alone and John and Sue ate pizzas together, we use a cover with the individuals $\mathbf{m}$ and $\{\mathbf{j}, \mathbf{s}\}$. There are 109 covers of the set for Mary, John and Sue, and each of them gives a possible interpretation for this sentence. For a subject with four members as in (48b) there are already 32297 possible covers, and the numbers grow fast (OEIS, 2010).

Quantificational treatments of distributivity may become richer than that. Consider the following example.

(51) Lennon and McCartney wrote *Help!*, *Imagine* and *Jet*.

Assume that *Help!* was written by the duo Lennon & McCartney, but each of the other songs was written by either John or Paul single-handedly. Under this historically plausible scenario, sentence (51) is judged true. To capture this kind of interpretation, Schwarzschild (1996) and others allow the context to contribute a *polyadic cover*. This cover determines *pairs* of sub-collections, where each pair has a sub-collection for the subject and a sub-collection for the object. For (51) our world knowledge induces a cover consisting of the following pairs: $\langle\{\mathbf{l}, \mathbf{m}\}, \mathbf{h}\rangle$, $\langle\mathbf{l}, \mathbf{i}\rangle$ and $\langle\mathbf{m}, \mathbf{j}\rangle$. Sentence (51) can now be treated by quantifying over these pairs, as in the following formula.

(52) $\forall\langle x, y\rangle \in \{\langle\{\mathbf{l}, \mathbf{m}\}, \mathbf{h}\rangle, \langle\mathbf{l}, \mathbf{i}\rangle, \langle\mathbf{m}, \mathbf{j}\rangle\}.\ \mathbf{write}(x, y)$

In words, the relation "*write*" holds for every pair in the historically salient cover: L&M-*Help*!, L-*Imagine* and M-*Jet*.

Many works adopt this generalization of distributivity operations, which allow them to range over elements of non-atomic, polyadic covers. Operators that quantify over non-atomic, polyadic collections are also referred to as *cumulativity operators*. For some works that adopt such cumulativity operators or cover-based quantification as an extension of Link's operator, see: Gillon (1987, 1990); Krifka (1989, 1992, 1996); Schwarzschild (1996); Verkuyl & van der Does (1996); Sternefeld (1998); Beck (2001); Beck & Sauerland (2001); Kratzer (2000, 2007); Ouwayda (2012); Nouwen (2013), among others. However, while Link's distributivity operator is well-motivated and does not suffer from serious over-generation problems,[17] Link (1998b, p.36) argues that only the "narrowly understood" distributive and collective interpretations are "well-entrenched in language, even if mathematically, both the collective and the distributive reading are but special cases of a more general cover interpretation". Like Link, we believe that the applications of covers have been over-extended.

First, many of the examples in the literature that were meant to show support for cover-based quantification actually concern cases where lexical reinterpretation alone may do the work. For instance, sentence (51) may be true not because some distributivity operators work at sentence-level or complex-predicate-level as in (52), but because of lexical information about the binary predicate for "*write*":

(53) For all individuals in $x_1, x_2, y_1, y_2 \in D$:
$\mathbf{write}(x_1, y_1) \wedge \mathbf{write}(x_2, y_2) \rightsquigarrow \mathbf{write}(x_1 \cup x_2, y_1 \cup y_2)$

In words, if the "*write*" relation holds for two pairs of (singular/plural) individuals, then it also holds of the respective unions. This is what some works call 'cumulative reference' (sec. 4.5), here expressed as a lexical property of the predicate "*write*".

Lexical assumptions similar to (53) also account for the interpretation of sentences (48a-b) above. As Winter (2000) points out, the logical analysis of such sentences, with bare plurals in the object position, may involve a collective analysis of that position (see also Zweig 2009). Specifically, for (48a):

(54) $\exists M \subseteq \mathbf{musical}.\ |M| \geq 2\ \wedge\ \mathbf{write}(\{\mathbf{r}, \mathbf{h}_1, \mathbf{h}_2\}, M)$

[17] McNally (1993) suggests to restrict Link's distributivity operator when analyzing interpretations of comitative ("*s*", 'with') constructions in Russian. Dalrymple *et al.* (1998a) argue that additional facts on Russian go against McNally's conclusions.

In words: for some plural individual M consisting of musicals, the plural individual for "*Rodgers, Hammerstein and Hart*" is in the relation **write** to M. The prominent interpretation of sentence (48a) follows from the lexical cumulativity assumption (53) about the predicate "*write*". Under this use of the lexical reinterpretation approach, how precisely the work on the musicals in M was divided in the trio $\{\mathbf{r}, \mathbf{h}_1, \mathbf{h}_2\}$ is not a matter for the compositionally derived analysis in (54). The lexical rule (53) makes sure that whenever the predicate "*write*" holds between $\{\mathbf{r}, \mathbf{h}_1\}$ and a set of musicals M_1, and between $\{\mathbf{h}_1, \mathbf{h}_2\}$ and a set of musicals M_2, analysis (54) turns out to be true for the trio $\{\mathbf{r}, \mathbf{h}_1, \mathbf{h}_2\}$ and the set of musicals $M_1 \cup M_2$. However, unlike the cover-based approach or cumulativity operators, no quantifier works compositionally to bring this about.

