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YOAD WINTER

WHAT MAKES CHOICE NATURAL?*

1 INTRODUCTION

The idea to use choice functions in the semantic analysis of indefinites has recently gained increasing attention among linguists and logicians. A central linguistic motivation for the revived interest in this logical perspective, which can be traced back to the epsilon calculus of Hilbert & Bernays (1939), is the observation by Reinhart (1992, 1997) that choice functions can account for the problematic scopal behaviour of indefinites and interrogatives. On-going research continues to explore this general thesis, which I henceforth adopt. In this paper I would like to address the matter from two angles. First, given that the semantics of indefinites involves functions, it still does not follow that these have to be *choice* functions. The common practise is to stipulate this restriction in order to get existential semantics right. However, a so-far open question is whether there is any way to derive choice function interpretation from more general principles of natural language semantics. Another question that has not been formally accounted for yet concerns the relationships between choice functions and the "specificity"/"referentiality" intuition of Fodor & Sag (1982) about indefinites. Is there a sense in which choice functions capture this popular pre-theoretical notion?

In order to answer these questions, this paper proposes a revision in the treatment of choice functions in Winter (1997), leaving its linguistic predictions unaffected but changing slightly the compositional mechanism. This modification opens the way for proving the following theorem: function variables in the analysis of the noun phrase must denote only choice functions and can derive only the standard existential analysis by virtue of the conservativity, logicality and non-triviality universals of Generalized Quantifier Theory as proposed in Barwise & Cooper (1981), van Benthem (1984), Thijsse (1983) and others. The same implementation also captures the "specificity" notion: indefinites with a non-empty restriction set denote principal ultrafilters in the revised formalization. These are the quantificational correlates to "referential" individuals. The conceptual point is empirically relevant, as it enables to classify the choice function interpretation of indefinites as "definite" and "strong" in a precise sense, and treat it on a par with proper names, definites and other "referential" noun phrases. Some implications of this point for the scope of indefinites in partitive constructions and *there*-sentences are briefly discussed.

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2 THE EMPTY RESTRICTION PROBLEM FOR CHOICE FUNCTION THEORY

Let us refer to the set denoting the N' in an indefinite NP by the name *restriction set*. For instance, the restriction set of the indefinite *some dog* is the set of dogs in a given model. Compositionally, there is no straightforward way to combine this set with the denotation of the predicate applying to indefinites. Generalized quantifier theory assumes that indefinites, like all noun phrases, denote quantifiers. The basic assumption in choice function semantics is that the denotation of indefinites is in fact an *individual* that is "picked up" from the restriction set. This "choice" process is carried out by a function applying to the restriction set. The natural type for this function is therefore $(et)e$. After such a function applies to the restriction set, of type et, we get an individual of type e. Sentences with indefinites are interpreted using existential quantification over such functions. This closure operation, and not the article or the numeral of the indefinite, is responsible for its existential interpretation.[1] For instance, we want the meaning of the elementary example (1) to be formalized along the lines of (2), with the intended meaning as roughly paraphrased in (3).

(1) Some dog barked.

(2) $\exists f_{(et)e}[\mathbf{bark'}(f(\mathbf{dog'}))]$

(3) There is a function such that the dog it chooses barked.

For a detailed linguistic argumentation for an analysis along these lines see Reinhart (1997) and Winter (1997). In a nutshell, the central argument these works use to motivate choice functions for the treatment of indefinites comes from their exceptional scopal behaviour. This is illustrated in the following example.

(4) If *some woman* arrives then John will be happy.

This sentence can be interpreted in one of two ways: either the arrival of *any woman* would satisfy John, or it is *some particular woman* John is expecting. In choice function theory this ambiguity is captured by the two possibilities to locate the existential quantifier over functions: either within the scope of the conditional operator or outside its scope. Other methods, including the standard analysis of indefinites using existential quantification over e-type individuals, must assume a syntactic representation where the restriction of the indefinite – the common noun *woman* – takes scope over the conditional. However, such a representation violates the Adjunct Island introduced by the conditional, which is an empirically motivated restriction on all theories of scope and extraction.

Getting back to the simple case in (1), the actual meaning representantion in (2) is very far from that of (1), which (3) is intended to paraphrase. For instance, (2), unlike (1), comes out necessarily true if some individual that is not a dog happened to bark. The reason is that any function mapping the set of dogs to this barking entity satisfies (2). Consequently, we must restrict the quantification over functions in (2) as in (5), where C is a predicate over $(et)e$ functions.

(5) $\exists f[C(f) \wedge \mathbf{bark'}(f(\mathbf{dog'}))]$

A proposition with this restriction can get closer to the meaning of (1). We assume that C defines the class of *choice functions*. A choice function assigns every non-empty set one of its members. Formally, C can be determined by the classical definition CH for choice functions in (6), so (5) is specified as (7).

(6) $CH_{((et)e)t} \stackrel{def}{=} \lambda g.\forall A \neq \emptyset [A(g(A))]$

(7) $\exists f [CH(f) \wedge \mathbf{bark}'(f(\mathbf{dog}'))]$

This significantly ameliorates the situation: assuming that the set of dogs is not empty, (7) correctly conveys the meaning of (1). But we still have a problem: when the set of dogs is empty sentence (1) is false. However, (7) is true in this case, provided further that some individual barked. This is because, according to (6), a choice function can map the empty set to *any* entity. Hence, a choice function that assigns a barking individual to the empty set (here, the denotation of *dog*) satisfies (7), which is therefore not the right meaning of (1) either. More generally, it is very hard to define C so that (5) respects the meaning of (1). In Winter (1997) it is shown that this is in fact *impossible* given standard logical assumptions. This problem for a formalization like (5) is referred to as the *empty restriction problem*.

