Locating Sets: Spatial Semantics of Indefinites and Collective Descriptions

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Abstract

This paper addresses systematic variations in the interpretation of indefinites and plurals when appearing within prepositional phrases and other spatial expressions. We show that these variations are explained when spatial processes in semantics locate the *sets* associated with properties and other set-based denotations of nominals. Starting with spatial indefinites we consider contrasts like the following.

(i) Michael is far from a gas station. (ii) Michael is close to a gas station.

In the appropriate context – e.g. a car race in the desert, when Michael's car is running out of gas – sentence (i) means that Michael is far from all gas stations. By contrast, sentence (ii) only has an existential interpretation, which states that there is at least one gas station near Michael. We show that the existential-universal alternation in (i)-(ii) marks two extremes in a much wider range of (pseudo-)quantificational interpretations of spatial nominals, which cannot be explained by any known theory of quantification over entities. By contrast, this behavior is consistent with *property-based* theories of indefinites (McNally 2009, "Properties and entity correlates", in Giannakidou & Rathert, eds., OUP). We propose that properties and other set concepts have a spatial dimension that is operational in locative sentences. Variations as in (i)-(ii) are explained by locating the set extension of the property denoted by the indefinite. Thus, sentences (i)-(ii) make a claim about Michael's distance from the spatial set concept 'gas station'. Such properties are located similar to other spatial objects. For example, being far from the property 'gas station' means being far from all of its 'parts' – individual gas station instances. By contrast, being close to a property means being close to at least one of its instances. Thus, the pseudo-quantificational variation in (i)-(ii) directly follows from meanings of spatial expressions and not from any compositional difference. This account of spatial indefinites naturally extends to spatial sentences with collective descriptions like 'the lakes' or 'the mountain chain', which show similar interpretative variability. At the same time, we observe some systematic differences between collective descriptions and singular indefinites in spatial constructions. We treat these differences as resulting from the referential cohesion of 'impure atom' entities denoted by plurals, as opposed to property denotations of indefinites. Spelling out formally the proposed spatial semantics leads to a general theory of set-based locative expressions.

1 Introduction

The traditional analysis of indefinite descriptions as existential quantifiers has led to important advances in their semantic treatment within the class of natural language nominal phrases. At the same time, it has often been observed that indefinites behave in ways that are apparently inconsistent with standard theories of quantification. In the last four decades, much research in formal semantics has been driven by the motivation to explain, or explain away, challenges for existential accounts of indefinites. Relevant phenomena that have been extensively studied are the interactions between indefinites and anaphors, their generic and kind readings, scope effects, existential *there* sentences, and indefinites in predicate positions.[1]

This paper addresses a problem for existential theories of indefinites that has received very little attention so far. We focus on the interpretation of *spatial indefinites*: indefinite descriptions appearing within prepositional phrases or other spatial expressions. As we show, the semantic behavior of spatial indefinites challenges quantificational theories of indefinites as well as theories of spatial reference. Consider for example the sentences below.

(1) a. Michael is far from a gas station.
    b. Michael is close to a gas station.

In the appropriate context – e.g. a car race in the desert, when Michael's car is running out of gas – sentence (1a) means that Michael is far from all gas stations. We informally say that sentence (1a) exhibits a universal interpretation. By contrast, sentence (1b) only has an existential interpretation, which states that there is at least one gas station near Michael. As we will show, interpretative alternations as in (1) appear with many spatial relations, correspond to some familiar problems with negative polarity *any*, and show expected differences between the *a* and *some* particles as well as parallelism with genericity effects. At the same time, despite their great semantic complexity, spatial readings of indefinites are distinguished from generic and other readings of indefinites in being strictly extensional. This allows us to concentrate on the relationships between in/definiteness and spatial expressions.

At first glance, the puzzle that sentence (1) illustrates may seem similar to other problems about the interpretation of indefinites. Apparently universal readings of indefinites also appear in generic sentences and in the scope of downward-monotone operators. However, we argue that spatial indefinites cannot be analyzed as wide-scope generics or narrow-scope existentials. Most importantly, the existential-universal alternation in (1) only marks two extremes in a wide range

[1] Some notable references on these problems, more or less respectively, are: Heim (1982), Kamp & Reyle (1993), Groenendijk & Stokhof (1991), Elbourne (2005); Carlson (1977), Krifka et al. (1995); Fodor & Sag (1982), Reinhart (1997), Kratzer (1998); Milsark (1977), Reuland & ter Meulen (1987); Landman (2004). For surveys on generalized quantifiers and general assumptions on natural language nominal/determiner phrases see Peters & Westerståhl (2006) and Szabolcsi (2010).

of interpretations available with various spatial relations. As we will show, spatial indefinites that involve non-monotonic distal statements – e.g. (*exactly*) *5km from a gas station* – show interpretations that are neither pure-existential nor pure-universal. To be 5km from a gas station is not to be 5km from all gas stations, nor is it to be 5km from some or other gas station – it is to be 5km from the *nearest* gas station. Analyzing spatial indefinites as wide-scope generics or narrow-scope existentials does not account for this and similar cases.

Notwithstanding these qualms, we will propose a framework that gives indefinites narrow-scope below spatial relations, which assumes a close connection between spatial interpretations of indefinites and their generic analysis. Alternations as in (1) are explained by analyzing the indefinite as a *property* or a *kind*, which is predicated over by the spatial relation. This follows a leading idea in much semantic research since Milsark (1977) that treats (some) indefinites as having predicative denotations, or their intensional guise as properties or kinds (Carlson 1977, Chierchia 1998). A property or a kind associated with an indefinite like *a bulldog* represents what different bulldogs have in common in different situations. This analysis constitutes a formal semantic correlate (Carlson 2009) of what cognitive scientists more generally refer to as *concepts* or *categories* (Margolis & Laurence 1999, Hampton 2012). We will henceforth consistently use the term *properties* for such representations. The link between properties and extensional predicates has led to a natural analysis of indefinites in predicate positions (e.g. *Fido is a bulldog*). This account was extended to cover existential and generic readings of indefinites in argument positions (*a bulldog is missing/usually friendly*), as well as other uses of indefinites in different languages.[2] We propose that properties, similarly to entity concepts, have a spatial dimension that is operational in locative sentences like (1a-b). Thus, just like the sentences *Michael is far from/close to London* make a statement about Michael's distance from the spatial entity *London*, the sentences in (1) make a claim about Michael's distance from the spatial property *gas station*.

In our property-based analysis of spatial indefinites we employ basic assumptions from other theories of locative expressions. The spatial region occupied by an entity x is referred to as x's *eigenspace* (Wunderlich 1991). Spatial relations between eigenspaces are intimately related to sub-part relations between the corresponding entities. For instance, being far from London means being far from all of its sub-parts. Conversely, being close to London means being close to at least one of its sub-parts. We show that the contrast in (1) follows from a similar consideration as soon as spatial semantics is tuned to deal with properties. We propose that the eigenspace for the a property-denoting indefinite like *a gas station* is the <u>union</u> of eigenspaces of its sub-parts: single gas station entities. With this assumption our proposal correctly expects sentence (1a) to require that Michael be far from the union of gas station regions. This entails that Michael is far from all gas stations.

[2] See Carlson (1977), Chierchia (1984), Zimmermann (1993), McNally (1998), van Geenhoven (1998), Dayal (1999), Condoravdi et al. (2001), Chung & Ladusaw (2003), van Geenhoven & McNally (2005), McNally (2009).

By contrast, sentence (1b) is analyzed as stating that Michael is close to the union of gas station regions. This statement means that Michael is close to at least one gas station, as intuitively required.

This account of spatial indefinites naturally extends to spatial sentences with *collective descriptions*. The following pairs of sentences show basically the same contrast as in (1).

(2) a. Michael is far from this group of gas stations.
 b. Michael is close to this group of gas stations.

(3) a. Michael is far from these gas stations.
 b. Michael is close to these gas stations.

Both singular descriptions like *this group of gas stations* and plural descriptions like *these gas stations* are analyzed as denoting semantic objects that are directly or indirectly associated with collections of entities (Lasersohn 1995, Schwarzschild 1996). We refer to such singular and plural NPs as *collective descriptions*. Similarly to spatial indefinites, the collective descriptions in (2) and (3) show a (pseudo-) universal interpretation when appearing with the spatial expression *far from*, but a (pseudo-)existential interpretation when appearing with the expression *close to*. We propose that the spatial origins of the contrasts in (1)-(3) are the same, and stem from the *set-based* nature of properties and collections. At the same time, we observe some systematic differences between collective descriptions and singular indefinites when they appear in spatial constructions. We treat these differences as resulting from general strategies for interpreting and locating collective descriptions, which are not active with property-denoting indefinites. First, interpreting plural descriptions involves a context-sensitive mapping from sets to 'impure atoms' (Link 1984, Landman 1996). Second, locating an atom denotation of a singular/plural description involves the 'functional hull' of this atom, which may be different than the union of its sub-parts' eigenspaces (Herskovits 1986). We propose that locating property denotations of indefinites does not involve these two processes. This accounts for the differences we observe between spatial collective descriptions and spatial indefinites.

The paper is organized as follows. Section 2 introduces our assumptions about spatial semantics and our proposal about its interactions with the property analysis of singular indefinites. Section 3 shows that these interactions correspond to more well-known problems about the interpretation of indefinites. Section 4 studies spatial interpretations of collective descriptions and their relations with the spatial interpretation of indefinites. Sections 5 and 6 develop formal details of the proposed semantics. This formal analysis crucially supports our main thesis, according to which locating sets of entities is an important ingredient of spatial meaning and part-whole relationships in language. By way of conclusion, section 7 elaborates on further implications of this thesis.

2 Spatial indefinites and the Property-Eigenspace Hypothesis

This section addresses existential and non-existential effects with spatial indefinites. We show that the variety of interpretations of spatial indefinites is inconsistent with existential or generic quantification over entities. Instead, we analyze spatial indefinites as property-denoting, with the property denotation as the direct argument of the spatial function. To account for the meaning effects that follow from this semantic structure, we introduce a new hypothesis about spatial interpretations of property-denoting indefinites. According to this hypothesis, the *Property-Eigenspace Hypothesis* (PEH), a property in a spatial sentence is analyzed using the *union* of regions for entities in the property's extension. We show how the PEH accounts for various interpretations of spatial indefinites.

2.1 Simple locatives and part-whole structure

In order to illustrate the basic assumptions of our analysis, let us reconsider the following simple locative sentences.

(4) Michael is far from/close to London.

As in other theories, we analyze spatial sentences as in (4) as establishing binary relations between 'locations' of entities, or abstract *regions* in a spatial ontology. In section 5 we will develop further the relevant aspects of this ontology. Without getting into technical details, at this stage it suffices to say that regions may be conceived of as sets of points in some spatial domain. A region associated with an entity is referred to as the entity's *eigenspace*.[3] Specifically, in (4) the entity denotations of *Michael* and *London* are associated with their respective eigenspaces in the spatial model. The locative expressions *far from* and *close to* contribute binary relations between these two regions. Suppose that the eigenspaces for *Michael* and *London* are the regions M and L, respectively. Sentences (4) are therefore assumed to denote the following propositions.

(5) a. $\textsc{far\_from}(M, L)$
b. $\textsc{close\_to}(M, L)$

Intuitively, these statements should be read as follows.

(6) a. The distance between the regions M and L is big.
b. The distance between the regions M and L is small.

Note the following simple properties of these readings.

[3]The term *eigenspace* is due to Wunderlich (1991). The German prefix *eigen* literally means "its own". Thus, when referring to the "eigenspace" of an entity x, we informally talk about x's "own region", with no mathematical connotation.

(7) a. Michael is far from London ⇔ Michael is far from *each* part of London.
b. Michael is close to London ⇔ Michael is close to *at least one* part of London.

In section 5 we will analyze formally the reasons that speakers accept the equivalences in (7). At an informal level, we note that these entailment patterns with *far from* and *close to* are expected by the intuitive paraphrases in (6). For instance, in (7a), if Michael is far from London then his distance from London is some distance d that in the context is judged to be sufficiently big. By simple properties of distance measuring, Michael's distance from *any* part of London is equal to d or bigger. By contrast, in (7b), if Michael is close to London, then his distance from London is some distance d that is judged to be sufficiently small. This small value measures Michael's distance to *some* part of London, but Michael's distance to further parts of the metropolis may be bigger. Thus, the '⇒' entailment in (7b) is only to an existential statement over subparts of London, but not to a universal statement.

2.2 The Property-Eigenspace Hypothesis

With this standard analysis in mind, let us now reconsider the contrast in (1), which is repeated below.[4]

(8) a. Michael is far from a gas station.
b. Michael is close to a gas station.

As we saw, sentence (8a) readily shows a universal interpretation, as paraphrased in (8'a) below. By contrast, sentence (8b) can only be interpreted existentially, as paraphrased in (8'b).

(8') a. '*Michael is far from each gas station*'.
b. '*Michael is close to at least one gas station*'.

As support for our claim about the contrast between sentences (8a) and (8b), let us consider two situations. Starting from sentence (8b), consider a situation where Michael is close to one gas station and far from many others. In this situation sentence (8b) is intuitively true, like its proposed existential paraphrase (8'b). By contrast, in the analogous situation where Michael is far from one gas station and close to many others, the salient interpretation of sentence (8a) is intuitively false, like its proposed universal paraphrase (8'a). Sentence (8a) can only be true in this

[4] Besides Mador-Haim & Winter (2007), the only work we are aware of that discusses non-existential phenomena as in (8a) is Iatridou (2003), and the later version Iatridou (2007). Iatridou attributes to Irene Heim (p.c.) the observation about the non-existential interpretation of 'gas station' sentences like (8a) (incidentally, the specific 'gas station' example in (8a) was independently suggested to us by Louise McNally, p.c.). The focus of Iatridou's work is the analogy with the temporal domain (see (17)-(18)) and it does not address the variety of non-existential effects with spatial indefinites that we analyze in this paper.

situation under a marked interpretation where the indefinite is understood as 'specific', similar to *a certain gas station* (see section 3.3). We conclude that a standard existential analysis of indefinites works well in sentence (8b) but fails with sentence (8a).[5]

Accounting for these observations, and following other theories, we propose that the indefinite in sentences (8a-b) denotes a *property*.[6] This property, like the entity for *London* in (4), is associated with an eigenspace. Crucially, we assume that a property P's eigenspace is the *union of eigenspaces of the entities in P's extension*. For instance, suppose that the indefinite *a gas station* in (8) denotes a property $\textit{GS}$, whose extension consists of three gas station entities. Suppose that the eigenspaces of these three entities are A, B and C. Our analysis takes the eigenspace of the property $\textit{GS}$ to be the union $A \cup B \cup C$ of the three entity eigenspaces. Supposing again that Michael's eigenspace is the region M, we analyze sentences (8a-b) as in (9a-b), respectively.

(9) a. $\textsc{far\_from}(M, A \cup B \cup C)$
b. $\textsc{close\_to}(M, A \cup B \cup C)$

This accounts for the linguistic contrast in (8), based on the intuitive contrast below:

(10) a. Being *far from* a union of regions means being far from *each* of the regions.
b. Being *close to* a union of regions means being close to *at least one* of the regions.

By virtue of the meaning of the relation $\textsc{far\_from}$, Michael's location M in (9a) must be far from each of the regions A, B and C, i.e. from all gas stations. By contrast, due to the meaning of the relation $\textsc{close\_to}$, in (9b) M must only be close to one of the regions A, B and C, i.e. to one of the gas stations.

This reasoning is parallel to the intuitive reasoning we examined in (7) above. Indeed, with some reservations (cf. sections 4 and 5), we propose that the contrast in (8) is accounted for by the same spatial principles that support the contrast in (7). The main new ingredient of this analysis lies in the spatial treatment of property-denoting indefinites. This is summarized in the following hypothesis.

Property-Eigenspace Hypothesis (PEH). *The eigenspace of a property P is the union of eigenspaces of entities in P's extension.*

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[5]In section 2.4 we will show that this remains the case also when the existential analysis is augmented with some additional stipulations.