How about the shoes and potatoes in (47a-b)? Here, lexical cumulativity as used above cannot lead to the desired interpretation. Let us see why. Under the assumption that the predicate "*cost $75*" takes plural individuals, there is no lexical assumption of 'cumulativity' that would account for the non-atomic distribution in (47a): it would be painfully wrong to assume that when two pairs of shoes cost $75 each, they also have the same price together. It is also hard to think of any cumulative inference with the lexical verb "*cost*" that could account for the non-atomic distribution in (47a). Similar problems would show up if we wanted to treat the predicate "*weigh 100kg.*" using some rule of lexical reinterpretation. The conclusion is that we would not derive the non-atomic interpretations of (47a-b) if we only gave them meaning representations like $\mathbf{cost\_\$75}(S)$ or $\mathbf{weigh\_100kg}(P)$, where S and P are the relevant collections of shoes and potatoes, respectively.

Does it mean that we have to add an operation of non-atomic distributivity to these representations? That may be too hasty. Suppose that you go shopping for shoes, and a shopkeeper tries to convince you to buy shoes by pointing out to you:

(55) These four shoes cost 75$.

Or, suppose that she said:

(56) Shoes A, B, C and D cost 75$.

Whatever the shopkeeper may mean by sentences like (55) and (56), she is unlikely to be giving you a price for each of two pairs of shoes. It is more likely that she is offering you a bargain deal of four shoes, or try to sell you single shoes for this price. Or, perhaps (see below), the shopkeeper may be quoting prices of four pairs of shoes, allowing each of the four shoes she mentions to act as a 'proxy' for a different pair. But is there any reason to assume here a pragmatically induced cover that would lead to a similar interpretation to the two-pair interpretation of (47a)? This is an empirical question that is currently unresolved, which leaves a noticeable gap in the cover-based approach. As we saw in section 3.1, different contexts may allow the sentence "*the children were separated*" (=(26)) to be interpreted with different separation criteria. In approaches that use pragmatically induced covers, the same analysis is invoked in (47). However, is there any motivation for admitting all covers in (55)?

In technical terms, unlike what we saw in sentence (47a), it is unclear if there is any context where the sentences in (55) show any non-atomic distribution.[18] Pointing out similar contrasts, Winter (2000) proposes that cases like (47a), where quantificational effects of non-atomic distributivity do appear, should not be derived by any distributivity operator. Rather, Winter suggests that such effects are related to some special properties of definite descriptions. As in other examples with definites (Nunberg, 1978; Jackendoff, 1992), when

[18] Lasersohn (1989, 2006) goes further than that, and challenges cover-based approaches using examples like "*the TAs earned exactly $20,000*" in situations where John, Mary, and Bill are the TAs, each of them earned $10,000, and the relevant cover involves $\{\mathbf{j}, \mathbf{m}\}$ and $\{\mathbf{m}, \mathbf{b}\}$. However, here the difference from (47a-b) is not formal, hence it is likely to be purely pragmatic. Indeed, in contexts where "*the TAs*" refers to these two groups, the relevant non-atomic interpretation seems to become more prominent, as in the examples discussed in section 3.1.

saying that "*this shoe costs $75*", a speaker may speak loosely of the price of a pair of shoes. In a similar way, "*the shoes*" in (47a) may be used to mean "*the pairs of shoes*"; "*the potatoes*" in (47b) may mean, in the right context, "*the baskets of potatoes*", and so on. Such metonymy is quite common with short general descriptions like *the shoes*, but it is much less salient when shoes are counted or enumerated as in (55). Thus, Winter suggests that a better understanding of 'metonymy' or 'dependency' processes with definites is required in order to analyze the pseudo-quantificational impression we get in cases like (47a-b). Like the pragmatic considerations of the cover-based approach, this proposal is also tentative because its pragmatic ingredient is not fully specified. However, Winter's proposal restricts the cases where pragmatics plays the key role to those cases for which there is evidence that this is needed: the analysis of anaphoricity/metonymy with singular and plural definites, which is independent of the study of plurals.

To conclude, there are systematic evidence for 'cumulative' processes as in (53) as part of lexical reinterpretation with certain predicates, e.g. "*write*". This leads to some non-atomic or polyadic distributive interpretations. Further, in some cases with simple plural definites non-atomic/polyadic distributivity may also seem to behave like a quantificational effect. However, like Link (1998b), we believe that there is little evidence that quantificational distribution over elements of general covers needs to be part of the compositional analysis of plurals. For further arguments and counter-arguments on this point see Lasersohn (1989, 1995); Gillon (1990); Winter (2000); Beck & Sauerland (2001); Kratzer (2007).

4.5 Notes on further issues

Cumulative reference and the classification of predicates

Many works stress the importance of inferences that are sometimes informally referred to as 'cumulative reference'. Consider the following examples.

(57) a. Mary is a girl, and Sue is a girl $\Rightarrow$ Mary and Sue are girls.
b. Mary and Sue are girls, and Debbie and Jane are girls $\Rightarrow$ Mary, Sue, Debbie and Jane are girls.
c. Mary (has) smiled, and Sue (has) smiled $\Rightarrow$ Mary and Sue (have) smiled.
d. Mary and Sue (have) smiled, and Debbie and Jane (have) smiled $\Rightarrow$ Mary, Sue, Debbie and Jane (have) smiled .

