The Semantics of Intensionalization*

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Abstract. This paper introduces a procedure that takes a simple version of extensional semantics and generates from it an equivalent possible-world semantics that is suitable for treating intensional phenomena in natural language. This process of *intensionalization* allows to treat intensional phenomena as stemming exclusively from the lexical meaning of words like *believe*, *need* or *fake*. We illustrate the proposed intensionalization technique using an extensional toy fragment. This fragment is used to show that independently motivated extensional mechanisms for scope shifting and verb-object composition, once properly intensionalized, are strictly speaking responsible for certain intensional effects, including *de dicto/de re* ambiguities and coordinations containing intensional transitive verbs. While such extensional-intensional relations have often been assumed in the literature, the present paper offers a formal sense for this claim, facilitating the dissociation between extensional semantics and intensional semantics.

1 Introduction

The simplicity and elegance of extensional higher-order logics make them attractive for treating many phenomena in natural language. The arguments for intensional (and hyper-intensional) semantics are of course compelling, but we would not like these considerations to complicate the analysis of properly-extensional phenomena. Unfortunately this is often the case, and especially in Montague's classical treatment in [1] (PTQ). In order to address this tension between extensional semantics and intensional semantics, this paper studies the relations between elementary extensional semantics and intensional semantics such as

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Montague's IL or Gallin's Ty2 in [2]. We propose a general process of *intensionalization* that maps an extensional framework to such an intensional framework, and illustrate its architectural benefits using a toy fragment. More generally, we argue that also in other frameworks, there are methodological and empirical reasons for taking intensionalization procedures to be a central part of the study of intensional phenomena.

The distinction between parts of a language that exhibit intensional effects and parts that do not can often be reduced to a simple distinction between two kinds of lexical items: those that create an intensional context and those that do not. In this paper, expressions like the verbs *seek*, *need* and *believe* and the adjective *fake*, which create an intensional context are called *intension-sensitive*. Expressions that do not create an intensional context, such as the verb *kiss* or the adjective *red*, are called *intension-insensitive*. We assume that an extensional semantics is sufficiently adequate for expressions that consist solely of intension-insensitive lexical items, while an intensional semantics is only needed for expressions with intension-sensitive lexical items.

In this paper we propose a modular approach to the architecture of intensional systems that is based on this assumption. We start out by introducing a simple grammatical framework with a standard extensional semantics, and then add intension-sensitive words to the lexicon. Since the intension-sensitive lexical items (semantically) select intensional objects, the extensional types and meanings of the intension-insensitive lexical items need to be shifted to intensional types and meanings. Following [3], we refer to this shifting process as *intensionalization*.

The strategy of intensionalization that we suggest can be traced back to the type shifting strategy of [4]. But, as far as we know, the only full-fledged intensionalization procedure in the literature, even if not under this label, was defined by Keenan and Faltz in [5]. A simple intensionalization procedure can be inferred from the introduction of intensional semantics in [6] (Ch. 12). This intensionalization procedure is similar to the "intensionality monad" that Shan defines in [7]. Shan partly follows an early version of [8], who uses a comparable architectural approach for treating natural language quantifiers using the notion of *continuations*.

The novel feature of our intensionalization procedure, as opposed to previous proposals, is that the shifting process that we employ from extensional types and meanings to intensional types and meanings is a general one, and applies uniformly to all syntactic categories and semantic types. Beside types and meanings of expressions, the intensionalization process changes very little in the extensional system. For example, *de dicto/de re* ambiguities and coordinations of intension-sensitive with intension-insensitive transitive verbs are treated as manifestations of purely extensional mechanisms in their intensionalized guise.

One central formal aspect of the intensionalization process is truth-conditional *soundness*. In order to preserve the insights of an extensional semantics, we need to guarantee that its intensionalized version is descriptively equivalent to it. In more exact terms: both the extensional semantics and its image under intensionalization should provably describe the same entailment relations between sentences. This paper sets the background for a soundness theorem, which is stated for the extensional framework we define. The details of the proof appear in [9].

The paper is organized as follows. Section 2 defines a setting for an extensional grammar, and illustrates it using a toy grammar. The intensionalization process is the subject of Section 3, which also states its soundness and demonstrates its application to the toy grammar of Section 2. Section 4 demonstrates how intension-sensitive lexical items are added to the intensionalized grammar, and illustrates the resulting grammatical interactions between such items and intension-insensitive expressions and extensional mechanisms.

2 Formalizing the extensional setting

To formalize an intensionalization procedure we first need to make explicit assumptions about the extensional semantics. We define a simple undirected type-logical grammar that includes the bare semantic details necessary for defining the intensionalization procedure. Syntactic, pragmatic and phonological details are ignored.

2.1 Extensional setting

The *extensional grammars* that we assume are of the form $\langle \Sigma, \mathbf{type}, \{\!\{\cdot\}\!\} \rangle$. Σ is the *lexicon* ('alphabet') – a finite set of *words* ('lexical items', 'terminal symbols'). Every word $\alpha \in \Sigma$ is assigned an *extensional type* $\mathbf{type}(\alpha)$. The extensional types that we assume are the standard functional types over basic types e and t. The set of all extensional types is denoted by $\mathcal{T}_{\text{ex}}$. The role of the third component of a grammar is to determine the possible interpretations for each word, as described below.