[6]For an introduction to intensional semantics, and in particular properties, see Gamut (1991). For relevant works on the analysis of indefinites as predicates or properties, see Zimmermann (1993), McNally (1998), van Geenhoven (1998), Dayal (1999), Condoravdi et al. (2001), Chung & Ladusaw (2003), van Geenhoven & McNally (2005). In our analysis we usually only rely on property *extensions*: sets of entities. However, we use the term 'property' to stress the connection we see between our treatment and intensional theories of indefinites, especially kind-based theories of their generic interpretations (Carlson 1977 and subsequent work).

The PEH directly accounts for the universal/existential alternation in (8). In a similar fashion, the PEH explains the contrast between sentences (11a) and (11b) below, which are paraphrased in (11').

(11) a. Michael is more than 20km from a gas station.
b. Michael is less than 20km from a gas station.

(11') a. '*Michael is more than 20km from each gas station*'.
b. '*There is a gas station less than 20km from Michael*'.

In section 5 we will further develop, and formally substantiate, our analysis of PPs with 'vague' modifiers as in (8) and 'precise' measure phrase modifiers as in (11).

2.3 More non-existential spatial indefinites

The non-existential interpretations of the spatial indefinites in (8a) and (11a) involve universal quantification. Let us move on to survey more examples of spatial indefinites and the way they are informally accounted for by the PEH. Consider first sentences (12) and (13) below.

(12) Fido is outside a doghouse.

(13) Fido is five meters away from a doghouse.

The intuitive truth-conditions that are expected by the PEH for these sentences are paraphrased below.

(12') '*Fido is outside each doghouse*'.

(13') '*The distance between Fido and the closest doghouse is five meters*'.

Let us first see how these statements are derived. For sentence (12), the PEH derives the universal analysis (12') because being outside a union of regions means being outside each of the regions individually. For sentence (13) the analysis in (13') is expected, because according to the PEH, the union of doghouse regions in (13) must be five meters away from Fido. This can only be true if Fido's distance to the closest region (or regions) in this union is five meters.[7] To see how these analyses are supported by linguistic intuitions, let us consider Figure 1. Consider first the truth-value judgements on sentence (12) in the two situations of Figure 1a. Sentence (12) is intuitively true in Figure 1a(i), where Fido is outside all the doghouses. However, (12) is false (or odd) in Figure 1a(ii), where Fido is inside one of the doghouses. The latter fact is not explained by the existential analysis of (12), which is true in both figures. By contrast, the universal statement (12') derived by the PEH is

[7] Here and henceforth, we assume that measure phrases like *five meters* in sentence (13) are interpreted as synonymous to *exactly five meters*. This assumption is convenient for our exposition but is quite innocuous for our theoretical purposes. The PEH-based analysis works equally well for measure phrases like *at least five meters* and *at most five meters* (see section 5).

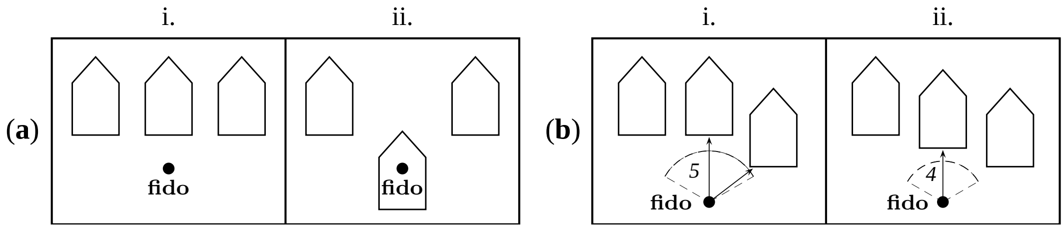

Figure 1: Situations supporting the PEH. *The pairs of figures in (a) and in (b) illustrate a contrast in truth judgements about sentences (12) and (13), respectively. Each of these sentences is true in the respective* (i) *situation but false in the respective* (ii) *situation. This behavior of indefinites in locative PPs supports the PEH over the standard existential analysis.*

true in Figure 1a(i) but false in Figure 1a(ii), similarly to the intuitive judgement. Moving on to sentence (13), we see that this sentence is intuitively true in Figure 1b(i), where Fido is five meters away from both doghouses that are closest to him. However, sentence (13) clearly has a false interpretation in Figure 1b(ii), where Fido is only four meters away from the closest doghouse.[8] Again, the existential analysis does not expect this contrast, because in both situations there exists one doghouse that is five meters away from Fido. Here also a universal analysis of the indefinite would fail, because obviously, sentence (13) does not require that Fido is five meters from every doghouse. By contrast, the PEH correctly describes the meaning of the sentence. Specifically, in Figure 1b(i) Fido is five meters away from the union of doghouse regions, but in Figure 1b(ii) he is only four meters away from this union. Therefore, the contrast in Figure 1b supports the PEH-based paraphrase (13') of sentence (13).

Consider now the following example (14), and the truth-conditions that the PEH expects for it, as reflected by the paraphrase (14').

(14) Fido is five meters outside a doghouse.

(14') '*Fido is outside each doghouse, and the distance between Fido and the closest doghouse(s) is five meters*'.

Note that the PEH compositionally analyzes sentence (14) as a combination of the universal statement of sentence (12) with the 'exact distance' interpretation of (13). Being five meters outside a union of regions U means being outside each region in U, while being five meters away from the closest region in U.[9] This PEH-based analysis is intuitively correct.

[8] There may be a possibility to understand sentence (13) existentially. Under this reading the sentence is true in both Figures 1b(i) and 1b(ii). However, the most prominent reading of (13) is the non-existential reading described above. In section 3.3 we suggest that all *a* indefinites in locative PPs may in principle show an ambiguity between an existential and a PEH-based reading.

[9] Semantically there is little difference between the statements conveyed by sentences (14) and (13). This is because measuring the distance to an entity x usually entails/presupposes being outside

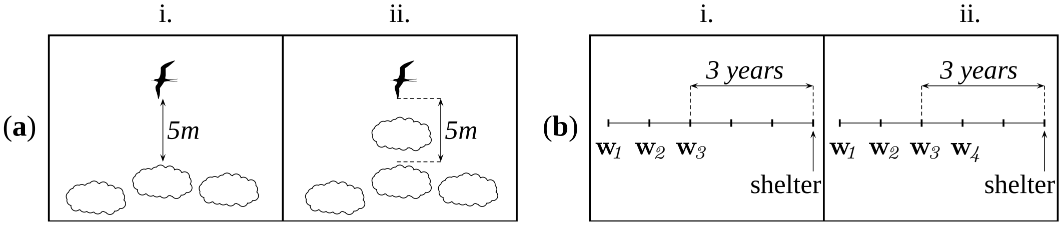

Figure 2: The PEH and spatial/temporal modification. *The contrast between Figures (a)(i) and (a)(ii), in which the truth-value of the spatial sentence (15) varies, illustrates a non-existential effect similar to the temporal sentence (17) in Figures (b)(i) and (b)(ii). Both cases are explained by the PEH.*

Consider now the following sentences.

(15) Tweety is five meters above a cloud.

(16) Tweety is five meters below a cloud.

Similarly to sentence (14), in sentences (15) and (16) a measure phrase modifies a locative prepositional phrase. However, in (15) and (16) the 'projective' prepositions *above* and *below* make a more noticeable contribution than *outside* to the interpretation of the sentence (see footnote 9). According to the PEH-based analysis of sentence (15), Tweety must be five meters above the union region of clouds. This means that Tweety is required to be above all the relevant clouds, and five meters above the nearest one among them. Indeed, as the PEH expects, sentence (15) is true in Figure 2a(i) but false in Figure 2a(ii). In both figures Tweety is five meters above one of the clouds. However, in Figure 2a(ii) this cloud is not the one that is closest to Tweety. Hence, the union-based analysis of the PEH captures the different status of sentence (15) in the two figures, whereas the analysis of indefinites as unambiguously existential does not. A similar point holds for sentence (16). Note that, as the PEH expects, the interpretation of sentences like (15) and (16) is clearly non-universal. For instance, sentence (15) may well true when there are many clouds in various distances and various directions from Tweety. What matters is Tweety's distance to the *closest cloud* that is located *below* her. We see this in the truth of sentence (15) in Figure 2a(i), where Tweety is not five meters above two of the clouds. This non-universal behavior of the spatial indefinite in cases like (15) and (16) will be further discussed in section 6, where it will be shown to follow from our formal treatment of the PEH.

The non-existential (and non-universal) interpretation of the spatial sentences (15) and (16) has a natural parallel with *temporal* sentences like the following ones (Iatridou 2003, 2007).

(17) The shelter was built three years after a war.

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x (cf. section 5.3). Thus, our PEH-based analysis in (13') entails the universal statement 'Fido is outside each doghouse'. Consequently, when the measure phrase *five meters* is combined with the preposition *outside* in (14), no strengthening of the distal meaning of (13) occurs.

(18) The shelter was built three years before a war.

Both sentences (17) and (18) similarly show non-existential interpretations. This can be illustrated by considering sentence (17) in Figures 2b(i) and 2b(ii), or (18) in similar situations. Without loss of generality, let us consider sentence (17) in Figures 2b(i-ii). Figure 2b(i) describes a situation where all the relevant wars happened three years or more before the shelter was built, and one of these wars, $\mathbf{w}_3$, happened exactly three years before that time. Sentence (17) is intuitively true in this situation. By contrast, when one of the wars, as in Figure 2b(ii), happened less than three years before the shelter was built, sentence (17) becomes false. This contrast is not explained by a purely existential analysis of sentence (17), which expects the sentence to be true in both situations. However, the PEH expects an interpretation of (17) where the union of 'war intervals' is three years before the building of the shelter. This intuitively leads to truth in Figure 2b(i), but to falsity in Figure 2b(ii). A similar point can be illustrated for sentence (18). Summarizing, we see that both sentences (17) and (18) are correctly paraphrased using the following universal statements, respectively.

(19) The time t in which the shelter was built satisfies: t is three years after/before the last/first war that happened before/after t.

Formal semantic treatment of these and similar phenomena with indefinites in temporal PPs is left for further research.[10] Another related phenomenon which we will have to put aside are (pseudo-)universal interpretations of indefinites in comparatives, as in *John is taller than a basketball player* (Joost Zwarts, p.c.). For recent work on this puzzle, see Aloni & Roelofsen (2011) and the references therein.

2.4 More on the inadequacy of quantificational approaches

For simple contrasts as in (8) and (11), we can try to entertain an alternative approach to the PEH. Suppose that some spatial relations are lexically *decomposed* into a complex expressions containing their antonym (Heim 2008). For instance, suppose that the meaning of the relation *far from* is decomposed to *not close to*, whereas the comparative *more than* is decomposed into *not at most*. With these decompositional assumptions, we may treat the indefinites in sentences like (8a) and (11a) using existential quantifiers that take narrow scope with respect to negation.[11] Thus, the contrast in (11) may be analyzed by using existential quantifiers when sentence (11a) is informally decomposed as in (20a) below, while (11b) is analyzed without decomposition as in (20b).

[10] In a more thorough account of indefinites in temporal PPs, the similarity between (17) and (18) would have to be contrasted with the different behavior of NPIs (cf. section 3.6) in cases like *the shelter was built before/#after any war*. We do not try to account for such contrasts here. For more on NPIs in temporal PPs see Condoravdi (2010) and references therein.

[11] For a broad overview of relevant interactions between indefinites and negation, see Blaszczak (1999).

(20) a. ‘*There does not exist a gas station x such that Michael is at most 20km from x*’.

b. ‘*There exists a gas station x such that Michael is less than 20km from x*’. (=(11b’))

This decompositional analysis with negation can be extended for cases like (11). However, simple existential-universal contrasts like (8) and (11) with antonymous items do not exhaust the range that we have seen of possible interpretations with spatial indefinites. Treating the spatial indefinites that were illustrated later in this section require other assumptions beyond a decompositional treatment of antonymy. For instance, reconsider sentence (14). As we saw, spatial indefinites with modified PPs, as in *5m outside a doghouse*, involve quantificational effects that are neither pure-existential nor pure-universal. It is unclear if the decompositional approach could derive such readings without further stipulations. Similar problems for the decompositional line would appear with examples (15)-(18). Thus, analyzing indefinites as narrow-scope existentials would not solve the problem of spatial indefinites, even when some spatial relations are decomposed using their antonyms.[12] Similar problems appear for another possible line (Joost Zwarts, p.c.), which would analyze both sentences in (8) as ambiguous between existential reading and a generic reading, involving roughly a universal quantifier. Like the narrow-scope existential analysis above, also the wide-scope generic analysis does not account for cases like (14)-(18) that are neither existential nor universal. To conclude, we do not know any treatment of indefinites that could use existential or generic quantification over entities to account for the wide range of interpretations we have seen with spatial indefinites.

The observations and considerations above justify a deeper study of the PEH and its implications for linguistic theory. One general question concerns the status of spatial indefinites in relation to other property-denoting indefinites. This is the subject of section 3. Another question concerns the implications of the PEH for the spatial interpretation of other, non-indefinite, noun phrases, especially collective NPs. This is the subject of section 4. These two questions are separate from one another. Accordingly, sections 3 and 4 below can be read independently of one another.

3 Spatial readings and other indefiniteness effects

In section 2 we surveyed a variety of spatial sentences with indefinites, as well as a couple of temporal parallels, which cannot be correctly paraphrased as existential

[12] A separate concern that we cannot discuss in detail involves the *direction* of the decomposition. Heim (2008) and others have only considered decomposition of the “negative” item in the antonymous pair (e.g. *short* in the pair *short-long*). We are not aware of any work that decomposes “positive” items like *far* or *more*, as seems necessary when analyzing spatial indefinites using decomposition.

statements. The PEH naturally accounts for these non-existential interpretations. In this section we discuss some of the more general semantic characteristics of spatial indefinites that put the PEH in a broader context of theories about indefiniteness.

3.1 Existence entailments of spatial indefinites

The PEH is not simply a logical strengthening of the standard existential analysis of indefinites. In some cases, the PEH analyzes spatial sentences without any existence entailment or presupposition. To see this, let us first reconsider example (8a), restated in (21) below.

(21) Michael is far from a gas station.

Following the PEH, we analyzed sentence (21) as establishing a binary relation between an entity for *Michael* and a property for *a gas station*. This analysis of sentence (21) is informally paraphrased in (22a) below, and contrasted with the standard existential analysis in (22b).

(22) a. '*Michael is far from the gas station property*'.
b. '*There is a gas station* x *s.t. Michael is far from* x'.

Without further assumptions, the PEH-based analysis in (22a) makes no claim about the existence of gas stations. Unlike the standard existential analysis (22b), statement (22a) is not automatically false when no gas stations exist, i.e. when the property *GS* denoted by the indefinite has an empty extension. More on the formal implications of this point will be said in section 5. As empirical background for this discussion, we should better examine spatial indefinites in contexts that increase the likelihood of an empty extension for the property. Consider for instance sentence (23) below in the context of (23c).

(23c) *During a car race in the desert, Michael's Ferrari F2002 is running out of oil. Unfortunately, the oil used by Michael's Ferrari is extremely hard to find.*

(23) Michael is far from a gas station that sells this type of oil.

Given the context (23c), it is not clear whether there are any gas stations that sell the oil needed for Michael's Ferrari. Now suppose that no such gas station turns out to exist. Would it make sentence (23) trivially false or trivially true? The speakers we consulted were not sure about this judgement. However, even if sentence (23) does not make an existence statement, the sentence surely *implies* that a gas station that sells the relevant type of oil exists somewhere. We found similar existence implications with all spatial indefinites that we considered.

Now let us consider an additional factor. Even the most prototypical of spatial PPs also have non-spatial usages. With such non-spatial usages of the prepositional expression *far from* it becomes easier to avoid an existential implication. Consider for instance the following sentence.

(24) Leonhard is far from a proof of his conjecture.