3 CHOICE FUNCTIONS OF THE DETERMINER TYPE

The empty restriction problem indicates that (5) is not the right way of using choice functions for linguistic purposes. Instead, in Winter (1997) I propose to define choice functions as mapping a set not to an e-type object, but rather to the quantifier corresponding to such an entity. This is standardly done using the Montague/Lambek type lifting. For instance, instead of choosing the entity $\mathbf{m}'$, for a dog named *Moses*, we pick up the quantifier $\{A : \mathbf{m}' \in A\}$, the *ultrafilter* that Moses generates. Thus, f is of the determiner type $(et)((et)t)$: a function from predicates to quantifiers. Formula (5) is replaced by (8). Now $f(\mathbf{dog}')$ has the generalized quantifier type $(et)t$ and CH^d is the set of choice function *determiners* defined in (9). This definition solves the empty set problem: it stipulates that a choice function determiner maps the empty set to the *empty quantifier*. Consequently, (10) is rendered false in case there are no dogs.

(8) $\exists f_{(et)((et)t)} [C^d(f) \wedge f(\mathbf{dog}')(\mathbf{bark}')]$

(9) $CH^d_{((et)((et)t))t} \stackrel{def}{=} \lambda D.D(\emptyset_{et}) = \emptyset_{(et)t} \wedge \forall A_{et} \neq \emptyset\ \exists x \in A[D(A) = \lambda B.B(x)]$

(10) $\exists f_{(et)((et)t)} [CH^d(f) \wedge f(\mathbf{dog}')(\mathbf{bark}')]$

More generally, (10) is equivalent to the standardly assumed meaning of (1): $\mathbf{dog}' \cap \mathbf{bark}' \neq \emptyset$. As shown in Winter (1997), this treatment provides a sound analysis of many scope and distributivity phenomena with indefinites. It can also be given a compositional implementation.

However, this approach scores rather poorly with respect to the general questions that are in the focus of the present paper. Except for the descriptive motivation, I know of no consideration that can justify the definition in (9). Many other logical possibilities besides CH^d are open for restricting the existential quantification over determiners in (8). For instance, consider a trivial alternative for CH^d, defined as the singleton containing the determiner **most**. Thus, we replace C^d in (8) by the restriction $\{\mathbf{most}\}$. With this restriction, sentence (1) would absurdly end up equivalent to the sentence *most dogs barked*. Absurd as it is, there is no general principle that blocks this possibility. In other words, assuming that the syntax and compositional interpretation specify for sentence (1) a reading like (8), they give no clue as for the value of C^d in this formula. The definition in (9) is simply an additional stipulation. Furthermore, even under this definition the "specificity" intuition is not captured. Since the restriction CH^d is imposed at a higher compositional level, there is no guarantee that $f(\mathbf{dog'})$, for instance, corresponds to a particular dog: the value for f can be any determiner (e.g. *most*). Thus, in order to compositionally know that the indefinite is "specific" we would have to impose somehow the CH^d condition on f at lower levels (by using some notation of sorted variables, for instance). This is an additional undesired mechanism that becomes necessary under the determiner treatment of choice functions in any theory that makes use of semantic properties of noun phrases. The proper way to characterize indefinites as "locally specific" is the focus of section 7 below.

4 AMENDMENT: INDEFINITES DENOTE GENERALIZED QUANTIFIERS BUT CHOICE FUNCTIONS ARE NOT DETERMINERS

Both problems mentioned above are due to the same reason: the determiner type assigned to the function in (8) is too high to impose significant restrictions on the set of functions quantified over. On the other hand, the solution to the empty restriction problem is completely well-motivated, as it follows the uniform treatment of NPs as generalized quantifiers. The problem is then, how can the output of a choice function still be a quantifier without the function being of the determiner type?

A way to do that is based on keeping to the $(et)e$ definition of choice functions. When the restriction set is not empty the formalization is straightforward. When the restriction set is empty it is mapped to the empty quantifier directly, with no interference of the choice function variable. There are various ways to implement this idea. One of the easiest is to use the lifting operator $\langle\ \rangle$ of $(et)e$ choice functions to determiners as defined in van der Does (1996):

(11) $$\langle\ \rangle_{((et)e)((et)((et)t))} \stackrel{def}{=} \lambda g.\lambda A.\lambda B.\exists x A(x) \wedge B(g(A))$$

Let us use the sugaring $\langle f \rangle$ instead of $\langle\ \rangle(f)$. Note that this definition guarantees the same results for lifted choice functions of the low type as for the choice functions of the determiner type defined in (9). When the restriction set is X and f is a choice function of type $(et)e$ we have:

1. If X is empty then $\langle f \rangle(X)$ is the empty quantifier, independently of the value of f.
2. Otherwise, $\langle f \rangle(X)$ is the principal ultrafilter generated by $f(X)$.

This means we can repeat the results of Winter (1997) using the lower type choice functions and the $\langle\ \rangle$ lifting. However, now we separate between the "choice" stipulation and the "empty case" stipulation: the restriction on the possible functions is as before in the existential quantification procedure; the stipulation that an empty restriction set is mapped to the empty quantifier is in the definition of the $\langle\ \rangle$ lifting operator. This separation will allow us to avoid the first stipulation. That is: to *derive* the restriction of the quantification to choice functions from independent principles. Thus, assuming that (12) is the right semantic analysis of (1), the question we are now facing is: what principles determine the interpretation of (12) so that it ends up with the existential meaning of (1) formalized in (13)?

(12) $\exists f_{(et)e}[C(f) \wedge \langle f \rangle(\mathbf{dog'})(\mathbf{bark'})]$

(13) $\mathbf{dog'} \cap \mathbf{bark'} \neq \emptyset$

This is not a trivial question. An *a priori* possible definition for C in (12) could have been given as follows. Consider the constant function that maps any set to the same entity, say, Charlie Chaplin. Let us call this $(et)e$ function the *Charlie Chaplin mapping*. Note that it is not a choice function: even sets that do not contain him are mapped to the great actor. Consider the definition of C as the singleton consisting only of the Charlie Chaplin mapping. Under this definition, (12) would oddly end up meaning *Charlie Chaplin barked and dogs exist*. What principle rules out such an unintuitive definition of C? Why should *choice* be the natural choice?

