Choice Functions and the Semantics of Indefinites

LOT Summer School, 21-25 June 2004

Yoad Winter
Technion/UiL OTS

email: winter@cs.technion.ac.il
WWW: http://www.cs.technion.ac.il/~winter/

1 Overview

1.1 Problems

1. *Scope*:
   John will be happy if some student I know arrives.

2. *WH in-situ*:
   Which teacher will be happy if which student arrives? John will be happy if Mary arrives.

3. *Functional questions*:
   Which woman does every man love? His mother.

4. *Distributivity*:
   Three students ate a cake.

1.2 General questions

1. *Language and Logic*: What logical principles do we need for the analysis of indefinite NPs? What kind of logical objects do indefinites denote?
   Traditional answers:
   - Existential quantifiers (Russell)
   - Epsilon-terms and choice/Skolem functions (Hilbert and Bernays)

2. *Syntax and Semantics*: In what ways does form determine meaning? Is there a direct mapping between "surface" form and meaning?
   Traditional answers:
   - Yes – compositionality (Montague Grammar)
   - No – representational theories (DRT)

1.3 Specific questions and proposed answers

1. *Is there a sense in which an indefinite like* "some person I met yesterday" *speaks about a particular person, or is it just an ordinary existential quantifier claiming there was such a person?*
   Answer: Both intuitions are correct. Indefinites refer to particular entities. However, this reference is only indirectly obtained via existential quantification over higher type objects called *choice functions* (CFs), which pick up such particular entities.

2. *Is there a need for existential closure of CF variables, or is their "deictic"/"widest scope" usage sufficient?*
   Answer: There are evidence that existential closure is needed at any compositional level. This allows us to obtain a theory of indefinites where they are uniformly treated using CFs.

3. *Given that, are there still linguistic evidence for the more general Skolem functions?*
   Answer: Yes – Skolem functions are useful for describing interactions between indefinites and anaphora, especially in cases of weak crossover and functional interpretation.

4. *What kind of scopal interactions do CFs add to the grammatical apparatus?*
   Answer: In the case of *plural* indefinites we can observe a clear separation between existential scope (derived by CFs) and the scope of distributive quantification (derived by QR).

5. *What is the class of "CF indefinites"?*
   Answer: Only the simple singular/plural indefinites (*a, some, several*) and the bare numerals (*three*), but not the complex numerals (*more than/less than exactly three*).

6. *What's the difference between* a *and* some*?*
   Answer: *Some* is at a higher level within the DP, which forces a CF interpretation.

1.4 Course plan

1. **Overview**

2. **Scope and indefinites**: the scope problem, standard scope mechanisms, scope islands, wide scope indefinites, Fodor and Sag's specificity, Reinhart's choice functions

3. **Existential closure vs. Parameterized CFs**: the distribution of intermediate scope, existential closure, "specific" CFs, formal definitions, Chierchia's observations, weak cross-over and CFs

4. **CFs in the analysis of plural indefinites**: notes on plurality, modificational vs. quantificational numerals, Ruys's observation and double scope, indefinites and collective coordination, choice functions and generalized quantifiers

5. **CFs as a flexibility operator**: DP structure and flexibility, restrictions on category shifting, *both...and*, accusative marking in Hebrew, verbless predication, appositional conjunction

6. **Wide scope indefinites and "functional" interpretation**: functional readings of questions and copular sentences, distributional and intuitive similarities with WS indefinites, unified Skolem-based variable-free mechanism, generalized quantifiers and functional readings, the distribution of generalized quantifiers with functional readings

7. **CFs and generalized quantifiers**

1.5 A short history of the problem

Russell (1919) puts indefinites in the group of "quantificational" noun phrases and attacks their classification as "referential", maintaining that *"the great majority of logicians who have dealt with this question"* were *"misled by grammar"*.

Hilbert and Bernays (1939): No quantifiers. Quantification is indirectly modelled using *epsilon terms*. A formula like $P(\epsilon x.Q(x))$ is used as equivalent to the standard Predicate Calculus formula $\exists x[P(x) \wedge Q(x)]$.

Montague (1973): In PTQ it is implicitly assumed that both Russell and the proponents of "referential" treatments were, so to speak, misled by formal logic. All NPs are treated as quantificational, the very distinction between "referential" and "quantificational" NPs is challenged.

In the 80s, however, DRT revives a difference between NPs (including indefinites) that evoke *discourse referents* and NPs that do not.