In contrast to sentence (23), sentence (24) does not make any clear statement or implication about the existence of a proof. If Leonhard's conjecture in (24) turns out to be disproved, that would not be in contradiction to sentence (24). For this reason, sentence (24) is correctly analyzed as the universal statement below, which is straightforwardly derived for (24) by the PEH, and which is standardly true if no proof exists to Leonhard's conjecture.[13]

(25) '*For each possible proof x of his conjecture, Leonhard is far from x*'.

These and similar examples suggest that indefinites in *spatial* PPs as in (23) have an existence entailment (or presupposition/implicature), also when the sentence they give rise to is not purely existential, i.e. not *equivalent* to an existence statement. By contrast, when no spatial location is assumed, as in (24), the indefinite may be purely non-existential, e.g. purely universal as analyzed in (25). This opposition between spatial and non-spatial PPs may seem like a complicating factor for the PEH, which does not assume any built-in existence entailment with spatial indefinites. However, as we shall see in section 5, this existence entailment is expected under standard formal assumptions on spatial expressions.

3.2 Extensionality of spatial indefinites

The variability in triggering existence entailments with indefinites in spatial and non-spatial PPs should be distinguished from their uniform *extensionality*. A well-known fact about indefinites is that they are highly sensitive to intensional contexts. A classical example is the intensional verb *look for*. Consider sentences (26a) and (26b), which only differ in the identity of the noun *lawyer* and *doctor*. The intensionality of these sentences is can be shown by observing that even in contexts like (26c), where the extensions of the two nouns are the same, one of the sentences (26a) and (26b) may be true while the other is false.

(26) a. John is looking for a lawyer.
     b. John is looking for a doctor.

(26c) Context: *All lawyers are doctors and all doctors are lawyers.*

With locative PPs we found no intensionality effects comparable to the classical one in (26). Consider for instance sentence (27) in the context (27c).

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[13]The same effect appears with universal quantification in modal environments as in the following example.

(i) Every proof of his conjecture must/will/would/should take much of Leonhard's time.

Similarly to (24), sentence (i) is infelicitous in contexts where it is known that the conjecture is false, but makes no existence assertion about the provability of the conjecture.

(27) a. John is far from a school.

b. John is far from a church.

(27c) Context: *All schools are churches and all churches are schools.*

In context (27c), the spatial sentences (27a) and (27b) become equivalent. Furthermore, this kind of equivalence also holds for non-spatial usages of *far from*. Consider for instance sentence (28), in the context of (28c).

(28) a. The world is far from a social revolution.

b. The world is far from a solution to the inequality between people.

(28c) Context: *Every social revolution is (or would be) a solution to the inequality between people, and every solution to the inequality between people is (or would be) a social revolution.*

Context (28c) claims that the two properties that are denoted by the descriptions *social revolution* and *solution to the inequality between people* are co-extensional. In this context, being far from one of the properties means being far from the other, i.e. sentences (28a) and (28b) become equivalent. These considerations justify our assumption in the PEH that when analyzing expressions such as *far from a gas station*/*school* or *far from a proof*/*revolution*, we should only consider the *extension* of the relevant properties.[14]

3.3 'Specific' spatial indefinites and the *a*/*some* distinction

The non-existential (or semi-universal) interpretation of spatial indefinites in sentences like (21) or (24) is not necessarily their only interpretation. As Iatridou (2003, 2007) points out, when their descriptive content is heavy enough, spatial indefinites may get a 'specific', wide-scope existential interpretation. Consider for instance sentence (29) below.

(29) We're far from a/some gas station that I read about in the guide.

The likely interpretation of sentence (29) is that (at least) one gas station that I read about in the guide is far away. The sentence does not necessarily mean that all gas stations that I read about in the guide are far away. This 'specific', or wide-scope existential interpretation is expected by most theories of indefinites.[15]

[14]Incidentally, note that despite their equivalence in the context (28c), which indicates extensionality, the non-spatial sentences (28a) and (28b), similarly to (24), make no claim about the existence of a social revolution or a solution to inequality.

[15]The discussion that follows presupposes an account of 'specific' indefinites as wide scope existentials (Ruys 1992, Abusch 1994, Reinhart 1997, Winter 1997). Other proposals (Fodor & Sag 1982, Farkas 1997, Schwarzschild 2002) assume domain restrictions in the context to account for such indefinites, or a combination of domain restrictions with a choice-function analysis (Kratzer 1998, Chierchia 2001, Winter 2004). Here we do not pursue the contextual/domain-restriction approach to 'specific' indefinites, and whether it can be combined with our analysis of spatial indefinites is a point for further research.

In (30) below we summarize the behavior of spatial *a* and *some* indefinites, with or without rich descriptive content. The notation '*' means "unavailable interpretation" and '#' means "unlikely interpretation".

(30) a. We're far from a gas station. (universal, # existential)
b. We're far from a gas station that I read about in the guide. (=(29)) (# universal, existential)
c. We're far from some gas station. (*universal, existential)
d. We're far from some gas station that I read about in the guide. (*universal, existential)

Also in other linguistic contexts, the *a*/*some* and descriptive content parameters affect systematic differences in their syntactic/semantic behavior (see e.g. Chierchia 2005, pp.145-6). For instance, the contrasts in (31) below show that also with respect to the generic/existential variation, indefinites in habitual sentences show a similar pattern to their behavior in spatial constructions as in (30).

(31) a. A dog barks. (generic, # existential)
b. A dog that I know barks. (# generic, existential)
c. Some dog barks. (*generic, existential)
d. Some dog that I know barks. (*generic, existential)

The *a* indefinite in (31a) prefers a generic interpretation, which is captured using properties or 'kinds' (Carlson 1977).[16] When more descriptive content is added as in (31b), the existential reading of the indefinite takes over. By contrast, the *some* indefinite in (31c-d) resists the generic interpretation, and only allows the existential reading. This pattern in (31) is identical to the pattern observed with the spatial indefinites in (30), where the *a* indefinite allows a property analysis and the *some* indefinite rules it out.

3.4 Spatial indefinites and kind-denoting nouns

In her property-based analysis of indefinites in *there* sentences like (32) below, McNally (1998) points out interesting relations between indefinites and 'kind'-denoting nouns. Consider for instance McNally's examples in (33a-b) in contrast with (34a-b).

(32) a. There was a doctor at the convention.
b. Martha has been a doctor.

(33) a. There was every kind of doctor at the convention.
b. Martha has been every kind of doctor.

(34) a. *There was every doctor at the convention.
b. *Martha has been every doctor.

[16]For more discussion of the restrictions on generic readings of *a* indefinites, see Cohen (2001) and references therein.

McNally proposes that the indefinites in (32) are licensed because they are property denoting, and *there* and copula constructions select for properties. As evidence for this proposal, McNally shows the acceptability of 'kind-denoting' NPs in (33), as opposed to NPs denoting ordinary entities in (34).

Spatial indefinites show the same relation with kind-denoting nouns as those pointed out by McNally for indefinites as in (32). Consider for instance sentence (35) below, in the context of Michael's race in the desert (23c).

(35) Michael is far from *the kind of* gas station that sells this type of oil.

Sentence (35), like (23) above, requires that Michael is far from all gas stations that sell this type of oil (assuming that such stations exist). As in McNally's proposal, this supports our treatment of spatial indefinites as property denoting. An analogous pattern, parallel to McNally's proposal and in agreement with theories of plurals, will be the basis for the analysis of plural definites and 'group'-denoting nouns in section 4.

3.5 Property-denoting indefinites: collective and non-collective readings

In our discussion above we have pointed out the parallelism between the spatial use of singular indefinites and their generic (kind-referring) interpretations. Let us consider another common feature of singular indefinites, including spatial ones – their resistance to collectivity, as opposed to plural indefinites.[17] A well-known collectivity distinction between generic *a* indefinites and bare plurals is illustrated by the following contrast (Krifka et al. 1995, p.89).

(36) a. #*A lion* gathers near acacia trees.
b. *Lions* gather near acacia trees.

Sentence (36a) makes the odd implication that a single lion may somehow 'gather'. By contrast, the bare plurals in sentence (36b) lead to a collective generic interpretation, which roughly states that *groups* of lions tend to gather near acacia trees.[18]

Spatial sentences show the same kind of contrast between singular and plural indefinites. Consider (37) and (38) below.

(37) a. *The house is between/among a lake.
b. The house is between/among lakes.

(38) a. The circle is in/inside a square.
b. The circle is in/inside squares.

[17]Contrasts in collectivity between singular and plural terms are of course a general fact about English, as in *#the/every teacher gathered in this room* vs. *the/all the teachers gathered in this room* (Winter 2002).

[18]Another familiar contrast that may belong in the same class is #*a unicycle has wheels* vs. *unicycles has wheels*.

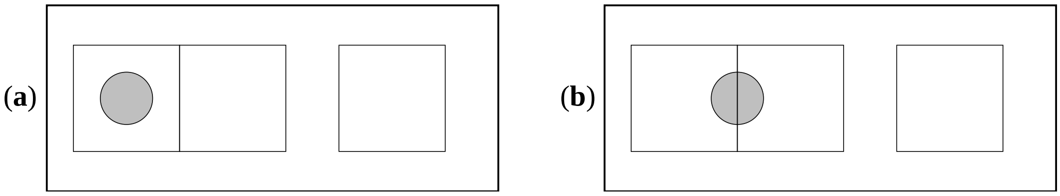

Figure 3: The PEH and singular/plural reference. *Sentence (38b), with the plural indefinite 'squares', is true in both figures (a) and (b). By contrast, sentence (38a), with the singular indefinite 'a square', is only true in figure (a).*

With the prepositions *between* and *among* in (37a), the contrast between singular and plural descriptions is well-known. A less familiar contrast with spatial indefinites is exemplified in (38). Sentence (38b), with the plural indefinite *squares*, allows the circle to be contained in the union region of some squares, with overlap between different squares (Figure 3b).[19] By contrast, in sentence (38a), the singular indefinite *a square* requires that there be at least one square that completely contains the circle. Thus, (38a) is true in Figure 3a, but false, or highly unacceptable, in Figure 3b.

Thus, on top of the requirement that the extension of spatial indefinites is non-empty (section 3.1), we see that singular spatial indefinites require a singular *witness* to the spatial relation. The PEH in its current formulation does not formally address this requirement, and leaves contrasts as in (37) and (38) untreated. We do not try to account here for this and other contrasts between singular and plural indefinites. Our point here is only to highlight the similar contrasts observed with other usages of property indefinites like the generic sentences in (36). One of the possibilities mentioned in the literature (Chierchia 1998) to account for such contrasts is to draw an ontological distinction between 'singular properties' and 'plural properties'.

3.6 Spatial negative polarity items and preposition monotonicity

Another aspect of the PEH is the possibility it opens for analyzing contrasts as in (39) and (40), with the spatial use of negative polarity *any* (Iatridou 2003, 2007).

(39) a. Michael is far from any gas station.
b. ?Michael is close to any gas station.

[19] The acceptability of sentence (38b) in Figure 3a is somewhat degraded when compared to Figure 3b. We believe that this effect, which also appears with plural definites (cf. section 4), is related to the complex implications of plurality (Zweig 2009). However, in a proper context, sentence (38b) is definitely OK in Figure 3a. For instance, consider the following discourse.

(i) *Squares represent areas covered by radars; the circle represents a huge flying object. At this point of time, the circle is inside (the) squares, hence the radar computer can fully describe the real shape of the object.*

By contrast, we have found no similar context that licenses sentence (38a) in Figure 3b.

(40) a. This park is outside any urban area.
b. ?This park is inside any urban area.

A well-known account (Fauconnier 1975, Ladusaw 1979) of negative polarity items (NPIs) like *any* describes them as being licensed by downward entailing environments. Using the PEH, the monotonicity of the prepositional expressions in (39) and (40) is indeed characterized as expected by the Fauconnier-Ladusaw Generalization. Consider for instance the following entailments.

(41) a. My country is far from Eurasia $\Rightarrow$ My country is far from Asia
b. My country is close to Eurasia $\not\Rightarrow$ My country is close to Asia

(42) a. My country is outside Eurasia $\Rightarrow$ My country is outside Asia
b. My country is inside Eurasia $\not\Rightarrow$ My country is inside Asia

With the spatial relations *far from* in (41a), being far from a region R (e.g. Eurasia) entails being far from any region contained in R (e.g. Asia). Following J. Zwarts and Winter (2000), we can characterize this inferential behavior of the spatial relation *far from* as *downward monotonicity*. By contrast, the relation *close to* is not downward monotone, as illustrated in (41b). Similarly, the (non-)entailments in (42) characterize the relation *outside* as downward monotone, and the relation *inside* as not downward monotone. The parallelism between (39)-(40) is immediately accounted for by the PEH and the Fauconnier-Ladusaw Generalization. To verify that it is indeed the case, consider the following simple version of the generalization (von Fintel 1999).

Fauconnier-Ladusaw Generalization. *An NPI is only grammatical if it is in the scope of an expression α such that α's denotation is downward monotone.*

The PEH uses a non-quantificational *property* denotation and analyzes spatial indefinites as properties in the scope of the spatial relation. Contrasts in (39)-(40) are directly explained by using the Fauconnier-Ladusaw Generalization and the monotonicity contrasts in (41)-(42). As further support for the interaction between the PEH and the Fauconnier-Ladusaw Generalization, consider the following entailment with spatial *a*-indefinites.

(43) a. Michael is far from a gas station $\Rightarrow$ Michael is far from a big gas station.
b. Michael is close to a gas station $\not\Rightarrow$ Michael is close to a big gas station.

(44) a. This park is outside an urban area $\Rightarrow$ This park is outside an industrial urban area.
b. This park is inside an urban area $\not\Rightarrow$ This park is inside an industrial urban area.

The entailment patterns of (41)-(42) involve proper names with a part-of containment relation (*Asia, Eurasia*). The PEH similarly analyzes the entailments of (43)-(44) as involving a containment is between the extensions of the properties *big gas station*/*gas station* and *industrial urban area*/*urban area*, hence also between their respective eigenspaces. A traditional existential analysis of the indefinites in (43)-(44) would not account for these (non-)entailments. For instance, in (43), if there is a gas station x s.t. Michael is far from x, it does not follow that there is such a *big* gas station: existential quantification is not downward-monotone, of course. Hence, the distribution of NPI spatial indefinites as in (39) and (40) would be problematic for an existential analysis.[20] In section 5 below we will analyze in more detail the monotonicity properties of locative relations and their relations with the property-based analysis of the PEH.[21]

4 Spatial collective descriptions and the Collection-Eigenspace Hypothesis

In section 2 we introduced basic assumptions of our proposed spatial semantics of property-denoting indefinites. In this section we focus on nominals like *the mountains* or *the mountain chain*, which are analyzed as referring to collections. Accordingly we classify such nominals as *collective descriptions*. As we will see, when collective descriptions appear within spatial PPs their interpretation shows non-trivial variability similar to that of spatial indefinites. Following our treatment of property-denoting indefinites, we hypothesize that collections can also be located, similar to properties, by having their eigenspace defined as the union of their members' regions. However, we also point out some systematic differences between spatial indefinites and spatial collective descriptions. We propose that these differences follow from a 'referential unity' of collections which is absent with properties. In the spatial domain this referential unity allows collections, in addition to their union eigenspace, to also be located at the 'functional hull' of their members' regions – this eigenspace of collections contains their members' union eigenspace, but may require some sort of spatial contiguity, e.g. geometric *convexity*. For instance, the eigenspace of a mountain chain is a region that contains the relevant mountains, but possibly also valleys that would not be classified as part of any mountain in the chain. We propose that the functional hull eigenspace of collective descriptions follows from their 'impure atom' denotation (Link 1984, Landman 1996). Following J. Zwarts and Winter (2000), we use this denotation together with

[20]For a related puzzle , see 'adversative licensing' of NPIs, illustrated by the following examples (von Fintel 1999).

(i) a. Max avoided any confrontation.
    b. ?Max faced any confrontation.

(ii) a. Max rejected any proposal to sell the company.
    b. ?Max accepted any proposal to sell the company.