5 ON THREE UNIVERSALS OF DETERMINERS

To answer the question, let us briefly review first some familiar results from generalized quantifier theory. I adopt notational conventions as summarized in van der Does and van Eijck (1996). A determiner denotation D on a domain E is a relation between subsets of E. We denote:

$$D_E \in \wp(\wp(E) \times \wp(E))$$

A celebrated observation on natural languages is the *conservativity* of their determiners. For any determiner D the following equivalence pattern holds:

(14) D dogs barked $\Leftrightarrow$ D dogs are dogs that barked

As a property of determiners in generalized quantifier theory, this is formally stated as follows:

(15) A determiner D is *conservative* iff
for all $A, B \subseteq E$: $D_E(A)(B) \Leftrightarrow D_E(A)(A \cap B)$.

The general hypothesis is:

(**U1**) All natural language determiners are conservative.

One remarkable property of conservativity is that it holds also of *complex* determiners like *four or five*, *all but one*, *more than three*, etc.

Another well-known property of determiners is *isomorphism invariance* (ISOM). Intuitively, a determiner being ISOM means that it does not concern the identity of individuals in sets it relates. More formally:

(16) A determiner D is ISOM iff
for every bijection f from E to E': $D_E(A)(B) \Leftrightarrow D_{E'}(f(A))(f(B))$.[2]

The corresponding universal is:

(**U2**) All natural language simple determiners satisfy isomorphism invariance.

This universal is not as general as (U1): it holds only of *simple* (lexical) determiners. A well-known potential counter-example is the case of genitive constructions like *John's*, whose meaning is not isomorphism invariant. Note however that the genitive construction is not lexical and, arguably, even not a syntactic determiner, hence it does not falsify universal 2.

A less familiar, yet not less sound, universal on determiners is *non-triviality*. Barwise & Cooper (1981, 181) consider the logically possible determiners that map any set either to the empty quantifier or to the power set quantifier $\wp(E)$. Let us call this property *right triviality*, which is formally defined as follows.

(17) A determiner D is *right trivial* iff
for all $A,B,C \subseteq E$: $D_E(A)(B) \Leftrightarrow D_E(A)(C)$.

Barwise & Cooper claim that no determiner in natural language is right trivial, but consider this universal itself trivial. Thijsse (1983, fn.12) disagrees with this contention. Without contradiction, I would like to agree with both claims. Barwise & Cooper certainly have a point, as right triviality of a determiner D would result in highly uninformative structures. For instance, with a right trivial determiner, the truth of the sentence *D dogs barked* could have been assessed with no respect to the set of barkers. Given that languages syntactically require sentential predicates, it would have been highly surprising if there existed lexical determiners that would make them semantically redundant in this way. However, Thijsse's remark is also warranted: non-triviality is not *logically* trivial. After all, languages *could* have been more uninformative than they in fact are. For instance, it is remarkable that non-trivial determiners are lexically eliminated but *complex* trivial determiners (see below) and other semantic trivialities (e.g. sentences with tautological/contradictory meanings) are not syntactically ruled out. This may be a relevant fact about the difference between the "autonomy of the lexicon" vs. the "autonomy of syntax" with respect to semantic considerations. Concluding, the non-triviality universal (U3) is not linguistically void.

(**U3**) No natural language simple determiner is right trivial.

Like (U2), this universal does not necessarily hold of complex determiners. For instance, the determiner *less than zero* is right trivial. Moreover, it is also left trivial (see below). Consequently, a sentence like *less than zero dogs barked* is a logical contradiction.

The symmetric property to right triviality is defined as follows.

(18) A determiner D is *left trivial* iff
for all $A,B,C \subseteq E$: $D_E(A)(B) \Leftrightarrow D_E(C)(B)$.

Note that a symmetric universal to (U3) using left triviality would have been redundant, as it follows from (U1) and (U3): a conservative non-right-trivial determiner is not left trivial either.[3] However, a determiner can be conservative and not left trivial while being right trivial.[4] This means that non-right-triviality, but not non-left-triviality, is a significant restriction on conservative determiners.

6 CHOICE IS DERIVED FROM UNIVERSALS ON DETERMINERS

Let us return to the question of how to deduce that the restriction C in (12) allows only choice functions. First, observe that the statement in (12) naturally specifies a determiner: the relation between **dog′** and **bark′**. More generally, the restriction C specifies the determiner D^C as follows:

(19) $D^C \stackrel{def}{=} \lambda A.\lambda B.\exists f[C(f) \wedge \langle f\rangle(A)(B)]$

This means that sentence (1) denotes the proposition $D^C(\mathbf{dog'})(\mathbf{bark'})$. Assume that the determiner D^C satisfies all known universals on determiners. If this were not the case, sentences like (1) could have ended up violating semantic restrictions on all other sentences of the same form. For instance, if D^C were not conservative the equivalence between (1) and the sentence *some dog is a dog that barked* would not have been maintained. In this way, universals (U1)-(U3) can affect the possible values for the restriction C. We will show now that given these universals, the value of C *must* be fixed in such a way that the determiner D^C is – as intuitively required – the existential determiner **some′** defined in (20).

(20) $\mathbf{some'} \stackrel{def}{=} \lambda A.\lambda B.A \cap B \neq \emptyset$

The first result below shows that (U1) implies that C is contained in the set of choice functions CH.