Lexical items are directly interpreted by models. Standardly, a *model* $\mathcal{M}$ is a pair $\langle \mathcal{F}_{\mathcal{M}}, \mathcal{I}_{\mathcal{M}} \rangle$, where $\mathcal{F}_{\mathcal{M}}$ is an *extensional frame* and $\mathcal{I}_{\mathcal{M}}$ is an *interpretation function*. The *extensional frame* is a collection of *domains* D_σ for every extensional type σ. Standardly, we assume that D_t is the set $\{0, 1\}$ of *truth values*, D_e is an arbitrary nonempty set E of *entities*, and $D_{(\tau\sigma)}$ is the set $D_\sigma^{D_\tau}$ of all functions from D_τ to D_σ. Thus, an extensional frame is uniquely determined by the set E of entities. For every E we refer to the induced frame as the *E-based (extensional) frame*. The *interpretation function* $\mathcal{I}_{\mathcal{M}}$ maps every word α to a member of $D_{\mathbf{type}(\alpha)} \in \mathcal{F}_{\mathcal{M}}$.

To derive interpreted sentences from interpreted lexical items, let us add a simple notion of a grammar, keeping in mind that, as before, we only introduce the bare semantic notions that are necessary in order to define an intensionalization process in a most general way. A derivation is either a lexical item (rule (1) below) or a compound derivation. Compound derivations are obtained from simpler derivations by one of two rules: *functional application* (rule (2) below) or *conjunction* (rule (3) below).

(1) a. Every lexical item α is a derivation of type $\mathbf{type}(\alpha)$ of the expression α.
b. For every model $\mathcal{M}$: $[\![\alpha]\!]^{\mathcal{M}} = \mathcal{I}_{\mathcal{M}}(\alpha)$.

(2) a. If Δ_1 is a derivation of type $(\sigma\tau)$ of an expression ε_1 and Δ_2 is a derivation of type σ of an expression ε_2, then $[\Delta_1\ \Delta_2]$ (respectively, $[\Delta_2\ \Delta_1]$) is a derivation of the expression $\varepsilon_1\ \varepsilon_2$ (respectively, $\varepsilon_2\ \varepsilon_1$) of type τ.
b. For every model $\mathcal{M}$: $[\![[\Delta_1\ \Delta_2]]\!]^{\mathcal{M}} = [\![[\Delta_2\ \Delta_1]]\!]^{\mathcal{M}} = [\![\Delta_1]\!]^{\mathcal{M}}([\![\Delta_2]\!]^{\mathcal{M}})$.

Regarding the conjunction rule, one of our main concerns is to enable a coordination of intension-insensitive words with intension-sensitive words, like in *Mary sought, found and ate a fish*. The syncategorematic introduction of conjunction that we use here is convenient for the sake of exposition of our intensionalization procedure, as it does not require adding a polymorphic entry to the lexicon or using several entries for an arbitrary number of types. Similar derivation rules can be formulated for other Boolean words such as *or* and *not*. For the purposes of this paper, however, it is enough to restrict attention to the conjunctive *and*. In order for two expressions to be conjoinable, they must be of the same type. Furthermore, this type must be *Boolean* (or *t-ending*). In a type system that contains the basic type t (e.g., our extensional types $\mathcal{T}_{\text{ex}}$), a type σ is called *Boolean* iff either $\sigma = t$ or $\sigma = (\sigma_1\sigma_2)$ for a Boolean type σ_2. For the semantic composition in the conjunctive rule (3) we use the well-known *Generalized Conjunction* operator from [4], denoted here '$\sqcap$'.

(3) a. If Δ_1 is a derivation of an expression ε_1 and Δ_2 is a derivation of an expression ε_2, both of a Boolean type σ, then $[\Delta_1 \text{ and } \Delta_2]$ is a derivation of type σ of the expression ε_1 *and* ε_2.
b. For every intended model $\mathcal{M}$, $[\![[\Delta_1 \text{ and } \Delta_2]]\!]^{\mathcal{M}} = \sqcap_{\sigma(\sigma\sigma)}([\![\Delta_1]\!]^{\mathcal{M}})([\![\Delta_2]\!]^{\mathcal{M}})$, where '$\sqcap_{\sigma(\sigma\sigma)}$' is recursively defined for Boolean types σ as follows:

$$\sqcap_{\sigma(\sigma\sigma)} = \begin{cases} \wedge \text{ (standard propositional conjunction)} & \sigma = t \\ \lambda X_\sigma \lambda Y_\sigma \lambda Z_{\sigma_1}. \sqcap_{\sigma_2(\sigma_2\sigma_2)} (X(Z))(Y(Z)) & \sigma = (\sigma_1\sigma_2) \end{cases}$$

Among all possible models for a grammar $G = \langle \Sigma, \mathbf{type}, \{\!\{\cdot\}\!\}\rangle$ we are interested only in those *intended* models that respect lexical restrictions on the meaning of individual words. This is obtained using the an operator $\{\!\{\cdot\}\!\}$ that assigns to every word $\alpha \in \Sigma$ a functional $\{\!\{\alpha\}\!\}$ that maps a frame $\mathcal{F}$ to the subset $\{\!\{\alpha\}\!\}^{\mathcal{F}}$ of $\bigcup \mathcal{F}$ that consists of all admissible interpretations for α. Formally, a model $\mathcal{M} = \langle \mathcal{F}_{\mathcal{M}}, \mathcal{I}_{\mathcal{M}}\rangle$ is an *intended model* for a grammar $G = \langle \Sigma, \mathbf{type}, \{\!\{\cdot\}\!\}\rangle$ iff $\mathcal{I}_{\mathcal{M}}(\alpha) \in \{\!\{\alpha\}\!\}^{\mathcal{F}_{\mathcal{M}}}$ for every $\alpha \in \Sigma$. The class of all intended models for a grammar G is denoted $\mathfrak{M}_G$.

For simplicity, we henceforth assume that for every lexical item α either (4) or (5) holds.