[21]One topic relevant for spatial indefinites that we could not address in this section involves *incorporated nominals*. These are cases where a nominal element becomes part of the verb rather than appearing as a prosodically separate argument (van Geenhoven 1998, Chung & Ladusaw 2003, Farkas & de Swart 2003, Carlson 2006). We believe that incorporation, and especially its studied connections with 'de dicto' indefinites (Zimmermann 1993, van Geenhoven & McNally 2005) can be theoretically linked to the behavior of spatial indefinites. However, the study of such possible links must be deferred for further research.

a convexity principle to account for the differences we observe between spatial collective descriptions and spatial indefinites.

4.1 Interpretative variability with spatial collective descriptions

Consider the contrastive pairs of sentences in (45) and (46) (=(3)) below.

(45) a. The house is far from the mountains.
b. The house is close to the mountains.

(46) a. Michael is far from these gas stations.
b. Michael is close to these gas stations.

In order for (45a) to be true, the house must be far from all the relevant mountains. By contrast, in (45b) the house may only be close to one of the mountains. A similar contrast appears in (46).

The contrasts in (45) and (46) involve spatial interpretations of plural definite descriptions. Similar contrasts appear with spatial interpretations of singular definites that refer to collections of entities. Consider for instance the pairs of sentences in (47) and (48) (=(2)).

(47) a. The house is far from the mountain chain.
b. The house is close to the mountain chain.

(48) a. Michael is far from this group of gas stations.
b. Michael is close to this group of gas stations.

The singular descriptions *mountain chain* and *group of gas stations* in (47) and (48) show a similar spatial interpretation to that of the plural nominals in (45) and (46). In order for the house to be far from the mountain chain in (47a) it has to be far from each individual mountain. But being close to the mountain chain in (47b) only requires being close to one mountain. A similar contrast appears in (48).[22]

The interpretative contrasts with the collective descriptions above are very similar to those that we have observed in section 2 with spatial indefinites. Our aim in this section is to extend the PEH in order to account for the interpretation of spatial collective descriptions. This will be done by using a principle similar to the PEH, which we call the *Collection-Eigenspace Hypothesis* (CEH). The collective descriptions that the CEH addresses are primarily plural definites (e.g. *the mountains*) and collective singular definites (e.g. *the mountain chain*).[23] Ignoring some

[22]Following our remarks in section 2 on the reasoning in (10), the same analysis above holds when the collections are described by proper names. Thus, a sentence like *the house is far from the mountain chain* has the same spatial interpretation as *the house is far from Sierra Nevada*. Both sentences entail that the house is far from any mountain in the relevant mountain chain. See section 7 for further discussion of this general point.

[23]We here ignore collective singular indefinites like *a mountain chain* as well as indefinite plurals like (*some*) *mountains*. For some remarks on such NPs see section 4.5.

important differences between these two kinds of NPs, we intuitively assume that both of them are associated with *sets* of entities. For instance, we assume that the noun phrases *the mountains* and *the mountain chain* both are associated with a set M, where each member of M is a single mountain.

Using these intuitive assumptions, we state the Collection-Eigenspace Hypothesis below, as a first approximation for treating spatial collective descriptions.

Collection-Eigenspace Hypothesis (CEH, v1). *The eigenspace of a collection C is the union of eigenspaces for C's members.*

This version of the CEH is almost identical to the PEH. As such, it accounts for the contrasts in (45)-(48) in the same way that the PEH accounts for the contrast in (8). Consider for instance examples (45) and (47). In sentences (45a) and (45b), the CEH treats the eigenspace of the set of mountains M as the union of eigenspaces for single mountains. For the house to be far from this union region it has to be far from all the single mountains. By contrast, in (45b), in order for the house to be close to the union region it only has to be close to one of the mountains. Our account of sentences (47a) and (47b) is analogous, since the denotation of *the mountain chain* is also associated with the set M of single mountains.

On the background of the discussion in section 2, this analysis seems natural enough. However, the above statement of the CEH and its description of simple contrasts like (45)-(48) ignore some of the challenges that collective descriptions introduce for spatial semantics. In the rest of this section we analyze more data and their implications for the conception of the PEH and the CEH. We will concentrate on some points where spatial collective descriptions differ from spatial singular indefinites. These phenomena will lead us to a revised statement of the CEH, which is still based on the PEH but takes into account the special features of collective reference.

4.2 Impure atoms and the context-sensitivity of collective reference

Consider sentence (49) below in the situation of Figure 4a.

(49) The road is ten meters away from the utility poles.

This sentence is easily interpreted as true in Figure 4a, where the poles form a line segment perpendicular to the road, at a distance of ten meters from it. The CEH correctly analyzes this effect: since the union region of the poles in Figure 4a is ten meters away from the road, the CEH models sentence (49) as true in this situation, as intuitively required. This interpretation could also be achieved using existential quantification, since only one pole in Figure 4a is ten meters away from it. Let us now consider sentence (49) in the situation of Figure 4b. This figure is constructed so that the distances from the road to the poles in it are the same as in Figure 4a. However, in contrast to Figure 4a, in Figure 4b it is hard to interpret sentence (49) as

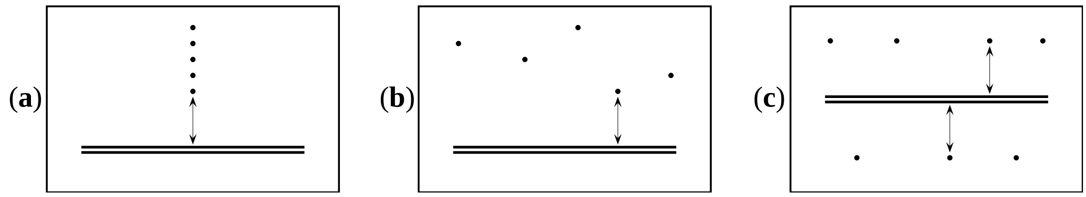

Figure 4: ‘The road is ten meters away from the utility poles’ (49). *The double arrows represent a distance of ten meters. In (a) the poles form a line, which is easily conceived of as an impure atom, ten meters away from the road. In (b) the chaotic arrangement of the poles makes it harder to interpret the plural as referring to an atom, and consequently the distributive reading becomes prominent, which makes the sentence false. In (c), an impure atom interpretation is still conceptually hard but the distributive reading is true, hence also the sentence.*

true. This fact is unexpected by both the existential analysis and the version of the CEH above, since the distances from the road to the poles remain the same. What is then the origin for the different judgements about (49) in the two figures? Intuitively, the only relevant difference between Figure 4a and Figure 4b is in the ‘referential unity’ of the set of poles. The arbitrary looking constellation of the poles in Figure 4b makes it harder to consider them as a contiguous unit. Reasonably, this lack of spatial contiguity rules out the union region interpretation of the plural definite *the utility poles* in (49).

But why should spatial contiguity be needed for a union region interpretation of plurals? In order to answer this question, let us briefly consider the semantics of plurality as treated, among others, by Link (1984) and Landman (1989, 1996). According to Link and Landman, one important origin for collectivity with plurals is their so-called ‘impure atom’ interpretation. According to this analysis, the collective interpretation of a plural like *the utility poles* is derived by associating its set denotation – a set of single poles – with a simple (‘atomic’) entity in the domain of singular entities. In sentence (49), this ‘impure atom’ represents the collection of poles and is co-referential with singular NPs like *the collection of poles* or *the line of poles*. The way in which a plural definite is associated with its impure atom denotation is highly sensitive to contextual factors. In contexts like Figure 4a, where the poles form a line, it is easy to associate them with an atomic entity. As a result, in Figure 4a, the plural definite *the utility poles* in (49) is easily associated with an eigenspace of its own. But the association of the plural with an impure atom is harder in contexts like Figure 4b, where the poles do not form any cognitively contiguous unit. Consequently, in Figure 4b the plural definite lacks an eigenspace of its own, and sentence (49) cannot be interpreted as true. As further evidence for this context-sensitivity of collective reference, Yael Seggev (p.c.) points out that sentence (49) becomes better if we zoom out on Figure 4b so that the poles can be conceived of as one contiguous ‘cloud’.

This context sensitivity of plural definites contrasts with the behavior of spatial singular indefinites. Consider for instance sentence (50) below in the context of Figures 4a and 4b.

(50) The road is ten meters away from a utility pole.

Unlike sentence (49), sentence (50) is equally interpreted as true in both Figures 4a and 4b. Thus, with the indefinite *a utility pole*, the spatial contiguity of the poles does not play a role. As expected according the PEH, what matters for the truth of (50) is only the distance between the road and the union region of the poles, without the effect of 'spatial contiguity' that we observed in (49). We propose that, in contrast to the context-dependency of impure atom reference with plurals, singular indefinites denote properties as their *basic denotation*. As a result, in sentence (50) contextual factors do not affect the association of the indefinite with a property and, consequently, with its union region eigenspace. Therefore, sentence (50) shows no truth-conditional difference between Figures 4a and 4b.

How are plural definites as in (49) interpreted when reference to an impure atom fails? The falsity of (49) in Figure 4b suggests that the sentence is interpreted distributively, similarly to *the house is ten meters away from <u>each of</u> the utility poles*. This distributive interpretation of (49) is false in Figure 4b. As a further observation about the role of distributivity in (49), consider Figure 4c. In contrast to Figure 4b, sentence (49) is easily interpreted as true in Figure 4c. However, intuitively there is little difference in the spatial contiguity of the poles between Figures 4b and 4c. In both situations, it is hard to see the poles as one 'referential unit'. Thus, the truth of sentence (49) is likely to have different origins in Figures 4a and 4c. In Figure 4a, the truth of the sentence must follow from the union eigenspace of an impure atom associated with the plural. By contrast, in Figure 4c, the intuitively true interpretation may simply be a matter of distributive quantification over single poles: unlike Figure 4a, in Figure 4c each pole is ten meters away from the road.

To summarize, the property-based analysis of singular indefinites and the collective interpretation of plural definites both involve sets of entities. However, we have observed two special properties of plural definites that affect their spatial interpretation.[24] First, a union eigenspace of plural definites requires that it be associated with an impure atom. This process is sensitive to contextual factors of referential unity. By contrast, deriving a union eigenspace for spatial indefinites involves no referential unity of the set of entities in the property's extension. Second, independently of their union eigenspace interpretation, plurals have a distributive interpretation. This distributive interpretation becomes prominent in contexts where reference to an impure atom fails, and it leads to an alternative analysis of the plural definite, roughly

[24] The discussion above ignores collective singular descriptions. Sentences like *the house is ten meters away from the <u>collection</u> of utility poles* are more easily interpreted as true in Figure 4b than sentence (49). In this case, the obligatory atom reference of the singular noun *collection* prevents a distributivity effect (de Vries 2012), and the explicit reference to an impure atom supports referential unity even in a messy constellation of poles. See section 4.3 below for more on the spatial effects appearing with singular collective nouns.

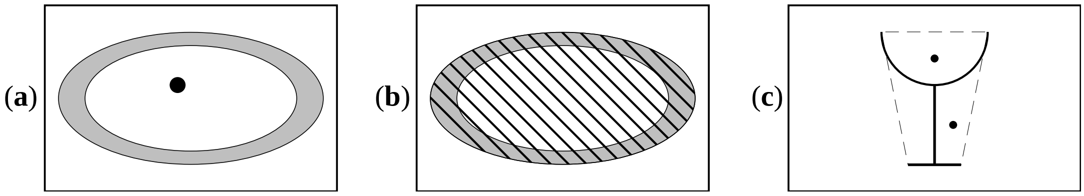

Figure 5: ***Convex hulls and functional hulls***. In (a) the point may be considered *inside the ring*, since its eigenspace is contained in the *convex hull* of the ring's eigenspace as depicted in (b). More generally, the *functional hull* of an entity's eigenspace may be different than its convex hull (Herskovits 1986): in (c) one of the points may be considered *inside the wine glass* while the other point is *outside* the glass, even though both points are contained in the glass's convex hull.

corresponding to universal quantification over the collection's members (Scha 1981, Dowty 1986, Brisson 1998, Winter 2000).

4.3 Impure atoms and eigenspace convexity

The previous section showed evidence that reference to 'impure atoms' leads to context-sensitivity of the eigenspace interpretation of plurals. Furthermore, we will now see that when it is available, the impure atom strategy also affects the geometrical shape of the eigenspace. In previous work on spatial expressions (e.g. Herskovits 1986, Jackendoff & Landau 1991, Zwarts & Winter 2000), it has often been observed that there is systematic vagueness surrounding the choice of the eigenspace with simple singular definites. Consider for instance Figure 5a. In this figure we can truthfully describe the point as either being inside the ring or outside the ring. Which of the two descriptions is preferred depends on pragmatic factors. For instance, suppose that the ring represents a pack of hounds chasing a fox, and the point represents the fox. In this case it is natural to describe Figure 5a using sentence (51) below.

(51) The fox is inside the ring of hounds.

By contrast, suppose that the ring represents a bagel and the point represents a bug. In this case it is more natural to describe Figure 5a using sentence (52) below, e.g. as support for the claim that the bagel is bug-free.

(52) The bug is outside the bagel.

A plausible conclusion from these and similar examples, is that descriptions like *the ring* or *the bagel* may be associated with two different eigenspaces. Under one strategy, as in sentence (51), the eigenspace of *the ring* covers the whole circular area that the ring surrounds. This analysis of (51) is forthcoming if the eigenspace for the ring of hounds is treated as a disc that contains the fox. Under the other strategy, as in (52), the eigenspace of a noun phrase like *the ring* or *the bagel* has the expected shape of a ring. The preposition *outside* in (52) requires that this ring eigenspace be disjoint from the location of the bug.[25]

But how can sentence (51) allow a disc shape to be used as the eigenspace of a noun phrase like *the ring*? Our proposal, following J. Zwarts and Winter (2000), is that the eigenspace R of a collective description can be freely shifted to R's *convex hull*. Intuitively, a convex hull of a set of points R is the smallest set that 'envelops' R and has a convex shape. More formally, the standard topological notions of *convexity* and *convex hull* are defined as follows.

Convexity and convex hull: *A set of points A is convex if the line segment between any two points in A is fully contained in A. The convex hull of a set of points A is the smallest convex set $\textsc{conv}(A)$ that contains A.*

Figure 5b illustrates the convex hull of the ring in Figure 5a. Using convex hulls, we assume that collective NPs that have a non-convex eigenspace are also associated with an alternative, convex eigenspace. For instance, an alternative eigenspace of the noun phrase *the ring of hounds* in sentence (51) has a disc shape, which accounts for the truth of this sentence in Figure 5a.

It should be remarked that the convex hull analysis that is illustrated by sentence (51) is only a simple example of the complex conceptual process that determines eigenspaces of objects. To see a case where the convex hull analysis is insufficient, consider the example by Herskovits (1986) that is reproduced in Figure 5c. Both points in this figure are within the convex hull of the wine glass. However, only one of the two points can be considered as *inside the glass*. The reason for that is unlikely to be geometrical. What determines the 'inside' of a glass here is function: the way glasses are normally used. A discussion of this point goes beyond the scope of the paper, but it should be kept in mind when considering the vagueness or ambiguity in the determination of eigenspaces. To take note of this general point, we occasionally refer to the convex hull analysis as an instance of a 'functional hull' analysis, whose details require further elaboration.

We have discussed the convex hull interpretation of sentence (51), which involves a singular spatial definite. As expected by the impure-atom analysis, similar effects also appear with plural descriptions. Consider for instance the following example (Herskovits 1986).

(53) The worm is in/inside the apples.

