Class 3

Plurals and Distributivity

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ESSLLI 2019, Riga

Challenges

I Plural individuals in the entity domain

II Distributive vs. collective predication

III Structured individuals

IV Non-atomic distribution

Challenge I - the entity domain

(1) The girl is/*are singing.

(2) The girls are/*is singing.

(3) Mary is/*are singing.

(4) Mary and Sue are/*is singing.

Intuitive idea:
Singular NPs (*the girl, Mary*) denote arbitrary entities: by default **atomic.**
Plural NPs (*the girls, Mary and Sue*) denote **collections** of such entities.

Problem:
The standard treatment of NPs using entities/quantifiers takes all NP denotations to range over arbitrary entities.

"arbitrary entities" = besides (non-)identity, no relation is given between entities
$\longrightarrow$ no entity represents a collection of other entities

Changing the domain of entities

Let E be a non-empty arbitrary set of entities. D_e is defined by:

$$\begin{aligned} D_{SG} &= \{\{x\} : x \in E\} \\ D_{PL} &= \{A \subseteq E : |A| \geq 2\} \\ D_e &= D_{SG} \cup D_{PL} = \{A \subseteq E : A \neq \emptyset\} \end{aligned}$$

D_{SG} and D_{PL} are the sub-domains of atomic/plural entities.

Lattice-theoretical notation:

i. Instead of '$\{x\}$' for atomic elements of D_{SG}, write 'x'.
ii. Instead of '$A \cup B$' for the union of sets $A, B \in D_e$, write '$A + B$'.
iii. Instead of '$\bigcup \mathcal{A}$' for the union of sets in $\mathcal{A} \subseteq D$, write '$\oplus \mathcal{A}$'.
iv. Instead of '$A \subseteq B$' for sets $A, B \in D_e$, write '$A \leq B$'.

Rationale: No empty set in D_e; we only use <u>unions</u> of entities.

$\rightarrow$ no intersection and complementation $\rightarrow$ D_e is a **join semi-lattice**.

Collective predication

Collective predicates:

meet, lift the piano together, be a nice team, like each other

(1) Sue and Mary met – $\mathbf{meet}(\mathbf{sue}+\mathbf{mary})$

$\mathbf{sue}, \mathbf{mary} \in D_{SG}$ $\mathbf{sue}+\mathbf{mary} \in D_{PL}$

$\mathbf{meet} \in D_{et}$

(2) The girls met – $\mathbf{meet}(G)$

$\mathbf{girl} \in D_t^{D_{SG}}$: characterizes a set with at least two girls

$[\![\textit{the girls}]\!] = G = \oplus\{x \in D_{SG} : \mathbf{girl}(x)\}$

$\mathbf{meet} \in D_{et}$

Note: the group/#girl met – $\mathbf{meet}(g)$, where $g \in D_{SG}$

Plural nouns and definites

Singular nouns	–	*girl, group*	–	denote in $D_t^{D_{SG}}$
Singular def. NPs	–	*the girl/group*	–	denote in D_{SG}
Plural nouns	–	*girls, groups*	–	denote in $D_t^{D_{PL}}$
Plural def. NPs	–	*the girls/groups*	–	denote in D_{PL}

Note: "f denotes in $D_t^{D_{SG}}$" $=$ $f(x)$ is undefined/trivially false for $x \notin D_{SG}$

We choose the former ("undefined"), hence assume $f \in D_t^{D_{SG}}$.

Example:

- **girl** characterizes the set $\{t, m, s\}$
- **girls** characterizes the set $\{t{+}m{+}s,\ t{+}m,\ t{+}s,\ m{+}s\}$
- **the**(**girl**) is undefined
- **the**(**girls**) $= G = t{+}m{+}s$

Plural nouns – formally

For every singular noun N_{sg} denoting a predicate $P_{et} \in {D_t}^{D_{SG}}$, the plural form N_{pl} denotes the predicate $*P \in {D_t}^{D_{PL}}$, defined by:

$$*P \;=\; \lambda y_e.\; y \in D_{PL} \wedge y \leq \oplus\{x \in D_e : P(x)\}$$

In words: N_{pl} denotes the predicate that holds of entities y made of at least two elements in the set characterized by N_{sg}'s denotation.