In particular Fodor and Sag (1982), in an influential paper, make the bold claim that there are indeed

two interpretations of indefinites: quantificational and referential.

Reinhart (1992) challenges Fodor and Sag's observations (in fact, using similar arguments to Farkas' (1981)), as well as the idea that there are "referential" indefinites. In effect Reinhart proposes to use the semantic correlate to Hilbert and Bernays' epsilon operator – choice functions – in order to deal with the problems that Fodor and Sag pointed out.

Later in the 90s: Choice functions become the center of a lively debate in linguistic theory, which involves semantic, syntactic and philosophical questions. This is where we start this course.

1.6 The broad picture

Fodor and Sag (1982) observe the contrast between examples like the following –

(1) John overheard the rumor that *a student of mine* had been called before the dean.

(2) John overheard the rumor that *each student of mine* had been called before the dean.

Why does the indefinite NP in (1) show a "wide scope reading" (over *the rumor that*) but the universal NP in (2) does not?

Where are standard theories unattractive?

1. **QR/Q-in/Q-storge**: need to add syntactic stipulations
2. **DRT**: using free existential closure alone, leads to the well-known "Donald Duck" problem (Reinhart (1992)):

   (3) Who will be offended if we invite which philosopher?
   Lucie will be offended if we invite some philosopher.

   (4) $\exists x[[\mathbf{we_invite}'(x) \wedge \mathbf{philosopher}'(x)] \rightarrow \mathbf{offended}'(\mathrm{l}')]$

   Since Donald Duck is not a philosopher, he can make the antecedent false, hence the whole statement true. And the problem is more general:

   (5) Mary likes exactly one student who invited a friend of mine.

   Hence we prefer a real "wide-scope" reading:

   (6) $\exists x[\mathbf{philosopher}'(x) \wedge [\mathbf{we_invite}'(x) \rightarrow \mathbf{offended}'(\mathrm{l}')]]$

   But the derivation of this reading in DRT is not more attractive than its derivation using standard scope mechanisms (QR etc.).

Three types of answers:

1. **Specificity**: Fodor and Sag, Geurts (using presupposition), Schwarzschild (using domain restriction).
2. **Island-free movement** (DRT/QR/other): Farkas, Szabolcsi, Abusch.
3. **Choice/Skolem functions**: Reinhart, Winter, Kratzer, Chierchia.

What is at stake here?

1. *Intermediate scope*: Do indefinites show non-widest scope beyond islands?

   (7) Every linguist knows every solution to some problem.

2. *Distributivity*: How do plural NPs behave beyond islands in terms of their distributivity?

(8) If three relatives of mine die I will inherit a fortune.

3. *WH in-situ*:

   (9) Who will be offended if we invite which philosopher?

4. *Simple interactions with anaphora*:

   (10) Every linguist knows every solution to some problem he studies.

5. *More complex interactions – functional readings*:

   (11) A woman that every man loves is his mother.

6. *DP typology*:

   (12) If (more than/less than/exactly) three relatives of mine die I will inherit a fortune.

7. *Intensionality*:

   (13) John believes that Jim will leave if some woman arrives.

8. *Sluicing*:

   (14) John talked to some woman, but I don't remember who.

9. *Definites vs. indefinites*

10. *Donkey anaphora*

11. *Interpretation of "a certain"*

12. *Interaction with weak crossover*

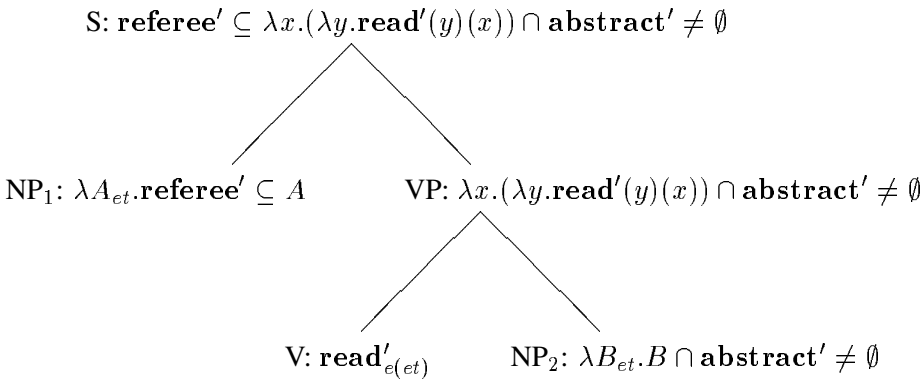
13. *The Scope of Disjunction*

2 Scope and indefinites

2.1 The scope problem

(1) Every referee read some abstract.