Suppose that the set of apples in sentence (53) is A, and that the apples in A form a heap. One possible but unlikely interpretation of (53) is that the worm is inside each apple in A. However, (53) can also be true when the worm is not inside each individual apple, but still within the boundaries of the heap. The worm may even be

[25]The discussion above relies on the natural analysis of the preposition *inside* as describing eigenspace *containment*, and of the preposition *outside* as describing eigenspace *disjointness*. See more on this standard analysis in section 5.1.

outside each apple in A, as long as it is within the heap boundaries.[26] We propose that this effect illustrates the 'impure atom' interpretation of plurals, under a functional hull eigenspace of the heap. The analysis goes as follows. In the proper context, the impure atom strategy interprets the definite *the apples* in (53) as an atomic entity a. The basic eigenspace of a is the union of the single apples' eigenspaces. This region is not convex, since there may be 'holes' between the apples. With this non-convex eigenspace of *the apples*, sentence (53) requires the worm to be inside the body of one or more apples. However, an alternative eigenspace of the entity a is the convex/functional hull of the apples' union region. This convex eigenspace allows an interpretation of (53) where the worm is in the space between the apples, but still outside each of them.

Further, note the contrast between the following sentences.

(54) a. The worm is outside the apples.
b. The worm is outside the heap of apples.

Let us again suppose that the apples in (54) form a heap. Sentence (54a), like (53), allows both a convex and a non-convex eigenspace for *the apples*. Under the convex interpretation, the worm in (54a) is required to be outside the heap of apples. Under the non-convex interpretation of (54a), the worm is only required to be outside each of the apples, but it might still be among them. Sentence (54b), unlike (54a), only allows the convex interpretation of *the heap of apples*: the worm cannot be among the apples, and its location must be disjoint from the heap shape enveloping the individual eigenspaces of the apples. This shows the crucial role of lexical choice for determining a description's eigenspace. With nouns like *ring* and *bagel* as in sentences (51) and (52), the eigenspace of the description may be non-convex or convex. By contrast, in (54b), the description with the noun *heap* is univocally convex. We conclude that nouns that describe physical shapes like *ring, bagel, heap* or *pile* put different lexical restrictions on the shape of the eigenspace with which collective descriptions may be associated.

Our proposal regarding impure atoms and convexity with collective descriptions is summarized below.

Impure atoms and convexity. *Under its impure atom interpretation* **a***, a collective description* EXP *may be associated with the union of* **a***'s subpart eigenspaces or any 'functional hull' of this union.*

1. *When* EXP *is associated with a non-convex eigenspace R, an alternative eigenspace of* EXP *is R's convex hull, denoted* $\textsc{conv}(R)$*. The choice between R and* $\textsc{conv}(R)$*, or any other functional hull, depends on contextual factors.*
2. *Collective nouns like* 'heap' *or* 'pile' *can only be associated with a convex eigenspace, due to their spatial lexical meaning.*

[26] A relevant context: suppose that you're a health freak who's afraid that a worm might touch one of his apples. Someone sees a worm crawling towards your favorite heap of apples, and shouts: "oh no, it is now inside the (heap of) apples!".

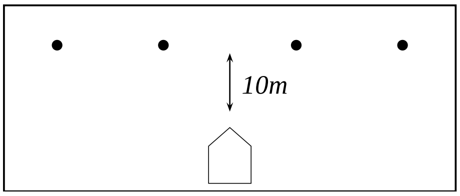

Figure 6: ‘***The house is (exactly) 10m away from the (row of) utility poles/a utility pole***’: with the collective description *the (row of) utility poles*, the sentence may be true in this picture, even though there is no pole that is ten meters away from the house. This is not the case for the spatial indefinite *a utility pole*, with which the sentence requires that there be a utility pole closest to the house and ten meters away from it.

Crucially, we assume that these convexity/functionality processes apply exclusively to impure atoms, and not to properties. This is the key factor that governs differences in spatial semantics between property-denoting indefinites and atom-denoting collective descriptions. Consider for instance the sentences (55) in the context of Figure 6.

(55) a. The house is (exactly) 10m away from the (row of) utility poles.
b. The house is (exactly) 10m away from a utility pole.

The poles in Figure 6 conceptually form a line that is ten meters away from the house. The sentences in (55a), with singular and plural collective descriptions, are both true in this situation. This is despite the fact that no single pole is ten meters away from the house. By contrast, sentence (55b) is intuitively false in Figure 6. This contrast between (55a) and (55b) is expected by our analysis above. The truth of the sentences in (55a) in Figure 6 cannot follow directly from the union eigenspace of the poles, since this union is more than ten meters away from the house. Thus, to make (55a) true in Figure 6, the eigenspace of the collective descriptions in (55a) must be a line segment representing the (row of) poles. This line is the convex hull of the poles’ eigenspaces, and its distance from the house is ten meters. The convex hull interpretation is available in sentence (55a) with the singular collective definite and plural definite, but unavailable for the singular indefinite in (55b). Our conclusion is that the convex/functional hull strategy is strongly associated with the impure atom interpretation of singular and plural definites. The property denotation of singular indefinites as in (55b) lacks the ‘referential unity’ of collections. This blocks the convexity operator or any other spatial operator generating a ‘functional hull’ of the property’s eigenspace.

A similar contrast is observed in (56) below (cf. (53)).

(56) a. The worm is in/inside the (heap of) apples.
b. The worm is in/inside an apple.

In (56a), the convexity of the heap allows the sentence to be true also when the worm is only in the space between the apples (cf. (53)). This convexity is possible due to the impure atom denotation of the collective descriptions. By contrast, in (56b), the property denotation of the singular indefinite *an apple* cannot be associated with a convex eigenspace. This is because whenever there are two or more apples, their union region is not convex, and the convex hull strategy is only available for impure atom denotations, not to property denotations. As a result, (56b) requires that the worm be inside one of the apples.

4.4 Revised CEH and summary

Given the observations and theoretical discussion above, we introduce the following revised version of the CEH.

Collection-Eigenspace Hypothesis (CEH, v2). *The eigenspace of an* (*impure*) *atom* **a** *is the union of eigenspaces for* **a***'s members, or the convex/functional hull thereof.*

This statement of the CEH leaves two interpretative effects to be determined by contextual and lexical factors: (i) Whether a plural description denotes an impure atom is determined by the context. (ii) The eigenspace of an impure atom is either the union of its members' regions, or their convex/functional hull. Which of the two candidate eigenspaces is chosen is determined by context, and, in the case of collective nouns, also by their spatial lexical meaning.

With these assumptions, we account for the systematic contrasts shown between singular indefinites and collective descriptions. As the CEH assumes, assigning a location to a plural noun phrase requires conceiving it as an impure atom. This 'referential unity' is not required when locating property-denoting singular indefinites. On the other hand, the revised version of the CEH allows shifting a union eigenspace to a convex eigenspace when locating impure atoms. This process is not allowed by the PEH for singular indefinites.[27] These theoretical principles are graphically summarized in Figure 7.

4.5 A note on bare plurals

Further work is required on the spatial behavior of *bare plurals*. As far as we were able to check, in the relevant aspects these NPs behave as might be expected, combining effects of indefiniteness and plurality. Consider for instance the following sentences.

(57) The road is ten meters away from utility poles.

(58) The house is ten meters away from utility poles.

[27] As mentioned in section 3.5, singular indefinites are also more restrictive than collective descriptions in their ability to refer to groups, due to independent restrictions on 'singular properties'.

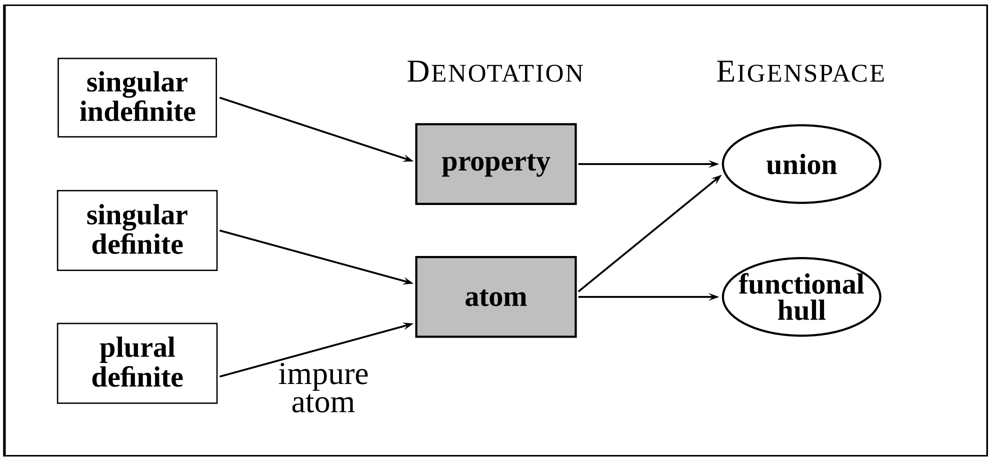

Figure 7: Set-based denotations and eigenspaces of singular indefinites and collective descriptions. *Singular 'a' indefinites denote properties, whose eigenspace is the union region for entities in their extension. Collective singular and plural definites may be associated with an 'impure' atom. The eigenspace of such an atom is either the union region for its member entities, or the convex/functional hull of this region.*

In sentence (57) the bare plural shows an existential effect. Consider for instance an 'accidental' arrangement of the poles as in Figure 4(b), but where two poles (rather than one) are exactly ten meters away from the road. The bare plural in sentence (57), like the singular indefinite in (50) and unlike the plural definite in (49), allows sentence (57) to be true. By contrast, in Figure 6, the bare plural in sentence (58) shows its plural guise: in this situation a convexity process is required for interpreting the sentence as true, and in this case the bare plural behaves more like the plural definite in (55a) than like the singular indefinite in (55b). Another kind of descriptions that may involve both indefiniteness and collectivity are collective singular indefinites like *a mountain chain*. As with bare plurals, we believe that these complexities are manageable, but we will not discuss them here.

5 Formal semantics of set-eigenspaces (1): topological and distal relations

The two hypotheses developed in sections 2 and 4 associate a property-denoting indefinite or a collective description with a set of entities. The spatial semantics of these expressions is based on locating this set of entities – determining its *eigenspace*. Turning the PEH and the CEH into a formal semantic framework amounts to specifying how eigenspaces are determined and composed with other denotations. This section develops such a formal semantics for locative relations like 'in' and 'far from', which pertain to the *topology* of regions and *distances* between them. Section 6 will extend this treatment for locative relations like 'above' or ‘to the north of’ that pertain to *directions*, especially when combined with distal modifiers like *five meters*. In both sections our analysis follows the same logical line. First, we analyze simple locative sentences without indefinites or collective reference. Second, we look at the implications of this analysis for sentences where eigenspaces are derived from sets of entities according to the PEH/CEH. In this way, we are able to show that central predictions of our analysis formally follow from the PEH/CEH within a conventional formal analysis of spatial meaning.

Subsection 5.1 analyzes topological and distal spatial relations as binary relations between sets of *points* in a metric topological space. This simple ontology can be easily translated to more complex formal analyses of spatial relations (Nam 1995, Zwarts & Winter 2000, Kracht 2002). However, it is sufficient for introducing, in subsection 5.2, some new observations about *additivity* and *anti-additivity* of topological and distal spatial relations. Subsection 5.3 further analyzes additivity in view of some related observations about ‘*outside*-entailments’ like *near London* ⇒ *outside London*. Consistently with previous accounts (Levinson 2000), we analyze such entailments as pragmatic. Subsection 5.4 adopts familiar assumptions about the association of entities with their eigenspaces (Wunderlich 1991, Zwarts & Winter 2000). Subsection 5.5 tunes this basic proposal with the PEH and the CEH by associating sets with the union of their members’ eigenspaces. This allows us to formally analyze the quantificational behavior of some basic cases of spatial indefinites.

5.1 Topological and distal relations

The basic spatial notion we use is *region*. We here formalize regions as sets of *points* in a topological space.[28] Officially:

Terminology (points and regions). *Let M be a topological space. Elements of M are referred to as* POINTS *and subsets of M are referred to as* REGIONS.

Locative expressions are treated as *relations between regions*. The locative relation INSIDE is simply defined as the subset relation between regions. Analogously, the relation OUTSIDE is analyzed as region disjointness.[29] Formally:

(59) For any two regions A and B:

$$\textsc{inside}(A, B) \Leftrightarrow A \subseteq B$$

$$\textsc{outside}(A, B) \Leftrightarrow A \cap B = \varnothing$$

[28] Given a set M, a *topological space* over M is a pair $\langle M, T\rangle$, where T is a subset of $\wp(M)$ that satisfies $\{\varnothing, M\} \subseteq T$, and that is closed under arbitrary unions and under finite intersections. The sets in T are called *open* and their complements in M are called *closed*. Here we sloppily refer to the set M itself as a “topological space”.

[29] We analyze *inside* and *outside* as equivalent to *completely inside*/*outside*. This is not always the case, wit. the felicity of *the car is inside*/*outside the garage, but part of it remains outside*/*inside the garage*. Such alternations between *completely* and *partially* interpretations are left beyond the scope of our formal treatment here.

We refer to *inside* and *outside* as *topological* prepositions. However, many of their basic uses, as well as those of verbs like *overlap* and *contain*, can be treated as in (59) using set-theoretical relations alone, without considering further properties of topological spaces. Here we concentrate on these set-theoretical uses.[30]

Relying on topology alone is not sufficient for describing *distal* concepts like those expressed by the locatives constructions *close to* and *far from*. In order to analyze such expressions as well, we endow the spatial ontology with a *metric function* ***d*** that helps modeling distance between regions.[31] Henceforth, we assume a metric space M with the natural topology as our spatial ontology.

Using the metric function ***d*** between points, we define below the natural *distance function* between closed regions.

Definition 1 (distance). *For any two non-empty closed regions* $A, B \subseteq M$ *that are mutually disjoint, we define the* distance *between* A *and* B *by:*
$\textsc{dist}(A, B) = min(\{\mathbf{d}(x, y) : x \in A \text{ and } y \in B\})$.

In words: $\textsc{dist}(A, B)$ is the minimal distance between points in A and points in B. For our purposes here we ignore regions A and B that are not closed or are not mutually disjoint.[32]

Using the $\textsc{dist}$ function, the spatial relations $\textsc{close\_to}$ and $\textsc{far\_from}$ are defined as follows.

(60) For any two non-empty closed regions A and B that are mutually disjoint:
$\textsc{close\_to}(A, B) \Leftrightarrow 0 < \textsc{dist}(A, B) < d_1$
$\textsc{far\_from}(A, B) \Leftrightarrow \textsc{dist}(A, B) > d_2$
where d_1 *and* d_2 *are positive reals s.t.* $d_1 \leq d_2$.

In words, the relation $\textsc{close\_to}$ holds between regions that are at a positive distance from one another, bounded from above by d_1. Analogously, the relation $\textsc{far\_from}$ holds between regions, with d_2 as the lower bound for the distance between them.

[30] Properly topological properties of these prepositions may be needed for describing fine-grained distinctions like the ones between the English prepositions *inside* and *in*. Similarly, the prepositions *between* and *among* (Zwarts & Winter 2000) invite using the geographical operation of *convex hull* (cf. section 4). Asymmetric usages of *outside* (e.g. *the cat is outside the box* vs. *#the box is outside the cat*) are also left untreated here. We believe that these asymmetries follow from the conventional implicature or presupposition *A might possibly be inside B* that is associated with the sentence *A is outside B*.

[31] A metric function over a set M is any function $\mathbf{d} : (M \times M) \to \mathcal{R}$ from pairs of elements in M to non-negative real numbers, which satisfies the following for all elements $x, y \in M$: $\mathbf{d}(x, y) = \mathbf{d}(y, x)$ (symmetry); $\mathbf{d}(x, y) + \mathbf{d}(y, z) \geq \mathbf{d}(x, z)$ (triangle inequality); $\mathbf{d}(x, y) = 0$ *iff* $x = y$ (identity of indiscernibles). The set M together with the metric ***d*** are called a *metric space*. Any metric space can be naturally defined as a topological space (Kelley 1955, p.119).

[32] We focus on closed regions because we want to simply analyze the distance between regions as the *minimal* distance between their points. The reason for our assumption that the two regions are mutually disjoint will become clear when we discuss what we call the 'outside' presupposition, in section 5.2.