Similar cumulative entailments are observed with mass nouns, as in (58) below (Lasersohn, 2011; Gillon, 2012).

(58) Puddle 1 is water, and puddle 2 is water $\Rightarrow$ puddles 1 and 2 are water.

Link's atomic distributivity operator directly accounts for cumulative entailments as in (57). Link's work also has relevance for the study of mass terms as in (58) (Bunt, 1985; Hinrichs, 1985; Krifka, 1989; Chierchia, 1998a). However, atomic distributivity alone does not expect the following kind of entailments.

(59) A and B met, and C and D met $\stackrel{?}{\Rightarrow}$ A, B, C and D met.

The question mark indicates that the entailment in (59) is much less obvious than those in (57). This is another piece of evidence against a non-atomic distributivity quantifier, which would expect entailments such as (59) to hold as generally as those in (57). Other non-entailments of this sort can be construed if we replace the predicate "*meet*" in (59) by predicates like "*are sisters*", "*cost 75$ together*", "*paint the box together*", "*are two engineers*", "*are outnumbered by the sheep*". We conclude that works like Krifka (1989); Kratzer (2007); Nouwen (2013), which introduce non-atomic cumulativity operators in the compositional analysis of plurals suffer from the same empirical problems that non-atomic distributivity operators suffer from. The same holds for common assumptions about the generality of polyadic cumulation. For instance, "*A and B are taller than X, and C is taller than Y*" does not entail "*A, B and C are taller than X and Y*".[19]

By arguing that many effects of distributivity, semi-distributivity, and 'cumulative reference' are accounted for by lexical reinterpretation of predicates we have only hinted at a rich topic of research for the theory of plurals: its relations with lexical semantics. For some works that have started to map this vast area see Dougherty (1970, 1971); Kroch (1974); Scha (1981); Hinrichs (1985); Dowty (1986); Roberts (1987); Higginbotham & Schein (1989); Taub (1989); Verkuyl (1993, 1994); Lasersohn (1995); Brisson (1998, 2003); Winter (2001, 2002); Hackl (2002b); Champollion (2010); Schwarzschild (2011); de Vries (2012, 2013); Mador-Haim & Winter (2014).

Reciprocal quantifiers and reciprocal predicates

Many predicates that trigger collective interpretations of plurals involve *reciprocal expressions*. This is the case in complex collective predicates like "*meet each other*" or "*meet one another*". As we have seen, collectivity effects also appear with lexical predicates such as "*meet*" in the sentence "*Mary and Sue met*". Like the predicate "*meet*", many other lexical predicates that trigger collectivity – e.g. "*fight*" and "*disagree*" – intuitively involve a reciprocal interpretation. Reciprocity may also appear with collective nouns, as in "*Mary and Sue are sisters (of each other)*".

In the case of "*Mary and John were separated*" (=(22a)), we relied on the following equivalences.

Mary and John were separated
⇔ Mary and John were separated from each other
⇔ Mary was separated from John, and John was separated from Mary

However, we should be careful not to draw hasty conclusions from such equivalences. First, not all cases of collectivity can be paraphrased by using overt reciprocal expressions or conjunctions of singular transitive sentences. For instance, in the sentence "*the soldiers surrounded the castle*" it is hard to find a reciprocal or transitive sentence with a related meaning. Similarly for the sentence "*Mary shuffled the cards*". Furthermore, even in cases where reciprocity is evident, as in "*Mary and John kissed*" it would be incorrect to assume that the collective interpretation is fully derived as in the corresponding reciprocal or transitive sentences. For instance, as Siloni (2012) extensively discusses, differences between the two cases show up when we consider the interpretation of "*Mary and John kissed five times*" *vis à vis* "*Mary and John kissed each other five times*".

Reciprocity with complex and lexical predicates has been the focus of much research. Some works concentrate on the syntax-semantics interface with overt reciprocal expressions (Higginbotham, 1980; Heim *et al.*, 1991; Williams, 1991). Other works concentrate on the relationships and differences between lexical reciprocity and complex reciprocal expressions (Siloni, 2001, 2012; Dimitriadis, 2008b). Yet another line of work analyzes the diversity of interpretations that reciprocal relations lead to (Langendoen, 1978; Dalrymple *et al.*, 1998b).

[19] This connection between distributivity operators and 'cumulativity entailments' is formally unsurprising. Any theory that assumes non-atomic polyadic distribution (e.g. via covers) expects the entailment pattern X_1 PRED *(Y_1), and* X_2 PRED *(Y_2)* $\Rightarrow$ X_1 *and* X_2 PRED *(Y_1 and Y_2)* to be valid: for each cover supporting the antecedent there is a corresponding cover supporting the consequent. More generally: the kind of covers we assume (atomic/non-atomic, monadic/polyadic) predicts the kind of cumulative entailments we expect.