(2) [$_{\mathrm{NP}_1}$ every referee] [$_{\mathrm{VP}}$ read [$_{\mathrm{NP}_2}$ some abstract]]



(3) $\forall x[\mathbf{referee}'(x) \rightarrow \exists y[\mathbf{abstract}'(y) \wedge \mathbf{read}'(y)(x)]]$

This analysis is sufficient for (1): sentence (4) entails (1), but also proposition (5) entails (3).

(4) Some abstract is one that was read by every referee.

(5) $\exists y[\mathbf{abstract}'(y) \wedge \forall x[\mathbf{refree}'(x) \rightarrow \mathbf{read}'(y)(x)]]$

However, the problem for the analysis of (6) becomes more serious.

(6) Some referee read every abstract.

(7) $\exists x[\mathbf{referee}'(x) \wedge \forall y[\mathbf{abstract}'(y) \rightarrow \mathbf{read}'(y)(x)]]$

Proposition (7) is not sufficient to account for all the entailments with sentence (6). Intuitively, sentence (6) can be used to convey the meaning of (8), as formalized in (9).

(8) Every abstract is one that was read by some referee.

(9) $\forall y[\mathbf{abstract}'(y) \rightarrow \exists x[\mathbf{referee}'(x) \wedge \mathbf{read}'(y)(x)]]$

More examples, with pragmatic preference for object wide scope (WS) readings:

(10) Some guard is standing in front of every building.

(11) Some cat is sitting inside every box.

(12) Some honest man was killed in every war.

(13) **Scope methodology 1**: A WS reading is easier to attest when it does not entail the NS reading.

(14) **Scope methodology 2**: A WS reading is easier to attest when it is pragmatically "more likely" than the NS reading.

2.2 Standard scope mechanisms

Quantifier Raising (roughly):

(15) Some referee read every abstract.

a. [some referee] λx.[[every abstract] λy.[x read y]

b. [every abstract] λy.[[some referee] λx.[x read y]

Semantic scope shifting mechanisms:

- Cooper (1975) Storage
- Hendriks (1993): Scope shifting as type shifting

2.3 Scope islands

Complex NP Constraint:

(16) a. John knows a girl from every country.

b. #John knows a girl who is from every country.

(17) $\forall x[\mathbf{country}'(x) \rightarrow \exists y[\mathbf{girl}'(y) \wedge \mathbf{from}'(x)(y)]]$

(18) a. Which country does John know a girl from?

b. *Which country does John know a girl who is from?

Coordinate Structure Constraint:

(19) a. Some woman gave birth to every man.

b. #Some woman gave birth to every man and will eventually die.

Therefore, while sentence (19a) has the WS reading in (20a), (19b) does not have any reading like (20b).

(20) a. $\forall x[\mathbf{man}'(x) \rightarrow \exists y[\mathbf{woman}'(y) \wedge \mathbf{give_birth_to}'(x)(y)]]$

b. $\forall x[\mathbf{man}'(x) \rightarrow \exists y[\mathbf{woman}'(y) \wedge \mathbf{give_birth_to}'(x)(y) \wedge \mathbf{will_die}'(y)]]$

(21) a. I wonder whom Mary gave birth to.

b. *I wonder whom Mary gave birth to and will eventually die.

Adjunct Island:

(22) a. #If every woman in the room gave birth to John then he has a nice mother.

b. $\forall x[\mathbf{woman}'(x) \rightarrow [\mathbf{give_birth_to}'(\mathrm{j}')(x) \rightarrow \mathbf{have_nice_mother}'(\mathrm{j}')]]$

c. For every woman in the room, if she gave birth to John then he has a nice mother.

(23) *Which woman did if give birth to John then he has a nice mother?

2.4 Wide scope indefinites

(24) If some building in Washington is attacked by terrorists then US security will be threatened.

(25) $[\exists x[\mathbf{building}'(x) \wedge \mathbf{attacked}'(x)]] \rightarrow \mathbf{threatened}'(\mathbf{security}')$

Contrast also (22a) with the following sentence.

(26) If some woman I know gave birth to John then he has a nice mother.

The problem: Why are indefinites more free in their scopal behavior than other NPs?

Is this really a problem?