Proposition 1 *Let D^C be conservative. Then $C \subseteq CH$.*

Proof: *Assume by negation $C \setminus CH \neq \emptyset$. Let $f_0 \in C \setminus CH$.*
By definition of CH, there is a set $A_0 \neq \emptyset$ s.t. $f_0(A_0) = a \notin A_0$.
By definition of D^C: $D^C(A_0)(\{a\}) \leftrightarrow \exists f[C(f) \wedge \langle f\rangle(A_0)(\{a\})]$.
The right side of this biimplication is true by the assumption on f_0.
Conclusion: $D^C(A_0)(\{a\})$.
By conservativity of D^C: $D^C(A_0)(A_0 \cap \{a\})$.
Because $a \notin A_0$ we have $D^C(A_0)(\emptyset)$. (i)
But notice that by definition of D^C: $D^C(A_0)(\emptyset) \leftrightarrow \exists f[C(f) \wedge \langle f\rangle(A_0)(\emptyset)]$.
By definition (11): $\neg\langle f\rangle(A_0)(\emptyset)$ for every f.
Conclusion: $\neg D^C(A_0)(\emptyset)$, in contradiction to (i).
We conclude $C \subseteq CH$.

This fact entails that if (12) is to express a "conservative statement" then only choice functions can be quantified over in the semantic process. However, this does not yet determine uniquely the meaning of (12). For instance, assume C is defined as the set of choice functions that assign Charlie Chaplin to every set in which he is contained. Consequently, (12) ends up meaning *some dog barked and [Charlie Chaplin barked if he is a dog]*. Furthermore, C could have been simply empty, making (12) contradictory. However, such unintuitive possibilities are ruled out by universals (U2) and (U3) on D^C. Given these additional constraints, (12) must express the existential proposition in (13). This is established in the following claim.

Proposition 2 *If D^C is conservative, isomorphism invariant and non-trivial then* $D^C = \mathbf{some}'$.

Proof:[5] *$D^C \subseteq \mathbf{some}'$ simply by proposition 1: D^C is conservative and therefore $C \subseteq CH$. By definition of D^C we conclude that $D^C(A)(B)$ implies $\exists f[CH(f) \wedge \langle f \rangle(A)(B)]$.*
By definition this implies $\mathbf{some}'(A)(B)$.
Let us show now $\mathbf{some}' \subseteq D^C$.
Note first that by definition of $\langle f \rangle$:
$D^C(A)(B) \Leftrightarrow \exists f[C(f) \wedge A \neq \emptyset \wedge B(f(A))]$. (*)
Assume $\mathbf{some}'(A)(B)$ holds and assume d is some element of the (non-empty) set $A \cap B$.
Since D^C is non-trivial, there are A' and B' s.t. $D^C(A')(B')$.
By (*) we conclude that there is $f' \in C$.
This f' trivially witnesses the proposition $\exists f[C(f) \wedge A \neq \emptyset \wedge f(A) = f'(A)]$.
Hence, by (*), we conclude $D^C(A)(\{f'(A)\})$. (i)
By conservativity of D^C: $D^C(A)(A \cap \{f'(A)\})$.
By (*): $\exists f[C(f) \wedge f(A) \in A \cap \{f'(A)\}]$.
Hence $f'(A) \in A$.
Let π be a permutation of the E domain that permutes d and $f'(A)$ but maps every other element to itself.
Since $f'(A) \in A$ and $d \in A$ we get $\pi(A) = A$.
By (i) and isomorphism invariance of D^C: $D^C(\pi(A))(\pi(\{f'(A)\}))$.
Or $D^C(A)(\{d\})$.
By (*) we conclude $\exists f[C(f) \wedge A \neq \emptyset \wedge f(A) = d]$.
But $d \in B$, therefore $\exists f[C(f) \wedge A \neq \emptyset \wedge B(f(A))]$.
We conclude that $D^C(A)(B)$ holds, as needed to be proved.

This result shows that the "logical form" (12) and universals (U1)-(U3) ensure that indefinites express classical existential quantification. However, this still under-determines C itself. There are certain logical possibilities besides CH that are not ruled out by propositions 1 and 2. For instance, consider the following definition:

$$C_1 \stackrel{def}{=} \lambda f_{(et)e}.CH(f) \wedge f(\lambda x.\bot) = f(\lambda x.\top)$$

In words: a function is in C_1 iff it is a choice function that assigns the same value to the empty set and to the whole domain of entities. Obviously $C_1 \neq CH$. On the other hand, it is easy to establish that $D^{C_1} = \mathbf{some}'$, hence there is no principle in generalized quantifier theory that rules out C_1.

This under-specification of C is linguistically innocuous. Proposition 2 shows that (U1)-(U3) do specify D^C as the existential determiner. Therefore, the meaning of sentences with existential quantification over functions *is* determined, so any remaining variation among speakers with respect to the value of C seems in principle impossible to detect.

7 ON CLASSIFYING INDEFINITES IN CHOICE FUNCTION THEORY

A prominent issue in semantic theory since Barwise & Cooper (1981) is the classification of NPs according to linguistically relevant denotational properties. In this section I briefly concentrate on two cases where the present choice function treatment can be used with the analyses in Ladusaw (1982) (of partitive NPs) and Keenan (1987) (of *there* sentences), so to capture some semantic properties of these constructions with respect to the scope of indefinites. This will also show some empirical reason to use the present modification of choice function semantics.

7.1 "Definite indefinites" and the partitive constraint

Barwise & Cooper (1981) propose an explanatory semantic analysis of the acceptability of various NPs in partitive constructions. The motivation of Barwise & Cooper (B&C) is to account for contrasts as the ones between (21) and (22) as a direct consequence of the semantics of the partitive construction.

(21) one of { *a*. the boys / *b*. these boys / *c*. the two boys }

(22) *one of { *a*. no boys / *b*. most boys / *c*. both boys }

Ladusaw (1982) improves upon B&C's proposal by considering also the semantics of plurals. In contemporary terminology, Ladusaw adopts the following standard assumptions about plurals in the domain E of individuals:

- $\mathsf{A} \subseteq E$ is the set of *atoms* in E.
- $\Pi : E \to \wp(\mathsf{A})$ is the *part-of* function mapping any $x \in \mathsf{A}$ to $\{x\}$ and any $x \in E \setminus \mathsf{A}$ to a set $X \subseteq \mathsf{A}$ s.t. $|X| \geq 2$.