(4) For every frame $\mathcal{F}$: $\{\!\{\alpha\}\!\}^{\mathcal{F}} = D_{\mathbf{type}(\alpha)} \in \mathcal{F}$.
(5) For every frame $\mathcal{F}$ there is $\varphi \in D_{\mathbf{type}(\alpha)} \in \mathcal{F}$ such that $\{\!\{\alpha\}\!\}^{\mathcal{F}} = \{\varphi\}$.

Whenever (4) (respectively, (5)) holds for a lexical item α we will say that α is a *nonlogical constant* (respectively, *logical constant*).[3]

[3] There are at least two other kinds of possible logical restrictions on the meaning of lexical items. First, there are lexical items α for which $\{\!\{\alpha\}\!\}^{\mathcal{F}}$ is a non-singleton

2.2 Example

As an example consider the simple grammar in Table 1. The determiners *every* and *a* are standardly treated as logical constants. For convenience, the single function that a logical constant is assumed to denote is described by a lambda-term in the 3rd column. On the other hand, nouns (both proper and common) and (in)transitive verbs, as well as other expressions of open lexical categories, are standardly treated as nonlogical constants. *Intersective* adjectives like *bald* or *red* are also treated as nonlogical constants of type (et). For the sake of the example, we assume that all intension-insensitive adjectives are intersective.

Table 1. Extensional lexical entries

word α	**Type**	$\{\!\{\alpha\}\!\}^{\mathcal{F}}$	λ-term
Mary, *John*	e	D_e	
red, *bald*	et	D_{et}	
king, *queen*	et	D_{et}	
smile, *jump*	et	D_{et}	
kiss, *eat*	$e(et)$	$D_{e(et)}$	
every	$(et)((et)t)$	$\{\text{EVERY}\}$	$\text{EVERY} = \lambda A^{et}\lambda B^{et}.\forall x^e[A(x) \to B(x)]$
a	$(et)((et)t)$	$\{\text{SOME}\}$	$\text{SOME} = \lambda A^{et}\lambda B^{et}.\exists x^e[A(x) \wedge B(x)]$

The toy grammar in (1)-(3), together with the lexical entries in Table 1, does not allow to derive all grammatical strings over the given lexicon. For instance, they do not allow to derive a transitive sentence like (6) below.

(6) A queen kissed every king.

Similarly, intersective modification with adjectives (e.g. *bald king*) and coordination of proper names with quantifiers (e.g., *Mary and every queen*) are not treated by the assumptions introduced so far. In order to deal with such examples without complicating too much the introduction of our proposed grammatical architecture, we use some phonologically-silent lexical items. These phonologically-silent lexical items (also called *empty words* or *operators*) are introduced in Table 2 as an *ad hoc* extension of the lexicon from Table 1. Note that all operators in this extension are treated as logical constants.

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proper subset of the domain of α's type. An example for such a lexical item is the TV *follow*. To account for the entailment from (ia) below to (ib), it is reasonable to assume that *follow* is interpreted as an arbitrary *transitive relation* in $D_{e(et)}$.

(i) a. Jack follows Cole and Cole follows Jessica.
b. Jack follows Jessica.

Second, there are lexical items for which $\{\!\{\alpha\}\!\}^{\mathcal{F}}$ is possibly the whole domain of its type, but there are restrictions on the relation between α's interpretation and the interpretation of other lexical items (e.g., *kill* and *die*).

Table 2. Extending the lexicon from Table 1 with empty words as type shifting operators.

word α	Type	$\{\!\{\alpha\}\!\}^{\mathcal{F}}$	λ-term
ϵ_{ONS}	$(e(et))(((et)t)(et))$	$\{$**ONS**$\}$	$\lambda R^{e(et)}\lambda F^{(et)t}\lambda x^e.F(\lambda y^e.R(y)(x))$ mapping a binary predicate between entities to a binary predicate between entities and quantifiers (the quantifier taking narrow scope)
ϵ_{OWS}	$(((et)t)(et))$ $(((et)t)(((et)t)t))$	$\{$**OWS**$\}$	$\lambda R^{(((et)t)(et))}\lambda F^{(et)t}\lambda Q^{(et)t}.$ $F(\lambda y^e.Q(\lambda x^e.R(\lambda A^{et}.A(y))(x)))$ mapping a binary predicate between entities and quantifiers to a binary predicate between quantifiers (the object quantifier taking wide scope)
ϵ_{lift}	$e((et)t)$	$\{$**LIFT**$\}$	$\lambda x^e\lambda A^{et}.A(x)$ lifting an entity to a quantifier
ϵ_{adj}	$(et)((et)(et))$	$\{$**ADJ**$\}$	$\lambda A^{et}\lambda B^{et}\lambda x^e.A(x)\wedge B(x)$ mapping a set to an intersective modifier

The operation of the empty words ϵ_{ONS} and ϵ_{OWS} can be demonstrated with sentence (6) above. This sentence can be derived in two different ways. In both derivations, (7a) and (8a), the operator ϵ_{ONS} is applied to the TV *kissed*. The difference is that derivation (8a) also uses the operator ϵ_{OWS}. In an intended model $\mathcal{M}$, derivation (7a) denotes the *object narrow scope* (ONS) interpretation (7b), and derivation (8a) denotes the *object wide scope* (OWS) interpretation (8b). To facilitate readability, we henceforth let **word** (in boldface) stand for $\mathcal{I}_{\mathcal{M}}(word)$ whenever *word* is a lexical item, $\mathcal{M}$ is an arbitrary model, and there is no other model in the context.