For further work on these topics see Sternefeld (1998); Beck (2001); Filip & Carlson (2001); Dimitriadis (2008a); Kerem *et al.* (2009); Dotlačil & Nilsen (2009); Sabato & Winter (2012); Mari (2014); Struiksma *et al.* (2014) as well as the collections Frajzyngier & Curl (1999); König & Gast (2008).

5 Plurals and quantification

So far in this chapter we have concentrated on referential plurals: noun phrases like "*Mary and John*", "*the girls*" and "*the girls and the boys*". However, as mentioned in section 2, many plural NPs are quantificational and cannot be easily treated as denoting individuals. In this section we discuss some of the problems in this domain and their proposed treatments. We start out by presenting the two main approaches to plural quantificational expressions, which analyze them as *modifiers* or as *determiners*. After introducing some problems for each of these approaches and their proposed solutions, we move on to discussing some other problems and theories related to cumulative, reciprocal and floating quantifiers, and their interaction with plurality.

5.1 Quantificational expressions

In (60) below we summarize some important classes of simple plurals.

(60)

a.	Bare plurals:	*girls, boys*
b.	Definites:	*the girls, the boys*
c.	Bare numerals:	*three girls, five boys*
d.	Modified numerals:	*more than three girls, at most five boys, exactly ten women*
e.	Other quantifiers:	*some girls, all the boys, no women, many cats*
f.	Partitives:	*most of the children, three of the girls*

In order to compositionally analyze the denotation of the plurals in (60) we first have to fix the denotation of the plural nouns within them. As in many works on plurals, let us treat plural nouns as one-place predicates applying to singular and plural individuals.[20] Such predicates are modeled by functions from the domain D to the set $\{0, 1\}$ of truth-values. Definition (61) below illustrates this analysis with the plural noun "*girls*".

(61) For every individual $A \in D$:
$[\![girls]\!](A)$ iff $\forall x \in A.\mathbf{girl}(x)$.

In words: the denotation of the plural noun "*girls*" holds of any singular or plural individual A that consists of singular individuals in the denotation **girl** of the singular noun "*girl*". Whenever this holds, we henceforth abbreviate and write: '$\mathbf{girls}(A)$'.

Getting back to the list in (60), let us first note the systematic variations that bare plural NPs as in (60a) show between existential and generic interpretations (e.g. "*dogs bark*" vs. "*dogs bit me yesterday*"). Treatments of both interpretations are often based on the denotation of plural nouns in (61) (Carlson 1977; Chierchia 1984, 1998b; Carlson & Pelletier 1995; Dayal 2011). However, the integration of theories of generic and existential bare plurals with the formal semantics of plurality has not been researched extensively, and it is beyond the scope of this review. By contrast, deriving the referential denotation of *definite* plurals as in (60b) is quite straightforward with noun denotations as in (61).[21] It is the interpretation of the properly quantificational NPs, exemplified in (60c-f), that we shall focus on now.

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[20] The question of whether plural nouns should indeed admit singular individuals goes beyond the scope of this chapter, and it is related to the problem known as 'dependent plurals': the fact that a sentence like "*all unicycles have* wheels" only claims that every unicycle has (at least) one wheel, and not (necessarily) more. Similarly, "*Sweet Jane or some other members of her gang are the* thieves *who stole La Gioconda*" may be true if Jane stole the Mona Lisa single-handedly, i.e. only one thief acted. See Zweig (2009) and references therein.

[21] See Sharvy (1980); Winter (2001) for techniques that unify the semantics of plural and singular definites.

5.2 QEs: modifiers or determiners?

Ignoring some syntactic complexities, we refer to all the pre-nominal elements in (60c-e) (e.g. *three, exactly ten, most of the*) as quantificational expressions (QEs).[22] When analyzing NPs as in (60c-e) in simple compositional frameworks, a critical decision is whether the QE within the NP denotes a *modifier* or a *determiner*.

The 'modifier' approach. In this approach, a QE is not the compositional source of quantification. The QE is analyzed as a *modifier*: its denotation composes with a predicative noun denotation as in (61) to derive another predicate. The QE denotation is assumed to select some of the individuals in the noun's denotation according to their cardinality. For instance, in the NP "*three girls*", the QE "*three*" selects the plural individuals with three elements from the denotation of the noun "*girls*". Modificational QE denotations do not change the semantic function of the noun in the sentence, as the NP still basically denotes a predicate. Accordingly, in the modifier approach quantificational effects are analyzed as external to the denotation of the QE.

The 'determiner' approach. In this approach, the QE maps the denotation of the noun to a *generalized quantifier* (Peters & Westerståhl 2006, chapter [**GQ**]). Under this analysis, the denotation of the QE itself is responsible for the quantificational interpretation of sentences with quantificational NPs.

In (63a-b) below we illustrate these two approaches by roughly paraphrasing their different analysis of sentence (62).

(62) At least three girls lifted the piano.

(63) a. *Modifier analysis*:
There is a plural individual A containing at least three girls, such that A lifted the piano.

b. *Determiner analysis*:
Counting singular girls who lifted the piano reveals at least three girls.