For $x, y \in E$ we say that *y is an atomic part of x* iff $y \in \Pi(x)$. Intuitively, then, each non-atomic entity is composed of two or more atomic ones.

Another technical notion central for Ladusaw's proposal (as for the present paper) is the Montagovian treatment of individuals as principal ultrafilters in the quantifier domain. Recall:

The *principal ultrafilter* I_a generated by $a \in E$ is the generalized quantifier $\{X \subseteq E : a \in X\}$.

Ladusaw, following B&C, defines the item *of* in partitives as a (partial) function from generalized quantifiers to sets. This definition should make sure, for instance, that the sub-expression *of the boys* in (21a) is interpreted as the predicate *boy*. Thus, whenever there are two or more boys the definite *the boys* is coherent and so the partitive *one of the boys* is equivalent to the indefinite *one boy*. Ladusaw's definition is as follows:

(23) *of* is a function from $\wp(\wp(E))$ to $\wp(E)$ defined by:
$$of(Q) = \begin{cases} \Pi(a) & Q = I_a \\ \text{undefined} & \text{otherwise} \end{cases}$$

Intuitively, whenever the NP following *of* is a principal ultrafilter, it is mapped to the set of atoms composing its generator. For instance, the NP *the boys* in (21a) is standardly treated in cases where there are more than two boys as the plural individual composed of all individual boys. These are assumed to be atomic entities. Therefore, in these situations the expression *of the boys* is coreferential with the predicate *boy*.

According to Ladusaw, the distribution of determiners[6] in NPs following *of* in partitives is directly related to the semantics of the partitive:

(24) *The Ladusaw Partitive Constraint* (PC): A determiner D is allowed in a partitive construction *of* $[_{NP}$D N'$]$ iff for all E, $A \subseteq E$: whenever $D_E(A)$ is defined, so is $of(D_E(A))$.

The attractiveness of this classification lies in its close relationship with the proposed semantics of the partitive: a noun phrase [D N] is allowed to follow *of* in a partitive if the determiner *D* guarantees that the semantics of *of* [D N] is well-defined whenever the semantics of [D N] is well-defined.

To exemplify how this proposal works, consider the Strawsonian definition of the definite determiner, following the proposal of Sharvy (1980) for a unified semantics of the singular/plural definite article:

$$\mathbf{the}(A) = \begin{cases} I_{\max(A)} & \max(A) \text{ exists} \\ \text{undefined} & \text{otherwise} \end{cases}$$
where $\max(A)$ is the $x \in A$ s.t. for every $y \in A$: $\Pi(y) \subseteq \Pi(x)$, if such x exists, undefined otherwise.

Every NP with the definite article satisfies Ladusaw's PC according to this definition and hence it is allowed in partitive constructions. Thus, both the acceptability and the meaning of such partitives are accounted for.[7] By contrast, the determiners in (22) give rise to non-ultrafilter quantifier in many situations, which accounts for their unacceptability.

Especially important for Ladusaw's proposal is the contrast (21c) vs. (22c), which is not explained by B&C's account. Ladusaw shows how this contrast follows from the collectivity of NPs like *the two boys* (as in *the two boys are a nice couple*) *vis-à-vis* the distributivity of *both boys* (cf. ?*both boys are a nice couple*). This fact is explained by identifying the first NP with the *principal ultrafilter* generated by a plural entity composed of two boys, whenever there are exactly two boys. However, *both boys* is equivalent to the universal quantifier *every boy* in these situations, which is not a principal ultrafilter. A common way to achieve this difference is to analyze numerals like *two* as *adjectives*: functions from predicates to predicates. The numeral *two*, for instance, maps the predicate *boys* to the set of plural entities consisting of two boys. See Winter (1997) for an implementation in a choice function analysis of indefinites.

One potential counter-example to (24) that Ladusaw mentions is the case of indefinites following the partitive *of*. Consider the following example from Ladusaw (1982):

(25) That book could belong to one of three people.

An indefinite NP like *three people* is analyzed by B&C as a quantifier that is not a principal ultrafilter. According to both B&C and Ladusaw, it should therefore be ruled out in partitives, contrary to fact. Ladusaw, however, notes that (25) is equivalent to (26), where *three people* is assigned "wide scope" over the partitive.

(26) There are three people such that that book could belong to one of them.

Consequently, Ladusaw tentatively proposes that (25) is "appropriately used only when the user has a particular group of people in mind". This is the same informal intuition of Fodor & Sag (1982) about the "specificity" of indefinites. If this idea can be made precise then (25) is not a counter-example to the PC because the indefinite is treated here as an individual (=principal ultrafilter), not as the B&C standard existential quantifier.

Using choice functions we can modify Ladusaw's proposal in a way that captures this intuition. The first step is to repeat the above definitions, but replacing the Strawsonian partial strategy with a Russellian approach to descriptions, more suitable to the present treatment of choice functions.