(7) a. [[*a queen*] [[ϵ_{ONS} *kissed*] [*every king*]]]
b. $\exists x^e[\mathbf{queen}(x)\wedge\forall y^e[\mathbf{king}(y)\rightarrow\mathbf{kiss}(y)(x)]]$

(8) a. [[*a queen*] [[ϵ_{OWS} [ϵ_{ONS} *kissed*]] [*every king*]]]
b. $\forall y^e[\mathbf{king}(y)\rightarrow\exists x^e[\mathbf{queen}(x)\wedge\mathbf{kiss}(y)(x)]]$

This use of (extensional) operators on predicates in order to derive ONS and OWS analyses essentially follows the (intensional) operators proposed in [10].

The empty word ϵ_{adj} is used to shift the set denoted by an intension-insensitive adjective to an intersective function of type $(et)(et)$. The latter can modify the set denoted by a common noun in the usual way.

The empty word ϵ_{lift} allows a proper noun like *Mary* to be of the same type as that of a quantified noun phrase like *every queen*. Such a typing is necessary for the treatment of coordinations like in *Mary and every queen smiled*.

3 A sound intensionalization procedure

Having defined a setting for an extensional semantics, we can now introduce our proposed intensionalization procedure for this setting and discuss its implications.

To enable the introduction of intension-sensitive lexical items we should let their arguments denote intensional objects. For example, it is well known that for a reasonable analysis of a sentence like *Mary sought a unicorn* the noun phrase *a unicorn* should not have an extensional meaning of the kind that was introduced in Section 2. In this section we introduce our proposed semantics of *intensionalization*, by which the extensional types and meanings of lexical items in an extensional grammar can be modified, so that the resulting grammar is equivalent to the original extensional grammar, and at the same time can be extended to a properly intensional grammar by only adding intension-sensitive items to its lexicon.

3.1 Intensionalization

The *intensional types* that we assume are the functional types over e, s and t. The set of all intensional types is denoted by $\mathcal{T}_{\text{in}}$. *Intensional frames* are defined similarly to extensional frames. But while an extensional frame is determined by a nonempty set E of entities as the domain of type e, an intensional frame is determined both by such a set E *and* a nonempty set W of *possible worlds* as the domain of type s. For a fixed choice of such E and W, the induced intensional frame is called the *E, W-based frame*. An *intensional grammar* is defined like an extensional grammar as a triple $\langle \Sigma, \mathbf{type}, \{\!\{\cdot\}\!\} \rangle$, where for every $\alpha \in \Sigma$: $\mathbf{type}(\alpha) \in \mathcal{T}_{\text{in}}$, and $\{\!\{\alpha\}\!\}$ maps every intensional frame $\mathcal{F}$ to a subset $\{\!\{\alpha\}\!\}^{\mathcal{F}}$ of $D_{\mathbf{type}(\alpha)}$. The derivation rules remain the same.

The intensionalization that we propose follows Van Benthem's typing recipe in [3]. Formally, the intensionalization procedure is defined as a mapping $\mathcal{I}$ from extensional grammars to intensional grammars. Given an extensional grammar $G = \langle \Sigma, \mathbf{type}_G, \{\!\{\cdot\}\!\}_G \rangle$, its intensionalization $\mathcal{I}(G) = \langle \Sigma, \mathbf{type}_{\mathcal{I}(G)}, \{\!\{\cdot\}\!\}_{\mathcal{I}(G)} \rangle$ is obtained by a systematic modification of the typing function $\mathbf{type}_G$ and the meaning functional $\{\!\{\cdot\}\!\}_G$. Following Van Benthem, we modify an extensional type σ by substituting (st) for every occurrence of t. The resulting intensional type is denoted $\ulcorner \sigma \urcorner$. Thus, for every $\alpha \in \Sigma$ we define $\mathbf{type}_{\mathcal{I}(G)}(\alpha) = \ulcorner \mathbf{type}_G(\alpha) \urcorner$.

Using this global type-change recipe, we should now intensionalize the *meanings* of lexical items so that $\mathcal{I}(G)$ is equivalent to G. To facilitate the intensionalization of meanings, we made the simplifying assumption that each lexical item is either a logical or a nonlogical constant. With this assumption, the intensionalization of meanings is defined so that (non)logical constants in G remain (non)logical constants also in $\mathcal{I}(G)$. The question of how to treat other kinds of lexical items may be more complicated, and is left for future research.[4]

[4] Makoto Kanazawa (p.c.), based on a discussion with Philippe de Groote and Reinhard Muskens, suggests a modification of our intensionalization procedure that does

The case of nonlogical constant is simple. If α is a nonlogical constant in G, we define $\{\!\{\alpha\}\!\}_{\mathcal{I}(G)}$ to be the functional that maps every intensional frame $\mathcal{F}$ to the domain $D_{\mathbf{type}_{\mathcal{I}(G)}(\alpha)}$ in $\mathcal{F}$. For a logical constant α in G to remain logical constant also in $\mathcal{I}(G)$, we need to define $\{\!\{\alpha\}\!\}_{\mathcal{I}(G)}$ in such a way that for every E, W-based intensional frame $\mathcal{F}$ there is an object $g \in D_{\mathbf{type}_{\mathcal{I}(G)}(\alpha)}$ such that $\{\!\{\alpha\}\!\}^{\mathcal{F}}_{\mathcal{I}(G)} = \{g\}$. It is expected that this object g is systematically derived from the unique interpretation of α in the corresponding E-based extensional frame $\mathcal{F}'$ using some mapping $L(\cdot)$ from $D_{\mathbf{type}_G(\alpha)} \in \mathcal{F}'$ to $D_{\mathbf{type}_{\mathcal{I}(G)}(\alpha)} \in \mathcal{F}$.