Without further assumptions, the modifier analysis in (63a) reflects a collective interpretation of (62), whereas the determiner analysis in (63b) reflects the distributive interpretation. The theoretical differences between the two approaches are considerable. The determiner approach tries to extend classical works that treat all NPs uniformly as generalized quantifiers (chapter [**GQ**]). The modifier approach follows a different tradition where many NPs, or at least some indefinite NPs, are initially treated as *predicates* (Milsark, 1974). In sections 5.3 and 5.4 below we elaborate on approaches to the semantics of plural QEs that emanate from these two different traditions.

5.3 The modifier approach

Consider first the bare numeral noun phrase "*three girls*". In the modifier approach the basic denotation of this NP is analyzed as follows.

(64) For every individual $A \in D$:
$[\![\textit{three girls}]\!](A)$ iff $\mathbf{girls}(A) \wedge |A| = 3$

In words: the basic denotation of the NP "*three girls*" admits any set of girls with three members.

[22] On the internal structure of noun phrases (and/or determiner phrases), especially in relation to QEs and their semantics, see Bartsch (1973); Verkuyl (1981); Abney (1987); Zwarts (1992); Zamparelli (1995); Hackl (2001, 2002a); Winter (2001) among others.

This analysis is compositionally obtained for the NP by letting the bare numeral QE within it denote a *predicate modifier*. Formally:

(65) For every one-place predicate P over the domain D, for every individual $A \in D$:
$([\![three]\!](P))(A)$ iff $P(A) \wedge |A| = 3$

In lambda notation, we assume that the numeral "*three*" denotes the following function:

$$\mathbf{three}^{\text{MOD}} = \lambda P.\lambda A.P(A) \wedge |A| = 3$$

In words: the function $\mathbf{three}^{\text{MOD}}$ sends every predicate P over individuals A to a predicate that only holds of the plural individuals that satisfy P and have three members. With this treatment of numerals, the modificational analysis of "*three*" compositionally derives the basic predicative meaning of "*three girls*" in (64). This kind of analysis is common in the literature on indefinites following Milsark (1974), where some, or all, indefinite NPs are basically analyzed as predicative. In terms of its linguistic broadness, this approach has various advantages. First, it gives a direct account of sentences like "*these are three girls*", where the indefinite plural appears in a predicate position. Second, it is compatible with many versions of discourse representation theory (Kamp & Reyle, 1993) and event semantics (Kratzer, 2007). Based on Milsark's initial motivation, the predicative approach to indefinites is also used to account for the distribution of NPs in *there* sentences (McNally, 2011). However, there are also some hard problems for this approach.

The modificational analysis of QEs takes the relevant plural NPs to denote predicates over collections. This still does not immediately account for quantificational interpretations of plurals in argument positions. To turn such nominal predicates into existential quantifiers, Partee (1987) introduces an existential operator into the compositional analysis.[23] In sentence (66) below, this leads to the analysis in (67).

(66) Three girls met.

(67) $\exists A.[\![three\ girls]\!](A) \wedge [\![met]\!](A)$ (introducing existential quantifier)
$\Leftrightarrow\ \exists A.\mathbf{girls}(A) \wedge |A| = 3 \wedge \mathbf{meet}(A)$ (by NP denotation in (64))

In words: sentence (66) is analyzed as asserting that there is some plural individual containing exactly three singular girls in the extension of the predicate "*meet*".

The analysis of collectivity in (66) is immediately extended for distributive interpretations of sentences with bare numerals such as (68) below. Whatever account of distributivity we adopt for referential NPs like "*the girls*" can be immediately used together with the modifier analysis. This is trivially so for the lexical reinterpretation approach to distributivity, which treats distributive predicates like "*smile*" on a par with collective predicates. Furthermore, this is also the case for Link's distributivity operator $\mathcal{D}$, as the analysis in (69) below illustrates.

(68) Three girls smiled.

(69) $\exists A.[\![three\ girls]\!](A) \wedge (\mathcal{D}([\![smiled]\!]))(A)$ (introducing ex. quantifier and $\mathcal{D}$)
$\Leftrightarrow\ \exists A.\mathbf{girls}(A) \wedge |A| = 3 \wedge (\mathcal{D}(\mathbf{smile}))(A)$ (by NP denotation in (64))
$\Leftrightarrow\ \exists A.\mathbf{girls}(A) \wedge |A| = 3 \wedge \forall y \in A.\mathbf{smile}(y)$ (by def. of $\mathcal{D}$ in (43))

In words: sentence (68) is analyzed as asserting that there is some plural individual containing exactly three singular girls, each of whom is in the extension of the predicate "*smile*".

[23] In some accounts of genericity and modality, the introduced quantifier may be a generic or a modal operator (Diesing, 1992). This can account for non-existential usages of numerals as in "*two people in love are always dependent on one another*". In other accounts of genericity, the predicate may also be analyzed as a *kind* or a *property* (Carlson & Pelletier, 1995). In event semantics, the existential quantifier may be a quantifier over events rather than individuals (Schein, 1993; Landman, 2000).

The analyses in (67) and (69) do not require that the exact number of girls who met or smiled be three. For instance, suppose that Mary, Sue and Jane met (or smiled), and that in addition, Joan and Linda had a separate meeting (or smiled, respectively). The analysis in (67) and (69) expects sentences (66) and (68) to be true in these situations. This is consistent with Gricean analyses of scalar implicatures with bare numerals, as well as more recent approaches to numerals and implicatures.[24] However, as van Benthem (1986, pp.52-53) warns, in many other cases the existential analysis is in a direct conflict with semantic intuitions.