Both d_1 and d_2 are contextually determined. The assumption $d_1 \leq d_2$ makes sure that regions are not simultaneously close to each other and far from each other in the same context, while leaving open the possibility to assume, if desired, that $d_1 = d_2$.[33]

Further, let us address some distal spatial relations involving *measure phrases* (MPs). As we have seen, a rich source of distal relations involves modification of the spatial expression (*away*) *from* by MPs like (*exactly*/*at most*/*at least*) *20 km*. This is illustrated in the following sentences.

(61) a. The house is (exactly) 20km from London.

b. The house is $\left\{ \begin{array}{l} \text{at most} \\ \text{less than} \end{array} \right\}$ 20km from London.

c. The house is $\left\{ \begin{array}{l} \text{at least} \\ \text{more than} \end{array} \right\}$ 20km from London.

Abstracting away from the measuring unit (meters, kilometers, etc.), let us assume that measure phrases denote subsets of the non-negative real numbers $\mathbf{R}^+$.[34] We thus assume the following natural denotations for the MPs in (61).

(62) a. $\text{20KM} = \{20\}$

b. $\text{AT\_MOST\_20KM} = \{r : r \leq 20\}$

$\text{LESS\_THAN\_20KM} = \{r : r < 20\}$

c. $\text{AT\_LEAST\_20KM} = \{r : r \geq 20\}$

$\text{MORE\_THAN\_20KM} = \{r : r > 20\}$

Note that in (62b) and (62c), the MP denotations are *downward monotone* and *upward monotone* sets of real numbers, respectively. By contrast, the assumed denotation of MPs like (*exactly*) *20km* is non-monotone.[35]

Generalizing the assumed denotations (60) of the distal relations *close to* and *far from*, we use the following denotation for locative relations of the form *MP from*. In this analysis and henceforth, we employ the notations 'MP' and 'MP_FROM' for the set of real numbers denoted by a measure phrase *MP*, and for the spatial relation *MP from* that is derived from it.

Definition 2 (**Measure-based spatial relation).** *Let* MP $\subseteq \mathbf{R}^+$ *be a set of non-negative real numbers. For any two non-empty closed regions* $A, B \subseteq M$ *that are mutually disjoint, the spatial relation* MP_FROM *is defined by:*

$\text{MP\_FROM}(A, B) \Leftrightarrow \text{DIST}(A, B) \in \text{MP}$.

[33] For scale-based semantics of antonymous adjectives like *close* and *far* see Kennedy (1999), references therein and subsequent works.

[34] Measuring units require multiplying these numbers by the proper constants, e.g. 0.001 for meters and 1 for kilometers, or any other constants in the proper ratio.

[35] A set $A \subseteq \mathbf{R}^+$ of positive reals is called *upward* (*downward*) *monotone* if for every number $r \in A$, for every $r' \in \mathbf{R}^+$, if $r' \geq r$ ($r' \leq r$, respectively), we have $r' \in A$. A set that is neither upward nor downward monotone is called *non-monotone*.

In words: the relation MP_FROM holds between the regions A and B if the distance between A and B is in the measure phrase denotation MP.

According to our assumptions above, we get the following denotations for the prepositional constructions in (61).

(63) a. $\text{20KM\_FROM}(A, B) \Leftrightarrow \text{DIST}(A, B) = 20$

b. $\text{AT\_MOST\_20KM\_FROM}(A, B) \Leftrightarrow \text{DIST}(A, B) \leq 20$

$\text{LESS\_THAN\_20KM\_FROM}(A, B) \Leftrightarrow \text{DIST}(A, B) < 20$

c. $\text{AT\_LEAST\_20KM\_FROM}(A, B) \Leftrightarrow \text{DIST}(A, B) \geq 20$

$\text{MORE\_THAN\_20KM\_FROM}(A, B) \Leftrightarrow \text{DIST}(A, B) > 20$

Using these simple assumptions about topological and distal relations, we can now move on and see their implications for our PEH/CEH-based analysis.

5.2 Additivity and anti-additivity of spatial relations

In section 3 we informally analyzed some monotonicity properties of locative relations and their interactions with licensing of negative polarity items. Two stronger properties than monotonicity, which affect existential/universal alternations with spatial indefinites and collective descriptions, are *additivity* and *anti-additivity* (F. Zwarts 1998, Nam 1994). Let $\mathcal{R}$ be a relation between subsets of a domain M, and let A, B_1 and B_2 be arbitrary subsets of M. Additivity and anti-additivity in the right argument of $\mathcal{R}$ are defined as follows.

(64) **Additivity**:
$\mathcal{R}(A, B_1 \cup B_2) \Leftrightarrow \mathcal{R}(A, B_1) \vee \mathcal{R}(A, B_2)$.

(65) **Anti-additivity**:
$\mathcal{R}(A, B_1 \cup B_2) \Leftrightarrow \mathcal{R}(A, B_1) \wedge \mathcal{R}(A, B_2)$.

Here and henceforth, we sloppily say that a relation $\mathcal{R}$ is '(anti-)additive' when referring to (anti-)additivity in its *right argument*.

In section 3 we discussed the intuitive entailments (41a) and (42a) with the locative expressions *outside* and *far from* that support their treatment as downward monotone relations. Further, we observe that these uni-directional entailments can be strengthened into the following equivalences.

(66) a. We're *outside* Eurasia ⇔ We're *outside* Europe <u>and</u> we're *outside* Asia.

b. We're *far from* Eurasia ⇔ We're *far from* Europe <u>and</u> we're *far from* Asia.

In the equivalences in (66) we assume that the eigenspace of Eurasia is the union of the eigenspaces of Europe and Asia. As we shall presently see, these equivalences support the treatment of *outside* and *far from* as anti-additive relations.[36] First, the

[36] Considering *determiners*, we repeat below some familiar illustrations for anti-additivity (F. Zwarts 1998, p.222).

equivalence in (66a) is expected by the following fact, which is a direct result of our definition of the relation OUTSIDE as set disjointness.

Fact 1. *For all regions* $A, B_1, B_2 \subseteq M$*:*

$$\text{OUTSIDE}(A, B_1 \cup B_2) \Leftrightarrow \text{OUTSIDE}(A, B_1) \wedge \text{OUTSIDE}(A, B_2).$$

In words: the relation OUTSIDE is anti-additive for all subsets of M. A similar fact holds of the spatial relation FAR_FROM, which is stated below.

Fact 2. *For all non-empty closed regions* $A, B_1, B_2 \subseteq M$ *s.t.* $A \cap (B_1 \cup B_2) = \varnothing$*:*

$$\text{FAR\_FROM}(A, B_1 \cup B_2) \Leftrightarrow \text{FAR\_FROM}(A, B_1) \wedge \text{FAR\_FROM}(A, B_2).$$

In words: the relation FAR_FROM is anti-additive over all subsets of M for which it is defined.

Also other distal relations that are downward monotone in their right argument are anti-additive over the subsets of M for which they are defined. For instance, consider the following equivalences, which are similar to the ones in (66) above.

(67) a. We're *at least 20km from* Eurasia ⇔ We're *at least 20km from* Europe <u>and</u> we're *at least 20km from* Asia.

b. We're *more than 20km from* Eurasia ⇔ We're *more than 20km from* Europe <u>and</u> we're *more than 20km from* Asia.

These equivalences are explained by the fact that the relations AT_LEAST_20KM_FROM and MORE_THAN_20KM_FROM in (63c) are anti-additive similarly to FAR_FROM. In general, anti-additivity holds for any relation MP_FROM for which the measure phrase denotation MP is *upward monotone*. Formally:

Fact 3. *Let* MP $\subseteq \mathbf{R}^+$ *be an upward monotone set of positive real numbers. For all non-empty closed regions* $A, B_1, B_2 \subseteq M$ *s.t.* $A \cap (B_1 \cup B_2) = \varnothing$*:*

$$\text{MP\_FROM}(A, B_1 \cup B_2) \Leftrightarrow \text{MP\_FROM}(A, B_1) \wedge \text{MP\_FROM}(A, B_2).$$

In words: the relation MP_FROM is *anti-additive* for every *upward monotone* set MP, over all subsets of M for which it is defined. The additional requirements in Facts 2 and 3 about the sets A, B_1 and B_2 make sure that we can apply the distance function DIST in our definition of distal relations.

Both Facts 2 and 3 follow directly from the following property of the function DIST over the closed regions.

Fact 4. *For all non-empty closed regions* $A, B_1, B_2 \subseteq M$ *s.t.* $A \cap (B_1 \cup B_2) = \varnothing$*:*

$$\text{DIST}(A, B_1 \cup B_2) = min(\text{DIST}(A, B_1), \text{DIST}(A, B_2))$$

(i) Everyone who is tall <u>or</u> happy smiled ⇔ Everyone who is tall smiled <u>and</u> everyone who is happy smiled.

(ii) No one who is tall <u>or</u> happy smiled ⇔ No one who is tall smiled <u>and</u> no one who is happy smiled.

(iii) No math teacher is tall <u>or</u> happy ⇔ No math teacher is tall <u>and</u> no math teacher is happy.

In words: the distance between a region A and a union region $B_1 \cup B_2$ is the minimal distance among the two distances from A to B_1 and from A to B_2.

Fact 2 follows from distance minimality as stated in Fact 4 and the intuitive definition of the relation $\textsc{far\_from}$ as imposing a lower bound on the distance between regions. Similarly, Fact 3 follows from distance minimality and the assumed upward monotonicity of the measure phrase in the definition of the relation $\textsc{mp\_from}$.

Distance minimality also immediately establishes the *additivity* of the spatial relation $\textsc{close\_to}$, and of spatial relations $\textsc{mp\_from}$ where the set $\textsc{mp}$ is *downward monotone*. This is formally stated in Facts 5 and 6 below.

Fact 5. *For all non-empty closed regions* $A, B_1, B_2 \subseteq M$ *s.t.* $A \cap (B_1 \cup B_2) = \varnothing$*:*

$$\textsc{close\_to}(A, B_1 \cup B_2) \Leftrightarrow \textsc{close\_to}(A, B_1) \vee \textsc{close\_to}(A, B_2).$$

In words: the relation $\textsc{close\_to}$ is additive over all subsets of M for which it is defined.

Fact 6. *Let* $\textsc{mp} \subseteq \mathbf{R}^+$ *be a downward monotone set of positive real numbers. For all non-empty closed regions* $A, B_1, B_2 \subseteq M$ *s.t.* $A \cap (B_1 \cup B_2) = \varnothing$*:*

$$\textsc{mp\_from}(A, B_1 \cup B_2) \Leftrightarrow \textsc{mp\_from}(A, B_1) \vee \textsc{mp\_from}(A, B_2).$$

In words: the relation $\textsc{mp\_from}$ is *additive* for every *downward monotone* set $\textsc{mp}$, over all subsets of M for which it is defined.

In the equivalences (66) and (67) above we have seen intuitive support for the anti-additivity expected by our definitions of the respective spatial relations. We should now like to examine whether also the *additivity* of distal relations like $\textsc{close\_to}$ and $\textsc{less\_than\_20km\_from}$ is warranted by the behavior of the corresponding spatial expressions. This question requires special attention to further details about these expressions and distal relations in general.

5.3 Additivity and 'outside' presuppositions

Let us focus on the locative expression *close to*. Given the additivity of the relation $\textsc{close\_to}$ (Fact 5), we expect the following equivalence to be intuitively valid.

(68) We're *close to* Eurasia $\overset{?}{\Leftrightarrow}$ We're *close to* Europe <u>or</u> we're *close to* Asia.

One direction of this equivalence surely holds:

(69) We're *close to* Eurasia $\Rightarrow$ We're *close to* Europe <u>or</u> we're *close to* Asia.

Sure enough, wherever we may be, the point in Eurasia that is closest to us is in Europe or Asia, or perhaps in both. Therefore if we are close to that point we are close to Europe, Asia or both. Consequently there is no question about the intuitive validity of the entailment in (69). But how about the other direction of the additivity equivalence in (68), as restated below?

(70) We're *close to* Europe <u>or</u> we're *close to* Asia $\overset{?}{\Rightarrow}$ We're *close to* Eurasia.

Suppose that the disjunctive antecedent of (70) is true. Does it follow that we are close to Eurasia? This is more questionable than the entailment in (69). To see that, let us without loss of generality consider the following entailment.

(71) We're *close to* Europe $\overset{?}{\Rightarrow}$ We're *close to* Eurasia.

Intuitive validity of the entailment in (71) would support the intuitive validity of (70), and vice versa. Since Europe and Asia are both contained in Eurasia, these entailments equally test the *upward monotonicity* of the expression *close to*. To see why these entailments are questionable, suppose that we are close to Europe and *within* Asia, say in the Asian part of Istanbul. In this case the antecedents of the entailments (70) and (71) are true, but it is hard to conclude that *we're close to Eurasia*: in fact, we are *in* Eurasia. This scenario suggests that the entailments (70) and (71), and consequently also the equivalence in (68), do not intuitively hold. But does it mean that our treatment of the distal expression *close to* using an additive (hence upward monotone) relation was somehow wrong?

We propose that the invalidity of the entailments in (70) and (71) is not a simple challenge for the truth-conditional analysis of distal relations, but involves their pragmatic behavior or their *presuppositions*. Thus, we tentatively propose that the consequent *we're close to Eurasia* in (70) and (71) requires that we be outside Eurasia, but this requirement is a presupposition rather than a truth-condition. This presupposition is not necessarily satisfied by the antecedent *we're close to Europe* in (71), or the disjunctive antecedent in (70). For instance, when we are in Asian Istanbul the antecedents of the entailments (70) and (71) are analyzed as true and felicitous. However, in this situation we are not outside Eurasia. Consequently a further semantic-pragmatic requirement of the consequent *we're close to Eurasia* is not met, hence its infelicity.

Somewhat sloppily, we say that the expression *close to* has an 'outside' presupposition, as more generally stated below.

(72) **The 'outside' presupposition**: Let P be locative prepositional relation. We say that P triggers an *'outside' presupposition* if sentences of the form NP_1 *is* P NP_2 presuppose that NP_1 *is outside* NP_2.

This statement is not meant as a full-fledged theory about 'outside' entailments with prepositions. As Levinson (2000) points out, similar phenomena with spatial expressions may reasonably be analyzed as purely pragmatic, conversational implicatures. By calling the outside-implications "presuppositions" we take one direction among other possible ones, which leave them beyond the core truth-conditional meaning of prepositions.

As pointed out by J. Zwarts and Winter (2000), most usages of locative prepositions like *far from, above, behind, below, among* and *between* show this 'outside' requirement. We now propose that with some prepositions, this requirement should be analyzed as differently than other truth-conditional effects with prepositions. As support for this claim, consider the contrast between sentences (73a-b) in the context of (73c).

(73) a. If we're *close to*/*less than 20km from* Europe we'll arrive by 9PM.

b. If we're *outside Europe* we'll arrive after 8PM.

(73c) Context: *On a flight from New York City to Zurich, we are interested in estimating our time of arrival by looking up our location.*

From the utterance of (73a) in the given context, we can infer that the speaker thinks we are outside Europe. In other words, the *outside* requirement of the spatial expressions *close to Europe* and *less than 20km from Europe* "projects" from the conditional clause to the matrix clause. This behavior is characteristic of presuppositions (Beaver & Geurts 2011). By contrast, the conditional in (73b) does not presuppose or imply that we are outside Europe. Thus, unsurprisingly, simple sentences with the preposition *outside* assert an *outside* statement rather than presupposing it.[37]

Our conclusion from this discussion is that when analyzing the formal semantics of expressions like *close to* and *less than 20km from*, we should start by assuming that they operate on disjoint regions, as required by their 'outside' presupposition. This restriction is encoded into our treatment of distal relations in (60) and Definition 2 above. As a result these relations are analyzed as additive over their domain, as stated in Facts 5 and 6.

5.4 Locating entities

So far we have informally used English examples to informally support our assumptions about spatial relations. In order to incorporate spatial relations like INSIDE, OUTSIDE, CLOSE_TO and FAR_FROM in a formal analysis of natural language sentences, we need to make an additional step and establish a connection between entities and their eigenspaces. To see the reason for that, consider the following sentences.