To motivate our proposed definition of this mapping, consider for example the determiner *every* as appearing in the extensional lexicon from Table 1. For this determiner, we need to map the object $\text{EVERY} \overset{def}{=} \lambda A^{et} \lambda B^{et}.\forall x^e[A(x) \to B(x)]$ in $D_{(et)((et)t)}$ to a unique member of the intensional domain $D_{\ulcorner (et)((et)t) \urcorner}$. The intensional denotation that we are after is similar to the denotation of *every* in PTQ.[5] This is the function that when applying to two properties P and Q, returns the proposition that is *true* in a world w just in case the predicate *extensions* in w of P and Q satisfy the containment requirement of EVERY. In symbols, we would like to end up with $L(\text{EVERY}) = \lambda w^s \lambda P^{e(st)} \lambda Q^{e(st)}.\text{EVERY}(P^w)(Q^w)$, where P^w is $\lambda x^e.P(w)(x)$ – the extension of the property P in a given index w – and similarly for Q^w.

To generalize this relatively simple example to any logical constant of any type, we first define an *extensionalization mapping* from intensional domains to extensional domains (Definition 3). We then use this notion in Definition 4 of the intensionalization mapping. To facilitate the definition of these mappings, we follow a tentative proposal in [3], and restrict our attention to the *quasi-relational* types of [11]. Definition 2 below of the quasi-relational types refers to the set $\mathcal{T}_e$ of *e-based types*.

Definition 1 (e-based types). *The set $\mathcal{T}_e$ of* e-based types *is the smallest set that satisfies:*

1. *$e \in \mathcal{T}_e$, and*
2. *If $\sigma_1, \sigma_2 \in \mathcal{T}_e$ then $(\sigma_1\sigma_2) \in \mathcal{T}_e$.*

Thus, the e-based types are simply the standard functional types over a single primitive type e of entities. A *quasi-relational* type is a Boolean type in which every argument is either quasi-relational or e-based. Formally:

Definition 2 (Quasi-relational types). *The set $\mathcal{T}_{\text{qr}} \subset \mathcal{T}_{\text{ex}}$ of* quasi-relational types *is the smallest set that satisfies:*

1. *$t \in \mathcal{T}_{\text{qr}}$, and*

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not need to stipulate different treatments for logical constants and non-logical constants. Furthermore, Kanazawa et al's proposal may be preferable to ours in some other important respects. We are currently studying the implications of their proposal.

[5] Determiners are introduced syncategorematically in [1], whereas here they are part of the lexicon. But this hardly matters for the semantic analysis.

2. If both $\sigma_1 \in \mathcal{T}_e \cup \mathcal{T}_{\mathrm{qr}}$ and $\sigma_2 \in \mathcal{T}_{\mathrm{qr}}$, then $(\sigma_1\sigma_2) \in \mathcal{T}_{\mathrm{qr}}$.

Note that the domain D_σ of a quasi-relational type $\sigma = (\sigma_1 \cdots (\sigma_n t) \cdots)$ is isomorphic to the cartesian product $D_{\sigma_1} \times \cdots \times D_{\sigma_n}$.

The definition of a w-extension of an object is as follows.

Definition 3 (w-extension). *Let $\mathcal{F}$ be an E, W-based intensional frame and $\mathcal{F}'$ the corresponding E-based extensional frame. Let $w \in W$, and $g \in D_{\ulcorner\sigma\urcorner} \in \mathcal{F}$ for some $\sigma \in \mathcal{T}_{\mathrm{qr}} \cup \mathcal{T}_e$. The* w-extension *of g is the object $g^w \in D_\sigma \in \mathcal{F}'$ that satisfies:*

1. *If $\sigma \in \mathcal{T}_e$ then $g^w = g$;*
2. *if $\sigma = t$ then $g^w = g(w)$;*
3. *if $\sigma = (\sigma_1 \cdots (\sigma_n t) \cdots)$, $n \geq 1$, then*

$$g^w = \lambda x_1^{\sigma_1} \cdots \lambda x_n^{\sigma_n}.\exists z_1 \cdots \exists z_n. \bigwedge_{i=1}^{n} ((z_i)^w = x_i) \wedge g(z_1) \cdots (z_n)(w)$$

In words, a tuple $\langle x_1, \ldots, x_n \rangle$ is in the w-extension of an intensional relation g, iff there is a tuple $\langle z_1, \ldots, z_n, w \rangle$ in g such that the w-extensions of the z_is are the x_is, respectively.

The intensionalization mapping $L(\cdot)$ is now defined as follows.

Definition 4 (Intensionalization mapping). *Let $\mathcal{F}$ be an E, W-based intensional frame and $\mathcal{F}'$ the corresponding E-based extensional frame. Let $f \in D_\sigma \in \mathcal{F}'$ for some $\sigma \in \mathcal{T}_{\mathrm{qr}} \cup \mathcal{T}_e$. The* intensionalization *of f is the object $L(f) \in D_{\ulcorner\sigma\urcorner} \in \mathcal{F}$ that satisfies:*

1. *if $\sigma \in \mathcal{T}_e$ then $L(f) = f$;*
2. *if $\sigma = (\sigma_1 \cdots (\sigma_n t) \cdots)$, $n \geq 0$, then*

$$L(f) = \lambda x_1^{\ulcorner\sigma_1\urcorner} \cdots \lambda x_n^{\ulcorner\sigma_n\urcorner} \lambda w^s.f((x_1)^w) \cdots ((x_n)^w)$$

In words, a tuple $\langle x_1, \ldots, x_n, w \rangle$ is in the intensionalization of a relation f, iff the w-extensions of $x_1, \ldots, x_n$ are in f.