Examples:

girl characterizes $\{t, m, s\}$	$\rightarrow$	$*$**girl** characterizes $\{t{+}m{+}s,\ t{+}m,\ t{+}s,\ m{+}s\}$
girl characterizes $\{t, m\}$	$\rightarrow$	$*$**girl** characterizes $\{t, m\}$
girl characterizes $\{t\}$	$\rightarrow$	$*$**girl** characterizes the empty set
girl characterizes the empty set	$\rightarrow$	$*$**girl** characterizes the empty set

Note: the commonly assumed star operator (cf. Champollion's *Distributivity in formal semantics*) admits atomic members $*P$ – replace "$y \in D_{PL}$" by "$y \in D_e$" in definition.

pro our approach:	straightforward (next slide)
cons:	*no girls arrived*, *either the girls or Dan are thieves*

Definites – formally

For every singular/plural noun *N*, the definite noun phrase *the N* denotes the *unique maximal element of* $[[\mathrm{N}]]$, if it exists. Otherwise, the denotation of *the N* is undefined.

$$\mathbf{the} \ = \ \lambda P_{et}. \begin{cases} x & P(x) \text{ and for every } y\text{: } P(y) \rightarrow y \leq x \\ \text{undefined} & \text{no such } x \text{ exists} \end{cases}$$

Examples:

$*$**girl** characterizes $\{t{+}m{+}s,\ t{+}m,\ t{+}s,\ m{+}s\}$ → **the**($*$**girl**) = $t{+}m{+}s$

$*$**girl** characterizes $\{t{+}m\}$ → **the**($*$**girl**) = $t{+}m$

$*$**girl** characterizes the empty set → **the**($*$**girl**) is undefined

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girl characterizes $\{t,\ m,\ s\}/\{t,\ m\}$ → **the**(**girl**) is undefined

girl characterizes $\{t\}$ → **the**(**girl**) = t

girl characterizes the empty set → **the**(**girl**) is undefined

Challenge II: Distributive predication

Distributive predicates: *sleep, wear a blue dress, have a baby, be vegetarian, be champions*

Problem – assuming that *sleep*, like *meet*, denotes in D_{et}, we get:

(1)	Sue and Mary slept	–	$\textbf{sleep}(\textbf{sue}+\textbf{mary})$
(2)	The girls slept	–	$\textbf{sleep}(G)$

Are these meanings adequate?

Approach taken here:

Distributive inferences from lexical predicates are a subtle matter – soft inferences from lexical meanings.

- Meanings in (1) and (2) are OK.
- Quantificational distributive meanings on top of (1) and (2) – obtained using operator (next...)
- Only distributive **nouns** obligatorily identify atoms (and allow the star operator); verbs and adjectives do not. This group is (#a) vegetarian

Lexical distributivity – alternative approach

- All distributive predicates – nouns, verbs and adjectives – basically range over entities in D_{SG}.
- Their actual lexical denotations are uniformly obtained by the star operator.

Example:

$\textbf{sleep} \in D_t^{D_{SG}}$

suppose that **sleep** characterizes $\{t, m, s, j\}$

$\textbf{the}(\textbf{girls}) = G = t{+}m{+}s$

$*\textbf{sleep}(G) = *\textbf{sleep}(t{+}m{+}s) = \textbf{sleep}(t) \wedge \textbf{sleep}(m) \wedge \textbf{sleep}(s)$

Challenge: the townspeople are asleep

Phrasal distributivity

Distributivity is not only a matter of lexical predicates like *sleep*.

(1) The girls sang or danced.

– True (at least) if *each* girl either sang or danced.

(2) The girls are wearing a blue dress.