(74) a. The car is inside the garage.

b. The car is outside the garage.

c. The car is close to the garage.

d. The car is far from the garage.

Suppose that the denotations for the noun phrases *the car* and *the garage* are some 'simple' entities $\mathbf{c}$ and $\mathbf{g}$, respectively. We derive the eigenspace for such entities using a *location function* that we denote 'LOC', which sends entities to regions (Zwarts

[37] Other English prepositions that unsurprisingly do not show any *outside* presupposition are the preposition *in* and its variants *inside* and *within*. Arguably, these locative concepts contradict the *outside* requirement as part of their lexical meaning, hence they are unlikely to show any presupposition entailing it.

& Winter 2000). Thus, the eigenspaces of the entities $\mathbf{c}$ and $\mathbf{g}$ are $\textsc{loc}(\mathbf{c})$ and $\textsc{loc}(\mathbf{g})$ respectively. In this way, sentences (74a-d) are analyzed as follows.

(75) a. $\textsc{inside}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\mathbf{g}))$
b. $\textsc{outside}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\mathbf{g}))$
c. $\textsc{close\_to}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\mathbf{g}))$
d. $\textsc{far\_from}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\mathbf{g}))$

This use of the location function guarantees that the NP complement within a PP can standardly refer to an entity.[38]

Summarizing our use of the LOC function for associating entities with their eigenspace, we introduce the following definition.

Definition 3 (eigenspace). *Let E be a non-empty set of locatable entities in a model and let M be a topological space. For every locatable entity $x \in E$, the* EIGENSPACE *of x is a non-empty closed region $\textsc{loc}(x) \subseteq M$.*

The set E of locatable entities is an arbitrary set defining the domain of the LOC function. Note the assumption that the eigenspace of any locatable entity is non-empty and closed, which will be important for our analysis of distal expressions like *close to* and *far from*. Note further that NPs derived from nouns like *proof* or *revolution* can also appear in locative PPs (cf. (24) and (28)) but do not necessarily denote locatable entities.

5.5 Locating objects associated with sets

Our assumptions in sections 5.1-5.4 above only concern spatial sentences as in (74), where the NP within the locative PP refers to a 'simple' entity.[39] We take it that similar assumptions at a comparable level or complexity must be adopted by any formal treatment of spatial expressions. In order to also deal with spatial indefinites and collective descriptions, the PEH and the CEH assign eigenspaces to objects that are associated with *sets* of entities. The PEH is about extensional eigenspaces of *properties*; the CEH is about eigenspaces of *impure atoms*. In both cases, the PEH and the CEH assign an eigenspace to an object x using a set of entities that is associated with x. For the sake of our formal discussion here, we only consider the 'union strategy' for locating such objects, which is shared by the PEH and the CEH.[40] To do that, we assume that properties and impure atoms are themselves

[38] J. Zwarts and Winter (2000) also use the LOC function in their compositional analysis of PPs as denoting sets of entities. This is needed in order to allow the treatment of PPs as ordinary predicates in post-copula positions (e.g. *is cheap and close to the airport*) and as ordinary modifiers in post-nominal positions (e.g. *a hotel close to the airport*). For the sake of our analysis here, the simpler but less compositional treatment in (75) is sufficient.

[39] See section 7 for some remarks that cast some doubt on this intuition of 'simplicity'.

[40] Addressing the 'convex hull' strategy of the CEH for locating impure atoms would add complexity to the formal analysis, which we prefer to avoid.

members of the set E of locatable entities, within a special subset of E that we call E'. Entities in E' are associated with sets of locatable entities in $E - E'$. Thus, we assume that properties and impure atoms are associated with sets of entities that are not themselves properties or impure atoms.[41] The PEH and CEH define eigenspaces for entities in E' based on the eigenspaces of the elements in the sets they are associated with. Formally:

(76) Let $E' \subseteq E$ be a set of locatable entities, where each element $x \in E'$ is associated with a non-empty set $A(x)$ of locatable entities in $E - E'$. According to the PEH and the CEH, for each $x \in E'$ we have:

$$\textsc{loc}(x) = \bigcup_{y \in A(x)} \textsc{loc}(y).$$

On the linguistic justification for the assumption that the set $A(x)$ is non-empty with property-denoting spatial indefinites, see section 3.1. Collective descriptions are associated with non-empty sets in most semantic theories of plurals (Winter 2001). From a formal point of view there is reason to avoid the complications that may ensue from associating empty sets with eigenspaces.[42]

The assumption in (76) allows us to analyze spatial indefinites as in the following examples.[43]

(77) a. The car is outside a gas station.
b. The car is far from a gas station.
c. The car is close to a gas station.

Suppose that the entity denotation for the noun phrase *the car* is $\mathbf{c}$. Suppose further that the indefinite *a gas station* denotes the property $\textit{GS}$. The analysis of the sentences in (77) is as follows.

(78) a. $\textsc{outside}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\textit{GS}))$
b. $\textsc{far\_from}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\textit{GS}))$
c. $\textsc{close\_to}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\textit{GS}))$

According to assumption (76), the eigenspace $\textsc{loc}(\textit{GS})$ of the property $\textit{GS}$ is the union of eigenspaces for entities in its extension $\check{\textit{GS}}$. In formula:

(79) $$\textsc{loc}(\textit{GS}) = \bigcup_{x \in \check{\textit{GS}}} \textsc{loc}(x)$$

By associating the set $\check{\textit{GS}}$ with the unary predicate $\mathbf{gas\_station}$ we get:

(80) $$\textsc{loc}(\textit{GS}) = \bigcup\{\textsc{loc}(x) : \mathbf{gas\_station}(x)\}$$

[41] This is a simplifying assumption. With NPs like *the clusters of utility poles* we may need to locate a collection C of collections. For defining such locations we would need to replace definition (76) by a recursive definition.

[42] As we saw, metaphorical uses of locative PPs as in sentence (24) do not trigger existence entailments. Without analyzing them formally here, we assume that these metaphorical usages of PPs involve other mechanisms besides the $\textsc{loc}$ function.

[43] Indefinites within *inside* PPs are ignored here because of the problems mentioned in section 3.5.

In words: the eigenspace of the property $\textit{GS}$ is the union of eigenspaces for entities that satisfy the predicate $\mathbf{gas\_station}$.

Note that when the extension $\check{\textit{GS}}$ is a finite set of entities, the region unions in (79) and (80) are finite. This gives us immediate analyses of the existential/universal alternations in sentences (77a-c), as shown below.[44]

For any locatable entity $\mathbf{c} \in E$ and a property $\textit{GS}$ associated with a non-empty finite extension $\check{\textit{GS}} \subseteq E$ of locatable entities:

(81) Universal analysis of '*outside*' example (77a):

$\text{OUTSIDE}(\text{LOC}(\mathbf{c}), \text{LOC}(\textit{GS}))$ ▷ compositional analysis (78a) of (77a)

$\Leftrightarrow \text{OUTSIDE}(\text{LOC}(\mathbf{c}), \bigcup\{\text{LOC}(x) : \mathbf{gas\_station}(x)\})$ ▷ eigenspace of property $\textit{GS}$ (80)

$\Leftrightarrow \forall x.\mathbf{gas\_station}(x) \rightarrow \text{OUTSIDE}(\text{LOC}(\mathbf{c}), \text{LOC}(x))$ ▷ anti-additivity of OUTSIDE (Fact 1) and non-emptiness and finiteness of $\check{\textit{GS}}$

(82) Universal analysis of '*far from*' example (77b):[45]

$\text{FAR\_FROM}(\text{LOC}(\mathbf{c}), \text{LOC}(\textit{GS})) \wedge \text{OUTSIDE}(\text{LOC}(\mathbf{c}), \text{LOC}(\textit{GS}))$ ▷ compositional analysis (78b) of (77b) and 'outside' presupposition

$\Leftrightarrow \text{FAR\_FROM}(\text{LOC}(\mathbf{c}), \bigcup\{\text{LOC}(x) : \mathbf{gas\_station}(x)\}) \wedge \forall x.\mathbf{gas\_station}(x) \rightarrow \text{OUTSIDE}(\text{LOC}(\mathbf{c}), \text{LOC}(x))$ ▷ eigenspace of property $\textit{GS}$ (80) and universal analysis of '*outside*' (81)

$\Leftrightarrow \forall x.\mathbf{gas\_station}(x) \rightarrow \text{FAR\_FROM}(\text{LOC}(\mathbf{c}), \text{LOC}(x)) \wedge \text{OUTSIDE}(\text{LOC}(\mathbf{c}), \text{LOC}(\textit{GS}))$ ▷ anti-additivity of FAR_FROM (Fact 2) and non-emptiness and finiteness of $\check{\textit{GS}}$, and 'outside' presupposition

[44]The assumption about the finiteness of $\check{\textit{GS}}$ is quite innocuous as far as naive intuitions are concerns. Without it, all sorts of topological subtleties would have to be dealt with.

[45]As argued in section 5.3, analyzing PPs often requires invoking an 'outside' presupposition.

(83) Existential analysis of '*close to*' example (77c):[45]

$\textsc{close\_to}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\textit{GS})) \wedge$
$\textsc{outside}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\textit{GS}))$ ▷ compositional analysis (78c) of (77c) and 'outside' presupposition

$\Leftrightarrow \textsc{close\_to}(\textsc{loc}(\mathbf{c}), \bigcup\{\textsc{loc}(x) : \mathbf{gas\_station}(x)\}) \wedge$
$\forall x.\mathbf{gas\_station}(x) \rightarrow \textsc{outside}(\textsc{loc}(\mathbf{c}), \textsc{loc}(x))$ ▷ eigenspace of property *GS* (80) and universal analysis of '*outside*' (81)

$\Leftrightarrow \exists x.\mathbf{gas\_station}(x) \wedge \textsc{close\_to}(\textsc{loc}(\mathbf{c}), \textsc{loc}(x)) \wedge$
$\textsc{outside}(\textsc{loc}(\mathbf{c}), \textsc{loc}(\textit{GS}))$ ▷ additivity of $\textsc{close\_to}$ (Fact 5) and non-emptiness and finiteness of $\check{}\textit{GS}$, and 'outside' presupposition

Let us further consider the following cases (cf. (13)).

(84) a. Fido is at least five meters from a doghouse.
b. Fido is at most five meters from a doghouse.
c. Fido is (exactly) five meters from a doghouse.
d. Fido is between three and five meters from a doghouse.

Sentences (84a) and (84b) are treated analogously to sentences (77b) and (77c). Thus, similarly to (82) and (83) we get the following analyses, with $\mathbf{f}$ the entity denotation of *Fido* and *DH* the property denotation of *doghouse*.

(85) a. Universal analysis of (84a):

$\textsc{at\_least\_5m\_from}(\textsc{loc}(\mathbf{f}), \textsc{loc}(\textit{DH})) \wedge \textsc{outside}(\textsc{loc}(\mathbf{f}), \textsc{loc}(\textit{DH}))$
$\Leftrightarrow \forall x.\mathbf{doghouse}(x) \rightarrow \textsc{at\_least\_5m\_from}(\textsc{loc}(\mathbf{f}), \textsc{loc}(x))$
$\wedge\ \textsc{outside}(\textsc{loc}(\mathbf{f}), \textsc{loc}(\textit{DH}))$

b. Existential analysis of (84b):

$\textsc{at\_most\_5m\_from}(\textsc{loc}(\mathbf{f}), \textsc{loc}(\textit{DH})) \wedge \textsc{outside}(\textsc{loc}(\mathbf{f}), \textsc{loc}(\textit{DH}))$
$\Leftrightarrow \exists x.\mathbf{doghouse}(x) \wedge \textsc{at\_most\_5m\_from}(\textsc{loc}(\mathbf{f}), \textsc{loc}(x))$
$\wedge\ \textsc{outside}(\textsc{loc}(\mathbf{f}), \textsc{loc}(\textit{DH}))$

The measure phrases (*exactly*) *5m* and *between 3m and 5m* in (84c) and (84d) are neither upward nor downward monotone, and as a result, the corresponding *MP from* relations are neither additive nor anti-additive. However, following Thijsse (1983), we note that the complex numerals in these MPs make them denote *intersections* of upward/downward monotonic sets. For instance, the measure phrase *exactly 5m* means *at least 5m* <u>*and*</u> *at most 5m*. Accordingly, the set $\{5\}$ that this MP denotes can easily be analyzed as an intersection of two monotone sets of reals: the set including 5 and all smaller reals, and the set including 5 and all greater reals. Similarly, the measure phrase *between 3m and 5m* is analyzed as the intersection of the two monotone denotations of the MPs *at least 3m* and *at most 5m*. When an MP denotation is an intersection, also the corresponding *MP from* spatial relation is an intersection. This is formally stated below.

Fact 7. *Let* $\text{MP}_1, \text{MP}_2 \subseteq \mathbf{R}^+$ *be sets of positive real numbers. For all non-empty closed regions* $A, B \subseteq M$ *s.t.* $A \cap B = \varnothing$*:*

$$(\text{MP}_1 \cap \text{MP}_2)\_\text{FROM}(A,B) \;=\; \text{MP}_1\_\text{FROM}(A,B) \wedge \text{MP}_2\_\text{FROM}(A,B).$$

In words: when MP is an intersection of two sets of real numbers MP_1 and MP_2, the corresponding distal relation MP_FROM involves a conjunction of the respective distal relations MP_1_FROM and MP_2_FROM.

In particular, when the sets MP_1 and MP_2 are downward monotone and upward monotone, respectively, the distal relation MP_FROM involves a conjunction of an additive distal relation and an anti-additive distal relation. Accordingly, for sentences (84c) and (84d) we get a mixed existential/universal analysis. For instance, consider the analysis below of sentence (84c).

(86) Existential/Universal analysis of (84c):

$$\begin{aligned}
&\text{EXACTLY_5M_FROM}(\text{LOC}(\mathbf{f}), \text{LOC}(\textit{DH})) \wedge \text{OUTSIDE}(\text{LOC}(\mathbf{f}), \text{LOC}(\textit{DH}))\\
&\Leftrightarrow \text{AT_MOST_5M_FROM}(\text{LOC}(\mathbf{f}), \text{LOC}(\textit{DH}))\\
&\quad \wedge\ \text{AT_LEAST_5M_FROM}(\text{LOC}(\mathbf{f}), \text{LOC}(\textit{DH}))\\
&\quad \wedge\ \text{OUTSIDE}(\text{LOC}(\mathbf{f}), \text{LOC}(\textit{DH}))\\
&\Leftrightarrow \exists x.\mathbf{doghouse}(x) \wedge \text{AT_MOST_5M_FROM}(\text{LOC}(\mathbf{f}), \text{LOC}(x))\\
&\quad \wedge\ \forall x.\mathbf{doghouse}(x) \rightarrow \text{AT_LEAST_5M_FROM}(\text{LOC}(\mathbf{f}), \text{LOC}(x))\\
&\quad \wedge\ \text{OUTSIDE}(\text{LOC}(\mathbf{f}), \text{LOC}(\textit{DH}))
\end{aligned}$$

This “mixed” effect in our analysis of sentence (84c) directly follows from the additivity of the relation AT_MOST_5M_FROM, the anti-additivity of the relation AT_LEAST_5M_FROM, and their analyses in (85).

6 Formal semantics of set-eigenspaces (2): projective relations and their distal modifiers

In this section we look further on sentences like (15), which is restated below.

(87) Tweety is five meters above a cloud.