We are now ready to define the intensionalization of the meanings of logical constants, which completes the definition of the intensionalization process. Let α be a logical constant in an extensional grammar G. For every intensional E, W-based frame $\mathcal{F}$ we define $\{\!\{\alpha\}\!\}^{\mathcal{F}}_{\mathcal{I}(G)} = \{L(f)\}$, where f is the single element in $\{\!\{\alpha\}\!\}^{\mathcal{F}'}_{G}$ for the extensional E-based frame $\mathcal{F}'$.

We claim that the intensionalization process that we have just defined is *sound*, in the sense that it preserves entailment between derivations of sentences. Entailment over an extensional grammar G is defined in the usual way: Δ_1 *extensionally entails* Δ_2 iff for every intended model $\mathcal{M} \in \mathfrak{M}_G$: $[\![\Delta_1]\!]^{\mathcal{M}} \leq [\![\Delta_2]\!]^{\mathcal{M}}$. Over an intensional grammar G, entailment is defined as a relation between derivations of type st: Δ_1 *intensionally entails* Δ_2 iff for every intended model $\mathcal{M} \in \mathfrak{M}_G$ and for every $w \in D_s$: $[\![\Delta_1]\!]^{\mathcal{M}}(w) \leq [\![\Delta_2]\!]^{\mathcal{M}}(w)$.

The soundness of the intensionalization process is formally stated in Theorem 1 below. The proof appears in [9]. For the proof we assume that whenever a nonlogical constant has a quasi-relational type, then all its arguments are of an e-based type. This restriction reflects our assumption that intension-insensitive relational nonlogical constants are basically relations between entities, or functions defined in terms of which.

Theorem 1 (Soundness of the intensionalization procedure). *Let G be an extensional grammar in which (i) every lexical item is either nonlogical or logical constant; (ii) every lexical item is of a quasi-relational or e-based type; and (iii) if a nonlogical constant has a quasi-relational type, then all its arguments are of an e-based type. Let Δ_1 and Δ_2 be two derivations of type t over G. Then Δ_1 extensionally entails Δ_2 over G iff Δ_1 intensionally entails Δ_2 over $\mathcal{I}(G)$.*

3.2 Example

To see the benefits of the proposed intensionalization procedure, let us get back to the lexical entries from Tables 1 and 2. The intensionalizations of these entries are shown in Table 3. For a logical constant α in this table, we write its constant interpretation as $L(f)$, where f is its constant extensional interpretation. A routine but somewhat tedious calculation shows that the relevant functions are as follows:

$$\begin{aligned}
L(\text{EVERY}) &= \lambda\mathcal{A}\lambda\mathcal{B}\lambda w.\forall x[\mathcal{A}(x)(w) \rightarrow \mathcal{B}(x)(w)] \\
L(\text{SOME}) &= \lambda\mathcal{A}\lambda\mathcal{B}\lambda w.\exists x[\mathcal{A}(x)(w) \wedge \mathcal{B}(x)(w)] \\
L(\mathbf{ONS}) &= \lambda R\lambda\mathcal{F}\lambda x\lambda w.\mathcal{F}^w(\lambda y.R(y)(x)(w)) \\
L(\mathbf{OWS}) &= \lambda\mathcal{R}\lambda\mathcal{F}\lambda\mathcal{Q}\lambda w.\mathcal{F}^w(\lambda y.\mathcal{Q}^w(\lambda x.\mathcal{R}^w(\lambda A.A(y))(x))) \\
L(\mathbf{LIFT}) &= \lambda x\lambda\mathcal{A}.\mathcal{A}(x) \\
L(\mathbf{ADJ}) &= \lambda\mathcal{A}\lambda\mathcal{B}\lambda x\lambda w.\mathcal{A}(x)(w) \wedge \mathcal{B}(x)(w)
\end{aligned}$$

In the lambda-terms above, x and y are of type e, w is of type s, A is of type et, $\mathcal{A}$ and $\mathcal{B}$ are of type $e(st)$, $\mathcal{Q}$ and $\mathcal{F}$ are of type $(e(st))(st)$, R is of type $e(e(st))$, and $\mathcal{R}$ is of type $(((e(st))(st))(e(st)))$.

The soundness of the intensionalization process is demonstrated with two simple examples. The sentences *every king smiled* and *every bald king smiled* have the derivations in (9a) and (10a), respectively. The denotations of these derivations are shown in (9b) and (10b), for the extensional grammar, and in (9c) and (10c), for the intensionalized grammar. These denotations support entailment from derivation (9a) to derivation (10a) both in the extensional grammar and in its intensionalization.

(9) a. [[*every king*] *smiled*]
b. $\forall x^e[\mathbf{king}(x) \rightarrow \mathbf{smile}(x)]$

Table 3. Intensionalization of the lexical entries from Tables 1 and 2.