– True (at least) if *each* girl is wearing a *different* blue dress.

(3) The girls won.

– True if each girl won (but also if one team of the girls won, or even if several teams of girls won).

– Indecisive evidence, under the approach taking here.

The distributivity operator

For every predicate P in D_{et}, the distributed predicate $\mathrm{D}(P)$ holds of every entity x s.t. P holds of every atomic member of x.

$\mathrm{D}(P_{et}) \; = \; \lambda x_e.\, \forall y \in D_{SG}.\; y \leq x \rightarrow P(y)$

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(1) The girls sang or danced.

a. $\textsc{or}^{et}(\textbf{sing}, \textbf{dance})(G)$ – collective
$\Leftrightarrow\; \textbf{sing}(G) \vee \textbf{dance}(G)$

b. $\mathrm{D}(\textsc{or}^{et}(\textbf{sing}, \textbf{dance}))(G)$ – distributive
$\Leftrightarrow\; \forall y \in D_{SG}.\; y \leq G \rightarrow (\textbf{sing}(y) \vee \textbf{dance}(y))$
$\Leftrightarrow\; \forall y \in G.\; (\textbf{sing}(y) \vee \textbf{dance}(y))$

The distributivity operator (cont.)

(2) The girls are wearing a blue dress.

a. $(\lambda y.\exists x.\textbf{blue\_dress}(x) \wedge \textbf{wear}(y,x))(G)$ – collective

$\Leftrightarrow\ \exists x.\textbf{blue\_dress}(x) \wedge \textbf{wear}(G,x)$

b. $\textsc{d}(\lambda y.\exists x.\textbf{blue\_dress}(x) \wedge \textbf{wear}(y,x))(G)$ – distributive

$\Leftrightarrow\ \forall y \in D_{SG}.\ y \leq G \rightarrow \exists x.\textbf{blue\_dress}(x) \wedge \textbf{wear}(y,x)$

$\Leftrightarrow\ \forall y \in G.\ \exists x.\textbf{blue\_dress}(x) \wedge \textbf{wear}(y,x)$

The distributivity operator (cont. 2)

(3) The girls won.

a. $\textbf{win}(G)$ – collective

b. $\text{D}(\textbf{win})(G)$ – distributive

In a specific model:

$\textbf{win}$ characterizes the set $\{t{+}m{+}s,\ t,\ s\}$

$G = t{+}m{+}s$

(3a) $= \textbf{win}(G) \Leftrightarrow \textbf{win}(t{+}m{+}s)$ – *true*

(3b) $= \text{D}(\textbf{win})(G) \Leftrightarrow \textbf{win}(t) \wedge \textbf{win}(m) \wedge \textbf{win}(s)$ – *false*

“Mixed” predicates

(1) Dylan wrote many hits.

(2) The Beatles wrote many hits.

(3) Simon and Grafunkel wrote many hits.

(4) Mary and Sue wrote many hits.

Sentence (4) is clearly true both if Mary and Sue wrote many hits as a team, or if each of them wrote many hits individually.

Many predicates in natural language are “mixed” in the same way.

Summary – morphological, lexical and phrasal distributivity

girls	morphological	$*\textbf{girl} \in D_t^{D_{PL}}$, where $\textbf{girl} \in D_t^{D_{SG}}$	
slept	lexical inference	$\textbf{sleep} \in D_{et}$	*the girl/group/townspeople slept*
met	-	$\textbf{meet} \in D_{et}$	*the group met* col. reading of *the two groups met*
won	-	$\textbf{win} \in D_{et}$	col. reading of *the girls won*
$[_{VP}$ slept$]$	phrasal	$\mathrm{D}(\textbf{sleep})$	*the two girls slept*
$[_{VP}$ met$]$	phrasal	$\mathrm{D}(\textbf{meet})$	dist. reading of *the two groups met*
$[_{VP}$ win$]$	phrasal	$\mathrm{D}(\textbf{win})$	dist. reading of *the girls won*

Related topics

Reciprocity: morphological (*friends*), lexical (*the boys hugged*), phrasal/derivational (*the boys hit each other*).