(27) $$of(Q) \stackrel{def}{=} \begin{cases} \Pi(a) & Q = I_a \\ \emptyset_{et} & \text{otherwise} \end{cases}$$

(28) $$\mathbf{the}(A) \stackrel{def}{=} \begin{cases} I_{\max(A)} & \max(A) \text{ exists} \\ \emptyset_{(et)t} & \text{otherwise} \end{cases}$$

In modifying Ladusaw's PC, we have to adapt it to these Russellian definitions. A first attempt is the following:

(29) PC (1st version): For all $E, A \subseteq E$: $D_E(A) \neq \emptyset_{(et)t} \Rightarrow of(D_E(A)) \neq \emptyset_{et}$

This modification correctly rules out many undesired determiners and rules in the definite description. It is unsatisfactory, however, because a trivial determiner like *less than zero* is not ruled out now, by contrast to Ladusaw's definition. The reason is that according to B&C this determiner is always defined, but it vacuously satisfies (29) because of its triviality. This is undesired of course, as partitives like **one of less than zero boys* are ruled out just like **one of less than five boys*. Therefore, in order to mend (29) we have to require that D is not trivial. Such an addition is not necessarily *ad hoc*, since the general question of how to derive relevant filters on NPs from triviality considerations such as B&C's or Ladusaw's is not completely settled at the moment (see e.g. Keenan 1987 for some reservations concerning this kind of reasoning). It may be that "triviality" should concern also vacuous satisfaction of plausible principles like (29). Let us therefore adopt the following modification:

(30) PC (2nd version): D is not trivial and for all $E,\ A \subseteq E$:
$$D_E(A) \neq \emptyset_{(et)t} \;\Rightarrow\; of(D_E(A)) \neq \emptyset_{et}.$$

The more substantial step in revising the PC is to relativize it to variable assignments in a way that allows the choice function treatment to account for cases like (25). This modification is natural, following the strategy of Heim (1987) (see below). We denote by D^{var} a determiner containing the (possibly empty) set of free variables in *var*.

(31) PC (final version): A determiner D^{var} is allowed in a partitive construction *of* $[_{\text{NP}}$ D N'$]$ iff D^{var} is not trivial[8] and for all $A \subseteq E$, for every assignment to the variables in *var*: $D_E(A) \neq \emptyset_{(et)t} \;\Rightarrow\; of(D_E(A)) \neq \emptyset_{et}$.

This definition of the PC accounts for the potential counter-example in (25), which is roughly formalized below.[9]

(32) $\exists f[CH(f) \wedge$ that book could belong to one $of(\langle f\rangle$(three people)$)]$

The determiner $\langle f\rangle$ satisfies the PC: by definition whenever $A \neq \emptyset$ we have $\langle f\rangle(A) = I_{f(A)}$, thus $of(\langle f\rangle(A)) = \Pi(f(A)) \neq \emptyset$. On the other hand, when $A = \emptyset$ the quantifier $\langle f\rangle(A)$ is empty. Consequently, the meaning (32) stands for can plausibly be paraphrased by (26).

Without getting into further details, I would like to make two general remarks. First, cases like (25) are not likely to be explained by standard scope mechanisms (Quantifier Raising, Quantifying-in) that do not treat the special scopal behaviour of indefinites. The reason is that indefinites, in the context of partitives as well, show the free scopal behaviour beyond syntactic islands, which does not appear with other NPs. For instance, sentence (33), where the antecedent of the conditional is an adjunct scope island, still has the wide scope reading (34).

(33) If that book belongs to one of three people I know, then we should keep it very carefully.

(34) There are three people I know such that if that book belongs to one of them, then we should keep it very carefully.

Second, Ladusaw's "specificity" intuition is subject to the same objections of Ruys (1992) (among others) against Fodor & Sag (1982). For instance, sentence (35) has the reading (36).

(35) Every book could belong to one of three people who admire it.

(36) For every book there are three people who admire it such that it belongs to one of them.

Here, as in Ruys's examples, there should be no three particular people with the relevant property. To wit, in a situation where for each book there are *different* three people who admire it, one of whom is the owner of the book, (35) is true. This "intermediate scope" of the indefinite is captured by the choice function mechanism (see Reinhart 1997 and Winter 1997).

The point in the analysis above that is most crucial for the purpose of this paper is the following. The above extension of Ladusaw's PC shows an advantage of the present treatment of choice functions over the one in Winter (1997). If choice functions are treated as in that paper using the determiner type, then there is no natural way to respect the PC: a function f of type $(et)((et)t)$ can potentially be assigned any determiner value, especially ones that violate the PC. By contrast, as we saw, the lifted function $\langle f\rangle$ respects the PC for every value of f.

7.2 "Strong indefinites" and the there*-sentence constraint*

Another well-known case where Barwise & Cooper's article initiated a research into denotational effects on grammaticality is the case of *there*-sentences. The contrastive cases (37) and (38) show some relevant examples.

(37) There is/are $\left\{\begin{array}{l}\text{some boy(s)}\\ \text{no boy(s)}\\ \text{two boys}\end{array}\right\}$ in the kitchen.

(38) *There is/are $\left\{\begin{array}{l}\text{the boy(s)}\\ \text{every boy}\\ \text{most boys}\end{array}\right\}$ in the kitchen.

Keenan (1987) specifies a set of "basic" determiners (whose definition is spared here) and defines a basic determiner as *existential* using the semantic definition below.

(39) A determiner D is *existential* iff
for all $A,B\subseteq E$: $D_E(A)(B) \Leftrightarrow D_E(A\cap B)(E)$.

Keenan argues for a semantics of *there* sentences where it follows that NPs with a basic determiner can be interpreted (existentially[10]) in *there* sentences if and only if the determiner is existential. This account captures many of the facts about the problem (with some familiar exceptions). Heim (1987) discusses the question of the scope of indefinites in *there*-sentences. Consider the contrast between (40) and (41), based on an example from Milsark (1977).

(40) Ralph believes that some lunatic is spying on him.

(41) Ralph believes that there is some lunatic spying on him.

According to Milsark and Heim, (40) has a reading that (41) lacks. This is the "wide scope" (*de re*) reading of the indefinite over the predicate *believe*:

(42) There is some lunatic such that Ralph believes that she is spying on him.