word α	**Type**	$\{\!\{\alpha\}\!\}^{\mathcal{F}}$
Mary, *John*,...	e	D_e
red, *sick*,...	$e(st)$	$D_{e(st)}$
king, *queen*,...	$e(st)$	$D_{e(st)}$
smile,...	$e(st)$	$D_{e(st)}$
kiss,...	$e(e(st))$	$D_{e(e(st))}$
every	$(e(st))((e(st))(st))$	$\{L(\text{EVERY})\}$
a	$(e(st))((e(st))(st))$	$\{L(\text{SOME})\}$
ϵ_{ONS}	$(e(e(st)))(((e(st))(st))(e(st)))$	$\{L(\mathbf{ONS})\}$
ϵ_{OWS}	$(((e(st))(st))(e(st)))(((e(st))(st))(((e(st))(st))(st)))$	$\{L(\mathbf{OWS})\}$
ϵ_{lift}	$e((e(st))(st))$	$\{L(\mathbf{LIFT})\}$
ϵ_{adj}	$(e(st))((e(st))(e(st)))$	$\{L(\mathbf{ADJ})\}$

c. $\lambda w^s.\forall x^e[\mathbf{king}(x)(w) \to \mathbf{smile}(x)(w)]$

(10) a. [[*every* [[ϵ_{adj} *bald*] *king*]] *smiled*]

b. $\forall x^e[(\mathbf{bald}(x) \wedge \mathbf{king}(x)) \to \mathbf{smile}(x)]$

c. $\lambda w^s.\forall x^e[(\mathbf{bald}(x)(w) \wedge \mathbf{king}(x)(w)) \to \mathbf{smile}(x)(w)]$

The second example involves the derivations (7a) and (8a) of the sentence *a queen kissed every king*, repeated here as (11a) and (12a) respectively. The respective extensional interpretations (7b) and (8b) of these derivations exhibit an extensional entailment from the ONS derivation to the OWS derivation. In the intensionalized grammar, derivation (11a) has the ONS interpretation in (11b), while (12a) has the OWS interpretation in (12b). From the soundness theorem it follows that these intensionalized interpretations preserve the extensional entailment, as can be independently verified.

(11) a. [[*a queen*] [[ϵ_{ONS} *kissed*] [*every king*]]]

b. $\lambda w^s.\exists x^e[\mathbf{queen}(x)(w) \wedge \forall y^e[\mathbf{king}(y)(w) \to \mathbf{kiss}(y)(x)(w)]]$

(12) a. [[*a queen*] [[ϵ_{OWS} [ϵ_{ONS} *kissed*]] [*every king*]]]

b. $\lambda w^s.\forall y^e[\mathbf{king}(y)(w) \to \exists x^e[\mathbf{queen}(x)(w) \wedge \mathbf{kiss}(y)(x)(w)]]$

4 Extending the intensionalized system

Our main reason to develop a sound intensionalization procedure is to allow a simple introduction of intension-sensitive entries into the lexicon, without any further modification in the intensionalized grammar. Van Benthem's typing strategy in [3] that we have followed enables a simple and natural introduction of intension-sensitive words like *seek* and *need* without modifying the grammar, while allowing these intension-sensitive TVs (ITVs) to be of the same type as (intensionalized) intension-insensitive TVs (ETVs) like *kiss*. In this section we demonstrate this by integrating ITVs into the lexicon of Table 3. As we shall see, *de dicto*/*de re* ambiguities and coordinations of ITVs with ETVs are simply treated in the extended grammar.

One simple way to add ITVs to the lexicon from Table 3 is to let them denote nonlogical constants of type $((e(st))(st))(e(st))$. By this we treat ITVs as in PTQ, where the object of such verbs is assumed to denote an *intensional quantifier*. It should be emphasized, however, that this is not an assumption of our intensionalization procedure but a simple way to accommodate ITVs into the toy lexicon that we are using for exemplification. The treatment of *de dicto/de re* ambiguities under this technique is demonstrated using the two derivation in (13) below of the sentence *Mary sought a king*.

(13) a. [*Mary* [*sought* [*a king*]]]
b. [[ϵ_{lift} *Mary*] [[ϵ_{OWS} *sought*] [*a king*]]]

(14) a. $\mathbf{seek}(\lambda \mathcal{B}^{e(st)}\lambda w^s.\exists y^e[\mathbf{king}(y)(w) \wedge \mathcal{B}(y)(w)])(\mathbf{mary})$
b. $\lambda w^s \exists y^e[\mathbf{king}(y)(w) \wedge \exists Q^{(e(st))(st)}[Q^w = (\lambda A^{et}.A(y)) \wedge \mathbf{seek}(Q)(\mathbf{mary})]]$

The denotation of (13a) is the *de dicto* interpretation in (14a) above. The denotation of (13b), on the other hand, is the *de re* interpretation in (14b). Note that the latter is created by the same mechanism that creates object wide scope interpretations in the extensional grammar (cf. (8)). This is similar to PTQ, where the *quantifying in* mechanism is responsible both for the creation of scope ambiguities and for the creation of *de dicto/de re* ambiguities. However, in distinction with the proposals by Montague, [10] and others, intensionalization spares us the need to define an intricate intensional version of the scope shifting mechanism.

The typing strategy that we follow also enables a simple treatment of coordinations between ITVs and ETVs, like in the sentence *Mary sought and kissed a king*. Each of the two derivations in (15) is equivalent to another reading of the intensional conjunct in the (ambiguous) paraphrase *Mary sought a king and kissed a king*. Derivation (15a) represents the reading in which Mary sought a king *de dicto*, while derivation (15b) represents the reading in which Mary sought a king *de re*.

(15) a. [*Mary* [[*sought* and [ϵ_{ONS} *kissed*]] [*a king*]]]
b. [*Mary* [[[ϵ_{OWS} *sought*] and [ϵ_{OWS} [ϵ_{ONS} *kissed*]]] [*a king*]]]

As mentioned above, our intensionalization process is not restricted to the Montagovian treatment of ITVs. An example of an alternative treatment is that of Zimmermann in [12], where the (in)definite object of an ITV is assumed to denote a property. To support Zimmermann's treatment, we assume that extensional indefinite NPs are of type et, and that the determiner *a(n)* is a logical constant of type $(et)(et)$, whose constant denotation is the identity function of this type. Thus, an indefinite like *a king* denotes the set denoted by the noun *king*.