Floating "each": *the boys each ate a pizza*

Multiplicity of events/Pluractional markers:

Kaqchikel (Mayan):	X-	in-	kan-	ala'	jun	wuj
	PERF-	1sS-	search-	PLURAC	a	book

"I looked for a book several times"
(Henderson 2011: 219)

"Pluractional adverbials": *Sue ate the cake piece by piece.*

Challenge III – structured individuals?

Problem 1:

(1) The girls and the boys were separated.

(2) The young children and the other children were separated.

We consider models where:

$G+B=C$: the girls and the boys are the children (C)

$YC+OC=C$: the young children and the other children are the same children

In these models: $\textbf{were\_separated}(G{+}B) = \textbf{were\_separated}(YC{+}OC)$

(1) and (2) do not seem equivalent under these conditions

the girls and the boys are just the children; the girls and the boys were separated
$\overset{?}{\Rightarrow}$ the young children and the other children were separated

Tentative conclusion 1: there are models such that $\textbf{girl} \cup \textbf{boy} = \textbf{child}$ but $[\![\textit{the girls and the boys}]\!] \neq [\![\textit{the children}]\!]$.

Challenge III – structured individuals? (cont.)

Problem 2:

(1) a. The girls are group A.
 b. The boys are group B.

(2) Group A and group B are of the same size.
 a. Group A is of the same size as group B.
 b. Group A and group B (together or separately) are of the same size mentioned earlier.

(3) The children are of the same size.
 a. Each child is of the same size.
 b. The children (together or separately) are of the same size mentioned earlier.

Tentative conclusion 2: there are models such that **girl**$\cup$**boy** = **child**, and groups A and B have the members in **girl** and **boy** (pre-theoretically), but $[\![$*group A and group B*$]\!] \neq [\![$*the children*$]\!]$.

Impure atoms

Impure atom principle: An NP denotation like [[the girls]] is a plural individual $G \in D_{PL}$, which can be freely mapped to a contextually determined member of the set of *"impure" atoms* in D_{SG} (groups, teams, committees, bands etc.) made of the members of G. We denote this selected impure atom by $\uparrow G \in D_{SG}$.

$\uparrow$ is used with precedence over other operators: $\uparrow A + B = (\uparrow A) + B$.

Impure atoms (cont.)

Example 1:

(1) The girls and the boys were separated.

a. **separated**$(G+B)$ "the children were separated"

b. **separated**$(\uparrow G+\uparrow B)$ "an impure atom made of G and an impure atom made of B were separated"

(2) The young children and the other children were separated.

a. **separated**$(YC+OC)$ "the children were separated"

b. **separated**$(\uparrow YC+\uparrow OC)$ "an impure atom made of YC and an impure atom made of OC were separated"

$G+B = YC+OC$, but $\uparrow G+\uparrow B \neq \uparrow YC+\uparrow OC$.

Readings (1b) and (2b) are prominent for (1) and (2), hence the perceived lack of equivalence.

Impure atoms (cont. 2)

Example 2:

(1) a. The girls are group A. $\uparrow G = \textbf{group\_A}$

b. The boys are group B. $\uparrow B = \textbf{group\_B}$

$[\![\textit{be of the same size}]\!] = \lambda x_e.\forall y, z \in D_{SG}.\ y, z \leq x \ \rightarrow\ size(y) = size(z)$

(2) Group A and group B are of the same size.

$\forall x, y \in \textbf{group\_A}+\textbf{group\_B}.size(x) = size(y)$

$\Leftrightarrow\ size(\textbf{group\_A}) = size(\textbf{group\_B})$

(3) The children are of the same size.

$\forall x, y \in C.size(x) = size(y)$

Challenge IV – non-atomic distribution?

See Champollion's article *Distributivity in formal semantics*