Formally treating such sentences involves phenomena that were not addressed by our analysis in section 5. First, we need to analyze *projective relations* like ‘above’, which pertain to directions from the reference object. Then, some properties of *distal modifiers* like ‘five meters’ must be re-assessed, in particular their reference to relevant directions from the reference object. Lastly, we need to pay special attention to *non-convex eigenspaces* like the collection of clouds described by the indefinite ‘a cloud’. As we will see, these phenomena are all present with ‘referential’ singular NPs like *the cloud*. When PPs like *5m above* <u>*the*</u> *cloud* are treated in full

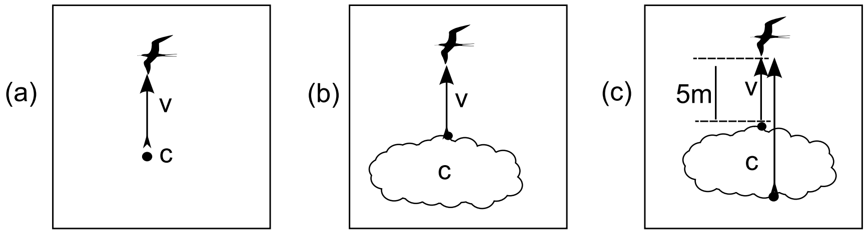

Figure 8: Tweety is (5m) above the cloud. *In (a), the vector v is 'above' the point* c*. In (b), the 'above vector' v is 'above' a boundary point of the region C. In (c), the vector v is a shortest vector among the 'above vectors' of the region C, and its length is five meters.*

generality, the PEH immediately leads us to a formal analysis of spatial indefinites as in sentence (87).

Locative relations like *above*, *behind* or *to the west of* presuppose directions from the reference object, which are determined by various factors involving lexical meaning, world knowledge and contextual information. We refer to such spatial relations as *projective*.[46] For instance, when locating an entity as being *above John's head*, we often take into account John's body's upright position. However, if John is lying supine, we may also consider a butterfly that is flying in front of John's nose as being *above John's head*. Thus, in order to treat projective relations, we need to specify some given directions from the reference object. One way of doing that is by postulating cone-shaped regions in the appropriate directions. For instance, consider a simple treatment of the prepositional phrase *above the cloud*, with a point c as the eigenspace of the cloud. We associate the point c with an 'up region', which is denoted $up(\mathsf{c})$. This region is an open, unbounded cone-shaped region having c as its center. A simple sentence like (88) below requires that Tweety's eigenspace is contained in $up(\mathsf{c})$.

(88) Tweety is above the cloud.

To simplify matters, let us assume that Tweety's eigenspace is also a point $\mathsf{t} \in M$. Under this assumption, sentence (88) requires that t is in the region $up(\mathsf{c})$.

For reasons that will unfold themselves, we state the requirement $\mathsf{t} \in up(\mathsf{c})$ in a somewhat roundabout way, using the geometric notion of *vectors* (J. Zwarts 1997, Zwarts & Winter 2000). We require that t is the end-point of a *vector* v starting at c and ending in the region $up(\mathsf{c})$. This is illustrated in Figure 8a.

Formally, a vector v here is a pair of points x and y in M, conceived of as a directed line segment between x and y. For any vector $v = \langle x, y\rangle \in M^2$, we denote $x =$ *s-point*(v) and $y =$ *e-point*(v). Postulating the region $up(x)$ for a point x, we define the set of *above-vectors* of x as follows.

(89) Let $x \in M$ be a point in a topological space M, and let $up(x) \subseteq M$ be the up region of x (e.g. a cone-shaped region in M with x as its center). The

[46]For two of many works in this vast area, see Herskovits (1986) and Logan (1995).

set of vectors $\mathbf{above}(x)$ is defined by:

$\mathbf{above}(x) = \{v \in M^2 : \textit{s-point}(v) = x \wedge \textit{e-point}(v) \in up(x)\}$

We refer to the vectors in $\mathbf{above}(x)$ as the *above-vectors of the point* x.

In words: the above-vectors of a point x are those vectors that start at x and end at the up region of x.

With this definition we tentatively analyze (88) as follows.

(90) $\exists v \in \mathbf{above}(\mathsf{c}).\textit{e-point}(v) = \mathsf{t}$

In words: t is the end-point of one of the above-vectors of the point c. This analysis is illustrated in Figure 8a.

In this analysis the eigenspace of the noun phrase *the cloud* in (88) is assumed to be a point c. Now we extend definition (89) for *regions* in M by using the set of *boundary points* of regions (Kelley 1955, p.45), which is denoted $boundary(A)$.[47] This is defined as follows.

(91) Let $A \subseteq M$ be a closed non-empty region in a topological space M. The set of vectors $\textsc{above}(A)$ is defined by:

$\textsc{above}(A) = \{v \in M^2 : \textit{s-point}(v) \in boundary(A) \wedge v \in \mathbf{above}(\textit{s-point}(v))\}$

We refer to the vectors in $\textsc{above}(A)$ as the *above-vectors of the region* A.

In words: the above-vectors of a region A are the above-vectors of boundary points of A.

This definition leads us to the non-tentative analysis below of sentence (88).

(92) $\exists v \in \textsc{above}(C).\textit{e-point}(v) = \mathsf{t}$

In words: t is the end-point of one of the above-vectors of the region C. This analysis is illustrated in Figure 8b.

Consider now sentence (93) below, with the distal MP *five meters* modifying the PP in sentence (88).

(93) Tweety is five meters above the cloud.

When measuring distances in a given direction from the cloud region C to a point t, we obviously need to only consider the *shortest* vectors from C to t. The shortest vectors v in a given set of vectors V are naturally defined by looking at their *norm* $|v|$. This is formally defined below.

(94) Let V be a set of vectors. We define $SH(V)$, the set of (per end-point) *shortest vectors* in V, by:

$SH(V) = \{v \in V : \forall v' \in V : \textit{e-point}(v) = \textit{e-point}(v') \rightarrow |v| \leq |v'|\}$

[47] For any subset A of a topological space M, the *interior* of A is the union of all open subsets in M contained in A. The *boundary* of A is the set of points in M interior neither to A nor to $M - A$.

(a) (b)

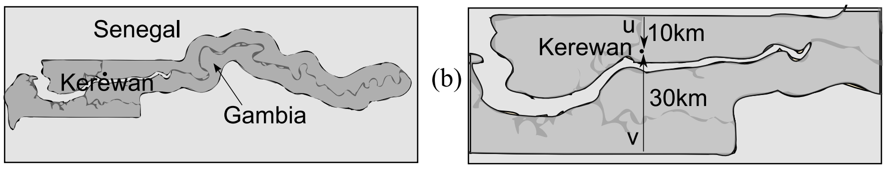

Figure 9: Gambia, Senegal, and the town of Kerewan Map (a) shows Gambia and the town of Kerewan; in (b), the vector v is the shortest vector from the southern border of Gambia to Kerewan, and u is the shortest vector from the northern border to Kerewan.

In words: the set $SH(V)$ of (per end-point) shortest vectors in V contains a vector v in V iff v's norm is minimal among the vectors in V with the same end point as v.

This definition of shortest vectors leads us to the following analysis of sentence (93).

(95) $\exists v \in SH(\textsc{above}(C)).\textit{e-point}(v) = \mathtt{t} \wedge |v| = 5$

In words: t is an end-point of one of the shortest above-vectors v of the region C, and v is five meters long. This analysis is illustrated in Figure 8c.

The analysis above of sentences with projective prepositions and their distal modifiers is a simplification of the proposal by J. Zwarts and Winter (2000). Zwarts & Winter show advantages of their vector space semantics for analyzing PP modification compositionally, and for stating linguistic universals about possible spatial concepts. For presentation sake we do not give here a full-fledged vector analysis. However, we should stress that Zwarts & Winter's framework can be used to implement our assumptions above compositionally, as well as the analyses in section 5.[48] Leaving the formal apparatus as tamed as possible, let us point out the advantages of the vector-based approach for our current purposes. These manifest themselves in cases that involve *non-convex* eigenspaces. Consider the geography of the West African country Gambia. As the map in Figure 9a illustrates, except for its short coastline on the west, Gambian territory is completely surrounded by the territory of another country, Senegal. On the map in Figure 9b we now consider the Gambian town of Kerewan, which is located about 10km from the Senegalese border to its north and about 30km from the Senegalese boarder to its south. Consider now the following sentence.

(96) Kerewan is 10km to the north of the Senegalese border.

This sentence is clearly false. This shows that in general, distal modification of projective PPs, e.g. *5m above* or *10km to the north of*, cannot be paraphrased as a conjunction of the distal relation and the projective relation. For instance, sentence (96) is not equivalent to the following sentence.

[48]Two modifications in our analysis that would be needed to achieve that are: (i) allowing PPs to denote sets of entities (footnote 38), and (ii) allowing MPs to denote sets of vectors rather than sets of real numbers as in section 5.

(97) Kerewan is to the north of the Senegalese border and 10km from the Senegalese border.

Unlike (96), sentence (97) is *true*. First, Kerewan, like all Gambian territory, is to the north of the Senegalese border (as well as to its south). Second, Kerewan is indeed 10km from the Senegalese border. However, sentence (97) is true just because the distance to the border also takes into account the part of the border that lies to the north of Kerewan. Importantly, this part of the border is irrelevant for assessing the truth of sentence (96). Expressions like *5m above* in (93) and *10km to the north of* in (96) must be analyzed as "5m in the 'up' direction from" and "10km in the 'north' direction from", respectively. Thus, sentences (93) and (96) do not involve simple statements about distances between two regions. The projective relations in them are inseparable from the analysis of their distal modifiers.

Our vector-based analysis, following Zwarts and Winter, captures this behavior of modified projective relations. Sentence (96) is analyzed in (98) below, analogously to our analysis of sentence (93) in (95).

(98) $\exists v \in SH(\textsc{north}(SB)).\textit{e-point}(v) = \mathsf{k} \wedge |v| = 10$

In words: k, the location of Kerewan, is an end-point of one of the shortest north-vectors v of the region SB of the Senegalese border, and v is 10km long. This analysis is illustrated in Figure 9b. In this figure, v is the shortest vector ending at Kerewan in the north-vector set for the Senegalese border. This vector is 30km long. Thus, the statement in (98) is false, like sentence (96) is. By contrast, the sentence *Kerewan is 10km to the <u>south</u> of the Senegalese border* is correctly analyzed as true, because the vector u in Figure 9b ends at Kerewan and is in the *south-vector set* for the Senegalese border.

Let us go back now to our initial puzzle, regarding the interpretation of spatial indefinites as in sentence (87) (= "*Tweety is five meters above a cloud*"). Our analysis of this sentence is identical to the analysis of sentence (93) in (95) above. All we have to do is to replace the eigenspace C of one single cloud by the union eigenspace $\bigcup\{\textsc{loc}(x) : \mathbf{cloud}(x)\}$ of the indefinite *a cloud* in (87). This is formally stated in (99) below.

(99) $\exists v \in SH(\textsc{above}(\bigcup\{\textsc{loc}(x) : \mathbf{cloud}(x)\})).\textit{e-point}(v) = \mathsf{t} \wedge |v| = 5$

In words: t is an end-point of one of the shortest above-vectors v of the union region $\bigcup\{\textsc{loc}(x) : \mathbf{cloud}(x)\}$, and v is five meters long.

From a topological point of view, there is a point of similarity between the property eigenspace for the indefinite *a cloud* in (87) and the eigenspace of the Senegalese border in sentence (96): both the set of clouds and Senegalese border are non-convex objects. Thus, our analysis, which is designed to describe distal modification of projective prepositions and non-convex eigenspaces, works equally well in sentence (96) and sentence (87). The advantages of our analysis of sentence (87) can be seen when reconsidering Figure 2a from section 2. In Figure 2a(i), the shortest above-vector of the union region of the clouds is five meters long. As a result, analysis (99) renders sentence (87) true in this situation, as intuitively required. In Figure 2a(ii), the shortest above-vector of the same union region is less than five meters long. Consequently, analysis (99) treats sentence (87) as false in this situation, which also conforms with intuition. As argued in section 2, this illustrates the *non-existential* nature of our PEH-based analysis. Further, adding clouds to Figure 2a(i) that are not below Tweety would not change the truth of the analysis (99) in this situation. This is because Tweety is not in the above-regions of such clouds. Similarly, the leftmost and rightmost clouds in Figure 2a(i) that are below Tweety but are more than five meters from her, do not change the truth of analysis (99): the above-vectors of these clouds may end at Tweety's location, but they are not shortest among the above-vectors of the clouds' *union region*. This further illustrates the *non-universal* nature of our PEH-based analysis, which is also in accordance with intuition.

7 Conclusion: subpart monotonicity and spatial meaning

Part-whole relations and spatial relations have attracted much attention in the philosophical literature and in semantic theory (Cruse 1979, Herskovits 1986, Winston et al. 1987, Iris et al. 1988, Moltmann 1997, Casati & Varzi 1999, Johansson 2004). Without getting into many of the relevant philosophical problems, this paper has concentrated on some linguistic puzzles about space and part-whole relations that seem to us of importance for semantic theories of indefinites, plurals and spatial expressions. We have focused on 'subpart' relations between three sorts of descriptions:

(100)
- Relations between descriptions of geographical units, e.g. between *Dordogne* and *France*.
- Relations between descriptions of entities and collections, e.g. between *Mt. Whitney* and *the mountains* or *the mountain range* (i.e. Sierra Nevada).
- Relations between entities and properties, e.g. between *Calcutta* and *a city*.

Intuitively, the three types of relations support spatial entailments like the following (Iris et al. 1988, p.435).

(101) Our friend is in Dordogne ⇒ Our friend is in France.

(102) The camp is in Mt. Whitney
⇒ The camp is in the mountains (of Sierra Nevada).
⇒ The camp is in the mountain range (of Sierra Nevada).

(103) Max is in Calcutta ⇒ Max is in a city.

Our analysis of spatial expressions started with the third type of subpart relations in (100), between properties and elements of their extension. Among the three types of relations in (100), the classification of this relation as a 'subpart' relation is probably the most suspectable, or surprising. This is because entailments as in (103) are also expected by the traditional existential analysis of indefinites (Montague 1973). However, without dismissing this standard existential analysis, we have also seen ample linguistic evidence for our classification. Treating the relation between indefinites and elements of their property extension on a par with other 'subpart' relations has desired results for the analysis of spatial indefinites.

Considering all the relations in (100) as instances of a general *subpart* relation, our accounts of all of these relations have consistently adopted the following assumption, which Casati and Varzi (p.15) dub "obvious".

(104) If y is subpart of x, then $\textsc{loc}(y) \subseteq \textsc{loc}(x)$.

In our terms, the assumption in (104) reflects a monotonicity of the eigenspace function $\textsc{loc}$ with respect to the subpart order on objects (cf. Casati & Varzi 1999, p.54). From this assumption we conclude the following:

(105) If the elements in a set Y are all subparts of x, then $\bigcup_{y \in Y} \textsc{loc}(y) \subseteq \textsc{loc}(x)$.

In our analysis of indefinites, we strengthened this subset relation into an equality (76), where Y is the extension of the property x. Thus, in the PEH we proposed that the location of a property *equals* the union of its extension's members locations. Similarly, the CEH analyzes the basic location of a collection as the union of its members' location. On top of these basic eigenspaces, some wholes may obtain spatial contiguity by virtue of additional assumptions on top of subpart monotonicity. The CEH allows collective descriptions to occupy any functional hull, especially the convex hull, of their basic eigenspace. We hypothesize that a similar principle is at work with the subpart-induced spatial relations between singular descriptions. For instance, the eigenspace for the term *Sierra Nevada* does not only need to contain mountains in this mountain range. It can also contain valleys, villages, lakes and other entities in the 'functional hull' of the mountains in Sierra Nevada. The mountains may be most prominent, but only their functional hull allows speakers to use the eigenspace of the expression *Sierra Nevada* in their communication. We believe that such factors that determine the eigenspace of 'simple' terms such as *Sierra Nevada* are similar to those that determine the eigenspace of complex collective descriptions like *the mountains of Sierra Nevada*, with subpart monotonicity as a limiting principle. These factors involve a complex array of principles involving lexical meaning, world knowledge and contextual information. Spelling out such principles as rigorous and psychologically plausible hypotheses and incorporating them into linguistic theory are major challenges. We believe that the facts and theoretical proposals that we have presented in this paper is a step in this direction, and will help in using and reassessing the formal semantics of the noun phrase also in some under-explored areas of spatial meaning.

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