Consider the NPs in (70) below, which are often classified as non-upward-monotone NPs (terminology borrowed from the determiner approach, cf. chapter [**GQ**]).

(70) Non-upward monotone NPs (nmNPs):
at most five boys, exactly ten women, no women, few dogs, less than five boys, between five and ten women, an odd number of dogs, less than a third of the cats

When nmNPs as in (70) are analyzed as in the modifier approach, existential analyses as in (67) and (69) become highly problematic. Using the distributive predicate "*smile*", let us illustrate what would happen if we tried to extend analysis (69) for sentence (71a) below. This would lead to the proposition in (71b).

(71) a. At most three girls smiled.

b. $\exists A.\mathbf{girls}(A) \wedge |A| \leq 3 \wedge \forall y \in A.\mathbf{smile}(y)$
In words: there is a plural individual A containing at most three girls, such that each girl in A smiled.

The analysis (71b) does not put any restriction on the maximal number of girls who smiled. For instance, suppose again that Mary, Sue and Jane smiled, and that in addition, Joan and Linda smiled. Sentence (71a) is clearly false, but the analysis in (71b) would take it to be true. This problem reappears with any simple combination of existential quantification with the modifier analysis of nmNPs.

A straightforward way of avoiding the problem with nmNPs is to avoid analyzing their QEs as modifiers. Some authors have pointed out empirical distinctions between bare numerals and modified numerals, which motivate a distinction between bare numerals and other QEs: bare numerals denote modifiers, whereas other QEs do not (Liu, 1990; Corblin, 1997; Winter, 2001; Szabolcsi, 2010). This leaves open the analysis of the other QEs in (60), but avoids the undesired effects of the modifier approach with nmNPs. Another direction in the literature is to analyze at least some of the QEs in nmNPs as modifiers, but to introduce more complicated quantificational processes into the sentential analysis beyond existential quantification. For works that attempt this line, see Landman (2000); Hackl (2001, 2002a); Fox & Hackl (2006); Geurts & Nouwen (2007); Nouwen (2010); Kennedy (2013).

5.4 The determiner approach

In the determiner approach, sentential quantification processes originate from the QE itself, which is analyzed as denoting a *determiner function*: a function from one-place predicates (noun denotations) to generalized quantifiers (NP denotations).[25] The classic work of Barwise & Cooper (1981) does not treat collective interpretations of plurals. Accordingly, Barwise and Cooper and many other studies of natural language quantification treat QEs as denoting functions from predicates over the domain D_{SG} of singular individuals to generalized

[24] See Horn (1972); Chierchia *et al.* (2011); Kennedy (2013), among others.
[25] In the terms of chapter [**GQ**], the functions that we here call 'generalized quantifiers' are isomorphic to *quantifiers of type* $\langle 1 \rangle$; 'determiner functions' are isomorphic to *quantifiers of type* $\langle 1, 1 \rangle$.

quantifiers over D_{SG}. In (72) below we give an analysis of the numeral "*three*" as denoting a determiner function over D_{SG}.

(72) For all one-place predicates P_1, P_2 over D_{SG}:
$([\![three]\!](P_1))(P_2) = 1$ iff $|\{x \in D_{SG} : P_1(x) \wedge P_2(x)\}| \geq 3$,
which we abbreviate by writing '$(\mathbf{three}^{\text{DET}}(P_1))(P_2)$'.

In words: the numeral "*three*" holds of the predicates P_1 and P_2 over singular individuals if there are at least three elements that are in the extensions of both P_1 and P_2.

In sentence (68), repeated below, this quantificational analysis directly leads to the analysis in (74).

(73) Three girls smiled.

(74) $(\mathbf{three}^{\text{DET}}(\mathbf{girl}))(\mathbf{smile})$
$\Leftrightarrow\ |\{x \in D_{SG} : \mathbf{girl}(x) \wedge \mathbf{smile}(x)\}| \geq 3$ (by QE denotation in (72))

In words: there are at least three singular individual girls in the extension of the predicate "*smile*". Note that similarly to the modifier analysis in (69), and following the same Gricean reasoning, in the determiner analysis (74) we treat the bare numeral "*three*" in sentence (73) as semantically equivalent to "*at least three*". This does not affect the main points of the discussion here.

The analysis in (74) uses the denotations **girl** and **smile** of the singular noun and the verb. These denotations range over singular individuals. Thus, while they allow us to easily capture distributive interpretations as in (73), they are not suitable for dealing with collective interpretations. Various adjustments of generalized quantifier theory have been proposed in order to deal with such interpretations of quantificational NPs.[26] Let us introduce two general techniques that have been proposed: an 'existential' and a 'neutral' approach. For concreteness, we again consider sentence (66), which is restated below.

(75) Three girls met.