To allow the composition of ETVs with predicative indefinites, it is customary to assume a process of *semantic incorporation* ([13],[14]). In this process, an ETV can compose with predicative indefinites by way of existential quantification. Formally, the extensional incorporation operator on ETVs is defined as follows.

(16) $\mathbf{INC} \stackrel{def}{=} \lambda R^{e(et)}\lambda P^{et}.\lambda y^e.\exists x^e[R(x)(y) \wedge P(x)]$

Thus, we assume that there is some phonologically-silent lexical item ϵ_{INC}, whose constant denotation is the function **INC**. The relevant addition to the extensional lexical entries from Tables 1 and 2 are given in (17) below.

(17)

word α	**Type**	$\{\!\{\alpha\}\!\}^{\mathcal{F}}$	λ-term
a(n)	$(et)(et)$	$\{\mathrm{A}\}$	$\lambda A^{et}.A$
ϵ_{INC}	$(e(et))((et)(et))$	$\{\mathbf{INC}\}$	$\lambda R^{e(et)}\lambda A^{et}\lambda x^{e}.\exists y^{e}[A(y) \wedge R(y)(x)]$

Since both the determiner *a(n)* and the empty word ϵ_{INC} are logical constants, they are both intensionalized using the operator $L(\cdot)$. The constant interpretations of these two items are as follows:

$$L(\mathrm{A}) = \lambda \mathcal{A}^{e(st)}.\mathcal{A}$$
$$L(\mathbf{INC}) = \lambda \mathcal{R}^{e(e(st))}\lambda \mathcal{A}^{e(st)}\lambda x^{e}\lambda w^{s}.\exists y^{e}[\mathcal{A}(y)(w) \wedge \mathcal{R}(y)(x)(w)]$$

With these intensionalized values, an ITV like *seek* can be added to the lexicon as a nonlogical constant of type $(e(st))(e(st))$, whose object argument is a property. Meaning derivation for coordinations like *sought and kissed a king* are now easily obtained using the incorporation operator. For example, derivation (18a) of the sentence *Mary sought and kissed a king* is interpreted as (18b).

(18) a. [*Mary* [[*sought* and [ϵ_{INC} *kissed*]] [*a king*]]]
b. $\lambda w^{s}.\exists y^{e}[\mathbf{king}(y)(w) \wedge \mathbf{kiss}(y)(\mathbf{mary})(w)] \wedge \mathbf{seek}(\mathbf{king})(\mathbf{mary})(w)$

Arguably, this account is as natural as the derivation in (15a), based on the Montagovian treatment of ITVs.

Like derivation (15a), derivation (18a) amounts to the reading of the sentence *Mary sought and kissed a king* in which Mary sought a king *de dicto*. A *de dicto* reading is also easily derived for simple transitive sentences without coordinations. For example, derivation (19a) of the sentence *Mary sought a king* is interpreted as (19b).

(19) a. [*Mary* [*sought* [*a king*]]]
b. $\mathbf{seek}(\mathbf{king})(\mathbf{mary})$

However, as Zimmermann notes, *de re* readings with ITVs, and more generally object-wide-scope readings, require a scope shifting mechanism for predicative indefinites. One such mechanism is the incorporation operator, which allows a property (e.g., the denotation of *a king* in (18a)) to take a wide scope over a relation between two entities (e.g. the denotation of *kissed* in (18a)). The same strategy can be used to derive a *de re* reading for a sentence like *Mary sought a king*, but for this the denotation of the ITV *seek* (type $(e(st))(e(st))$ must be shifted to a suitable relation between two entities (type $e(e(st))$). This shifting can be achieved by the intensionalization of the operator **AL** in (20) below. In the definition of the extensional **AL**, **indent** is the operator $\lambda x^{e}\lambda y^{e}.y = x$ from e.g., [15].

(20) $\mathbf{AL} \stackrel{def}{=} \lambda R^{(et)(et)}\lambda x^{e}.R(\mathbf{indent}(x))$

Suppose that in the extensional system there is a zero morphology word ϵ_{al} that denotes **AL** in every model. With this operator, derivation (21a) of the sentence *Mary sought a king* is interpreted as (21b).

(21) a. $[Mary\ [[\epsilon_{\text{INC}}\ [\epsilon_{\text{al}}\ sought]]\ [a\ king]]]$
b. $(L(\mathbf{INC}))((L(\mathbf{AL}))(\mathbf{seek}))(\mathbf{king})(\mathbf{mary}) =$
$\lambda w^s.\exists x^e[\mathbf{king}(x)(w) \wedge \exists P^{e(st)}[P^w = (\lambda y^e.y = x) \wedge \mathbf{seek}(P)(\mathbf{mary})]]$

In words, according to this interpretation the sentence is true in w just in case there is a king in w such that Mary sought a property that uniquely defines this king in w. It should be noted that the derivation of **AL** from **indent** is a result of hypothetical reasoning that is embodied in undirected type logical grammars as proposed in [16] and [17].

5 Conclusion

So far, the study of intensionalization has not been a central part of the massive semantic literature on intensionality. In this paper we argued that such a process is necessary if we want to understand better the separation between extensional semantics and intensional semantics. We propose that intensionality phenomena are lexically driven, and that it is mostly this fact that allowed Montague to use essentially extensional mechanisms for treating long-standing puzzles like *de dicto/de re* ambiguities. The study of intensionalization provides a missing link in this story: it explains what is "extensional" in those mechanisms. By doing that, it articulates the lexically-driven nature of intensionality phenomena in natural language.

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