This empirical claim seems correct, though somewhat subtle. I would like to strengthen it using a similar example from the Hebrew, replacing the predicate *believe* by the predicate *toha* ("wonder"):[11]

(43) sara toha ha'im eyze mešuga še-animakir nimca ba-bayit šela
Sara wonder whether some lunatic that-I know is in-house her

"Sara wonders whether some lunatic I know is in her house"

(44) sara toha ha'im yesh eyze mešuga še-animakir ba-bayit šela
Sara wonder whether there some lunatic that-I know in-house her

"Sara wonders whether there is some lunatic I know in her house"

Consider a situation where Sara thinks there is a lunatic in her house, but she doesn't know who. I am a police officer who knows *Jack the Ripper*, and I suggest to Sara that this particular lunatic is in her house. Sara wonders if this can be the case but nevertheless, she believes I know the lunatic in her house. Sentence (44) is false: Sara does not question my acquaintance with some or other lunatic who is in her house. By contrast, in this situation (43) is true, or at least has a true reading, the *de re*/wide scope reading of the indefinite: there is a particular lunatic (namely Jack the Ripper) whose presence in her house Sara questions.[12]

According to Heim, such contrasts are accounted for assuming that a wide scope indefinite leaves behind an *e*-type variable ("trace"). In (44), this construal would make the *there*-sentence ungrammatical at LF. The reason is that there are variable assignments for which the variable denotes an individual, or a principal ultrafilter, a quantifier that is not licensed in *there*-sentences according to most treatments.

Heim's conclusion holds also with respect to the choice function treatment of the scope of indefinites and Keenan's definition of existential determiners. Reconsider sentence (41). Its wide scope reading using choice functions is roughly as given below.

(45) $\exists f[CH(f) \wedge \text{Ralph believes that there is } \langle f\rangle(\text{lunatic})(\text{spying on him})]$

For many models and variable assignments the determiner $\langle f\rangle$ is not existential. For instance, consider $A=\{a,b\},\ B=\{b\},\ f(A)=a,\ f(B)=b$ where $a \neq b$. In this case $\langle f\rangle(A)(B)$ does not hold because $B \notin \langle f\rangle(A)=I_a$. However, $\langle f\rangle(A\cap B)(E)$ holds, as $E \in \langle f\rangle(A\cap B)=I_b$. If Heim's reasoning is correct this fact rules out the wide scope reading of the indefinite.

The time is ripe for dealing with an elementary question: what guarantees that indefinites ever have an existential ("weak") narrow scope interpretation? This is of course needed to rule in *there* sentences with indefinites of a *narrow*(*est*) scope construal. According to Reinhart (1997), indefinites have, in addition to their choice function treatment, also a traditional generalized quantifier reading. This immediately answers the question using Keenan's definition. However, as argued in Winter (1997), as soon as we adopt a choice function analysis, there is no evidence that an analysis of the indefinite article as an existential determiner is still necessary. But this does not mean that the determiner in an indefinite cannot end up denoting the existential determiner after all, when the rest of the compositional mechanism is considered. The compositional proposal in Winter (1997) allows the derivation in (46) below, where the existential quantifier over choice functions composes directly with the determiner $\langle f \rangle$. This happens because $\langle f \rangle$ is actually defined (without free variables) as a function from $(et)e$ functions to determiners. More details on this mechanism are given in Winter (1997), which can be easily adjusted to the present modification.

(46) $\lambda X_{((et)e)t}.\exists g[CH(g) \wedge X(g)]$ (existential quantifier over choice functions)
$\lambda g.\lambda A.\lambda B.\langle g \rangle(A)(B)$ (mapping the function g to the determiner $\langle g \rangle$)
$\Rightarrow\ \lambda A.\lambda B.[\lambda X.\exists g[CH(g) \wedge X(g)]](\lambda g.\langle g \rangle(A)(B))$
$=\ \lambda A.\lambda B.\exists g[CH(g) \wedge \langle g \rangle(A)(B)]\ =\ \mathbf{some}'$

Since the choice function mechanism derives in this way the standard reading of the indefinite determiner, it still has an existential denotation in the narrowest scope construal of the existential quantifier over choice functions.

I believe the discussion above, however superficial, shows one point in which the "specificity" intuition is relevant to the choice function mechanism. Whenever the semantic properties of noun phrases play a role in a linguistic theory, as it is the case in partitives and *there*-sentences, some facts indicate that the "free choice function variable" reading of an indefinite NP is to be classified as similar to "referential" NPs like definite NPs. To wit, it is "definite" (= allowed in partitives) and "strong" (= disallowed in *there*-sentences). With the higher type analysis for choice functions in Winter (1997) this classification is problematic without further stipulations, as the variable ranges over *all* possible determiner values. However, in the lower type analysis adopted here the expression $\langle f \rangle$ maps any non-empty set to a principal ultrafilter, no matter what the value of f is. This agrees with the "specificity" intuition, although, of course, the analysis does not hinge on this informal notion.

8 A SPECULATIVE REMARK

Two aspects were central to the argument in favour of the revised mechanism of choice functions: (i) the conceptual argument of section 6, based on deriving the choice restriction from universals on determiners. (ii) The sound classification of indefinites containing function variables although these are not specified locally as choice functions.

While both arguments involve the standard motivation to reduce the amount of assumptions in the theory, the general nature of the first one deserves some further attention. Using Chomskyan terminology, the choice function approach seems to assume that the introduction of function variables in indefinite NPs, as well as the existential quantification over choice functions, are part of human linguistic knowledge (innate or acquired). A similar position is taken in Barwise & Cooper (1981, 200) with respect to the semantic universals of generalized quantifier theory. In this paper I tried to show one point where the two perspectives are complementary: a speaker, having the knowledge of principles of generalized quantifier theory and existential quantification over functions, can *deduce* the “choice” restriction. The speaker does not have to learn it separately or be given it (innately) in addition to other principles. This idea may have psycholinguistic implications. For instance, it is expected that failure to obey the principle of conservativity should immediately affect the interpretation of indefinites. Of course, it remains to be seen whether and how this prediction can be tested, let alone verified.

NOTES

1 Syntactically complex numerals like *more than three, less than three* or *exactly three* are still treated as in generalized quantifier theory. Arguments for this treatment are given in Reinhart (1997) and Winter (1998).