We have assumed that the noun "*girls*" and the verb "*meet*" in (75) both denote one-place predicates over the domain D of singular and plural individuals. The two ways of paraphrasing the counting in (75) are given below.[27]

(76) a. *Existential analysis*:
There is a set A consisting of exactly three girls, s.t. the set A had a meeting.
Formally: $\exists A \in D.\mathbf{girls}(A) \wedge |A| = 3 \wedge \mathbf{meet}(A)$

b. *Neutral analysis*: (term due to van der Does 1992)
There are at least three singular individuals x s.t. x is a girl and x took part in some or other meeting.
Formally: $|\{x \in D_{SG} : \mathbf{girls}(x) \wedge \exists A \in D.x \subseteq A \wedge \mathbf{meet}(A)\}| \geq 3$

Based on these paraphrasing techniques, we can derive determiner functions over singular and plural individuals that properly mimic them (van der Does, 1992, 1993). While this approach is technically sound, in terms of empirical adequacy, there are open questions for both the existential and the neutral analyses. The existential analysis suffers from the same

[26] See Scha (1981); Lønning (1991); van der Does (1992, 1993); van der Does & Verkuyl (1995); Verkuyl (1997); Dalrymple *et al.* (1998b); Winter (2001); Ben-Avi & Winter (2003); Lønning (2011); Szymanik (2010).

[27] In addition, Scha (1981) and van der Does (1992) postulate a distributive analysis of plural QEs. This is may not be necessary if we have a distributivity operator on predicates, which is optional on top of the existential and neutral analysis.

problems with nmNPs that we saw in section 5.3 for the modifier approach, especially with distributive predicates. The neutral analysis does not suffer from these problems, but it has to face some other problems. First, the neutral analysis in (76b) makes no claim about any meeting of any group of girls. Rather, it only says something about individual girls. However, many speakers do not accept sentence (75) as true if three girls each participated in a *different* meeting. Counter-intuitively, statement (76b) expects sentence (75) to be judged as true in such situations. Second, even if this possible problem is avoided,[28] the neutral analysis illustrates a "non-atomic" approach. This leads to similar questions to the ones pointed out in section 4.4 for the analysis of distributivity using non-atomic covers. Consider for instance sentence (77) below.

(77) Exactly three girls drank a whole glass of milk together.

The adverbial *together* favors here at least one existential effect: the reported group of there girls has to act together as a team. The neutral analysis of sentence (77) is more permissive, and allows interpretations where the girls do not act together, e.g. when each of the three girls belongs in a different team that drank milk. It is unclear if sentence (77) admits such interpretations. For instance, suppose that Mary and Sue drank a whole glass of milk together, and so did Sue and Jane. If these are the only groups that drank a whole glass of milk together, the neutral analysis is true, but it is questionable if sentence (77) can be accepted.

These and other problems complicate the analysis of quantification with plurals. For some solutions and further problems see van der Does (1992, 1993); Dalrymple *et al.* (1998b); Winter (2001); Ben-Avi & Winter (2003); Peters & Westerståhl (2006). In the current stage of the research on plural quantification, we believe that it is still hard to see which variant of the determiner approach to collectivity may lead to the best results, and whether, and in which cases, it may be supplemented or replaced by the modifier approach. Only when sufficient empirical evidence are accumulated and analyzed, may it be possible to decide on the most promising theoretical direction. Works on related problems can be found in studies of plurals and events (Schein, 1986, 1993; Krifka, 1989; Landman, 2000; Kratzer, 2000, 2007) and plurals in discourse (Kamp & Reyle, 1993; van den Berg, 1996; Nouwen, 2003; Brasoveanu, 2011).

5.5 Further problems with plurals and quantification

In this section we very briefly mention two other problems of quantification with plurals and their relations to the problems we have discussed, and refer to some works in these domains.

Cumulative quantification

So far, we have assumed that quantificational NPs are analyzed as unary quantifiers which must have scope over each other (chapter [**SCO**]). The scope relations between the quantifiers may potentially give rise to further ambiguities. A good first approximation is that there is a preference for the quantifier order that corresponds to the left-to-right order of the NPs in the sentence, with widest scope for the quantifier corresponding to the leftmost NP (Scha, 1981). Other orders are optional; they may come to the fore because of stress patterns, discourse priming, or simply because of their better real-world plausibility. Note that quantifiers which result from the lexical reinterpretation of "referential plurals", as discussed above (4), always

[28] This can be done by paraphrasing (76b) as follows: "counting singular girls who took part in meetings *of girls* reveals at least three girls". Formally: $|\{x \in D_{SG} : \mathbf{girls}(x) \wedge \exists A \in D.x \subseteq A \wedge \mathbf{girls}(A) \wedge \mathbf{meet}(A)\}| \geq 3$. See van der Does (1992); Winter (2001).

have narrow scope with respect to the quantifiers which correspond to the "quantificational NPs".

In some sentence interpretations, however, it seems that quantifiers do not take scope over each other. Consider the following examples.

(78) a. Exactly one student greeted exactly one teacher.
"Exactly one student greeted a teacher and exactly one teacher was greeted by a student."

b. Exactly two students greeted exactly two teachers.
"Exactly two students greeted a teacher (or teachers) and exactly two teachers were greeted by a student (or students)."