2 Recall that a function $f: S \to S'$ is a *bijection* (isomorphism) iff for all $x, x' \in S$: $f(x) = f(x') \Rightarrow x = x'$ (f is an injection) and for every $y \in S'$ there is $x \in S$ s.t. $f(x) = y$ (f is a surjection). For every function $f: S \to S'$, for any $A \subseteq S$ we denote: $f(A) = \{f(x) : x \in A\}$.

3 **Proof**: Assume by negation D is conservative and not right trivial but left trivial. For any domain E, left triviality implies $\forall A \forall B[D_E(A)(B) \leftrightarrow D_E(\emptyset)(B)]$. By conservativity $\forall B[D_E(\emptyset)(B) \leftrightarrow D_E(\emptyset)(\emptyset)]$. Conclusion: $\forall A \forall B[D_E(A)(B) \leftrightarrow D_E(\emptyset)(\emptyset)]$. Especially D is right trivial. Contradiction.

4 For instance, the determiner D s.t. $D_E(A)(B)$ holds iff $A \neq \emptyset$.

5 This proof was greatly simplified thanks to a proposal by an anonymous reviewer.

6 In fact, Ladusaw (1982, 238f) states the PC in terms of full NPs rather than determiners. Although I believe this direction is desired, it faces certain complications with complex NPs (e.g. coordinations) that I would like to avoid here. Therefore, I keep to the B&C practice of concentrating on simple NPs containing an overt determiner.

7 Note that partitives like **one of the book* are not semantically ruled out. Such effects may reasonably attributed to pragmatics. As Ladusaw mentions, the partitive *some of the book*, by contrast, is fine when *the book* is understood as a mass term.

8 The definition of triviality for determiners containing free variables is a natural extension of this notion using universal quantification over variable assignments. For instance, D^{var} is right-trivial iff for all $A, B \subseteq E$, for every variable assignment to the variables in *var*: $D_E(A)(B) \Leftrightarrow D_E(A)(C)$.

9 Recall that in the choice function treatment of numerals, the expression *three people* does not denote a generalized quantifier but a set of plural individuals (each consisting of three singular members).

10 A subtlety: for Keenan, sentences as in (38) are not necessarily ungrammatical, but rather, unlike (37), they do not convey existential statements, in a sense that he explicitly defines.

11 English favors with such verbs a negative polarity item instead of the indefinite, which complicates the test.

12 Moreover, in this situation the following is not contradictory:

(i) sara toha ha'im eyze mešuga še-ani makir nimca ba-bayit šelaaval hi lo
Sara wonder whether some lunatic that-I know is in-house her but she not
toha ha'im yesh eyze mešuga še-ani makir ba-bayit šela
wonder whether there some lunatic that-I know in-house her
"Sara wonders whether some lunatic I know is in her house but she doesn't wonder whether there is some lunatic I know in her house"

REFERENCES

Barwise, J. & Cooper, R. 1981. Generalized Quantifiers and Natural Language. *Linguistics and Philosophy 4*, 159-219.

van Benthem, J. 1984. Questions about Quantifiers. *Journal of Symbolic Logic 49*, 443-466.

van der Does, J. 1996. Quantification and Nominal Anaphora. In: K. von Heusinger & U. Egli (eds.). *Proceedings of the Konstanz Workshop Reference and Anaphoric Relations*. Arbeitspapier 79. Fachgruppe Sprachwissenschaft, Universität Konstanz, 27-56.

van der Does, J. & van Eijck, J. 1996. Basic Quantifier Theory. In: J. van der Does & J. van Eijck (eds.). *Quantifiers, Logic and Language*. Stanford/CA: CSLI Publications, 1-45.

Fodor, J. & Sag, I. 1982. Referential and Quantificational Indefinites. *Linguistics and Philosophy 5*, 355-398.

Heim, I. 1987. Where Does the Definiteness Restriction Apply? Evidence from the Definiteness of Variables. In: E. Reuland & A. ter Meulen (eds.). *The Representation of (In)definiteness*. Cambridge/MA: MIT Press, 21-42.

Hilbert, D. & Bernays, P. [1939] 1970. *Grundlagen der Mathematik*. Vol. II. 2nd ed. Berlin; Heidelberg; New York: Springer.

Keenan, E. 1987. A Semantic Definition of 'Indefinite NP'. In: E. Reuland & A. ter Meulen (eds.). *The Representation of (In)definiteness*. Cambridge/MA: MIT Press, 286-317.

Ladusaw, W. 1982. Semantic Constraints on the English Partitive Construction. *Proceedings of the First West Coast Conference on Formal Linguistics (WCCFL 1)*, 231-242.

Milsark, G. 1977. Towards an Explanation of Certain Pecularities in the Existential Construction in English. *Linguistic Analysis 3*, 1-30.

Reinhart, T. 1992. Wh-in-situ: An Apparent Paradox. In: P. Dekker & M. Stokhof (eds.). *Proceedings of the Eighth Amsterdam Colloquium*. Institute for Logic, Language and Computation (ILLC), University of Amsterdam, 483-491.

Reinhart, T. 1997. Quantifier Scope: How Labor is Divided Between QR and Choice Functions. *Linguistics and Philosophy 20*, 335-397.

Ruys, E. 1992. *The Scope of Indefinites*. Ph.D. dissertation. University of Utrecht.

Sharvy, R. 1980. A More General Theory of Definite Descriptions. *The Philosophical Review 89*, 607-624.

Thijsse, E. 1983. On some Proposed Universals of Natural Language. In: A. ter Meulen (ed.). *Studies in Modeltheoretic Semantics*. Dordrecht: Foris, 19-36.

Winter, Y. 1997. Choice Functions and the Scopal Semantics of Indefinites. *Linguistics and Philosophy 20*, 399-467.

Winter, Y. 1998. *Flexible Boolean Semantics: Coordination, Plurality and Scope in Natural Language*. Ph.D. dissertation. University of Utrecht.