These interpretations are known as *cumulative* interpretations. Such interpretations cannot be accounted for by unary quantifiers that take scope over each other (Peters & Westerståhl 2006, p.351, chapter [**GQ**]). As (78a) illustrates, this "non-linear scope" behavior is not restricted to plurals. Such sentences may be analyzed by constructing a complex quantifier that ranges, for instance, over student-teacher pairs, selects the pairs that satisfy the greet-relation, and applies the cardinality-requirement expressed by the determiners to the first elements and the second elements of these pairs, respectively. See Scha (1981), and, for a more elaborate discussion, Landman (2000, pp. 129-140).

Some authors have suggested that such polyadic quantification is unnecessary, observing that in one sense, cumulative quantification is similar to nested existential quantification over collections. Both kinds of quantification are "scopeless" in that the relative scope of the two quantifiers is irrelevant for the truth-conditions. Thus, it has often been maintained that existential quantification over collections suffices to account for cumulative readings (Roberts, 1987), and similarly, using an event-based approach, in Schein (1993). In some examples such approaches work. However, as we saw, with existential analyses these successes are limited to examples that involve upward monotone quantifiers. As Landman (2000) points out, they fail in the general case.

Another alternative is to assume that the problematic quantificational interpretations of NPs, like the non-monotone or downward monotone quantifiers, should be derived from weaker, upward monotone interpretations, through some process of maximization (Scha, 1991). The ambiguity of a numeral ("*three*": *exactly* three or *at least* three) may be a matter of focus. In this way we may attempt to derive the cumulative interpretations from weak, upward monotone analyses, by means of a maximization process. Landman (2000) develops this line, taking an event-based treatment as a point of departure.

Floating quantifiers and collectivity/distributivity adverbials

The QEs in the examples in (60) all appear before the noun. Some QEs can also appear in other positions in the sentence. For instance, consider the following examples from Hoeksema (1996).

(79) a. We *all* should have been drinking tea.

b. We should *all* have been drinking tea.

c. We should have *all* been drinking tea.

QEs like "*all*", which show this syntactic flexibility, are often referred to as floating quantifiers. In English, also "*each*" and "*both*" are QEs that can appear floating in similar ways. One obvious question is how meanings of floating QEs compose in their various positions, and whether this variation has implications for their semantic analysis. A less obvious question is whether there is a relation between the semantics of floating QEs and covert distributivity operators like those that we discussed in section 4. These questions have been addressed in various works, especially in relation to the complex syntax of floating QEs in different languages (Bobaljik, 2003; Fitzpatrick, 2006; Cirillo, 2009), but also in relation to their semantic effects (Dowty & Brody, 1984; Beghelli & Stowell, 1997; Hoeksema, 1996).

Another important phenomenon that we can only mention is the interpretation of certain adverbials. Especially central is the item "*together*" as in "*we drank tea together*". The interesting semantic property of "*together*" is that it collectivizes not only typically 'mixed' predicates like "*lift the piano*" but also apparently distributive predicates like "*be happy*". Other "mereological" adverbials appear in sentences like "*the circles completely/mostly/partially cover the square*". For more on such adverbials, especially in relation to part-whole structures, see Schwarzschild (1994); Lasersohn (1990a); Moltmann (1997, 2005, 2004).

6 Conclusion

We have reviewed some of the most well-studied problems about the formal semantics of plurals, and discussed some approaches to their solution. While we have tried to remain neutral on some dilemmas, we believe that some conclusions emerge from this critical survey. First, as we extensively discussed in section 3, the decision between flat domains and nested domains depends on the treatment of various distributive, semi-distributive and reciprocal/symmetric interpretations. We believe that there have been important advances in our understanding of these interpretations and their possible sources. However, the decision on the structure for the domain of plural individuals is also informed by the behavior of group nouns as in "*the girls are the committee(s)*" or "*the group(s) is/are running*", which is still a major problem. Second, there is considerable evidence that distributivity operators should be used at some level of the compositional analysis. At the same time, on the face of the richness of the lexical sematic effects on distributivity, distributivity operators may reasonably be considered as a last theoretical resort in compositional semantics. While the evidence given so far for atomic-unary distributivity operators is quite solid, this is not the case for more intricate forms of distribution, especially the non-atomic polyadic approach of cover-based distributors. More work on lexical semantics of predicates and its interaction with plurals is crucial for deepening our understanding of distributivity.

Further, the treatment of collective interpretations of plural quantifiers may depend both on empirical research into the semantic status of neutral and non-monotonic analyses of numerals and other quantificational expressions, also in relation to cumulative quantification. Work in this area may help in analyzing some of the hard problems we have pointed out for the analysis of plural quantification. Since the neutral analysis of quantifiers is consistent with cover-based approaches, this may also shed some light on the general nature of distributivity.

Finally, in consistency with our general line, we would like to reiterate the importance that we see for a rigorous theory about the lexicon and the pragmatics of plurals, especially in relation to collectivity, distributivity and reciprocity of predicates. Under the lexical reinterpretation approach to distributivity, this may be the main area where plurals are related to group descriptions and to part-whole structure in language. More general and precise theories of these lexical and pragmatic domains will surely shed more light also on the formal semantics of plurality.

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