Schwarzschild (2002) thinks it is not. For Schwarzschild, "WS indefinites" are really only *singleton indefinites*: indefinites with a singleton restriction set. This position means that when indefinites in sentences like (24) or (26) show a "WS" effect there should only be one building in Washington, one woman I know, etc. This is unlikely, and moreover: the same effect of singleton NPs should have appeared with other NPs besides simple indefinites, but evidently it does not (cf. Liu (1990), Corblin (1997), Beghelli (1995), Kamp and Reyle (1993:ch.4)):

(27) #If every/exactly one woman I know gave birth to John then he has a nice mother.

2.5 Fodor and Sag's specificity

Indefinites are ambiguous between two interpretations:

1. A standard reading as existential *quantifiers*.
2. A *referential/specific* reading where the indefinite functions as 'a 'private' pointing gesture within the mind of the speaker" (Fodor and Sag (1982:381)).

Advantage: "Wide scope" indefinites do not require stipulative any modification of standard scope mechanisms.
Problems: What is the proper definition of this "private pointing gesture"? Why are only indefinites ambiguous in this way and not other NPs?

2.6 Reinhart's choice functions

Reinhart (1997) - three assumptions:

(A1) Indefinites lack quantificational force of their own. An indefinite basically denotes a *predicate*, henceforth the *restriction* predicate.

(A2) An indefinite NP in an argument position, however, can end up denoting an *individual*. This is because its semantics involves a free function variable that assigns an individual to the restriction predicate.

(A3) This function variable is "closed" using an existential quantifier, together with the requirement that it is a choice function. This *quantificational* procedure can apply at any compositional level.

Example:

(28) Some woman smiled.

(29) $\exists f[CH(f) \wedge \mathbf{smile}'(f(\mathbf{woman}'))]$

Definition 1 (choice functions) *Let E be a non-empty set. A function $f : \wp(E) \rightarrow E$ is a* choice function *iff for every $A \subseteq E$: if A is not empty then $f(A) \in A$.*

(30) $CH_{((\tau t)\tau)t} \stackrel{def}{=} \lambda f_{(\tau t)\tau}.\forall P_{\tau t} \neq \emptyset[P(f(P))]$

(31) $[\exists f[CH(f) \wedge \mathbf{attacked}'(f(\mathbf{building}'))]] \rightarrow \mathbf{threatened}'(\mathbf{security}')$

(32) $\exists f[CH(f) \wedge [\mathbf{attacked}'(f(\mathbf{building}')) \rightarrow \mathbf{threatened}'(\mathbf{security}')]]$

(33) The US security will be threatened if there is *any* function choosing from the set of buildings in Washington one that was attacked by terrorists.

(34) There is a choice function such that US security will be threatened if the building it picks is attacked by terrorists.

Advantages:

1. No syntactic movement or other stipulative changes in standard scope mechanisms.
2. Reduction of ”free scope” to ”predicative nature”.
3. More semantic explicitness.

3 Existential closure vs. Parameterized CFs

3.1 The intermediate scope problem

Intermediate scope:

(35) Every professor will rejoice if a student of mine cheats on the exam.

a. NS: $\forall x[\mathbf{prof}'(x) \rightarrow [\exists y[\mathbf{student}'(y) \wedge \mathbf{cheat}'(y)] \rightarrow \mathbf{rejoice}'(x)]]$

b. IS: $\forall x[\mathbf{prof}'(x) \rightarrow \exists y[\mathbf{student}'(y) \wedge [\mathbf{cheat}'(y) \rightarrow \mathbf{rejoice}'(x)]]]$

c. WS: $\exists y[\mathbf{student}'(y) \wedge \forall x[\mathbf{prof}'(x) \rightarrow [\mathbf{cheat}'(y) \rightarrow \mathbf{rejoice}'(x)]]]$

(36) Each teacher overheard the rumor that a student of mine has been called before the dean.

Fodor and Sag claim that the IS interpretation is not there, and conclude that indefinites are ambiguous between two readings:

1. *A quantificational (existential) reading restricted by islands.*
2. *A referential (specific) reading that has an effect similar to the WS-existential reading.*

The challenge – are there IS readings?:
Farkas (1981):

(37) Each student has to come up with three arguments that show that some condition proposed by Chomsky is wrong.

(38) Everybody told several stories that involved some member of the Royal family.

Ruys (1992) and Abusch (1994):

(39) Every professor will rejoice if a student of *his* cheats on the exam. (Ruys)

a. NS: For every professor: if there is a student of the professor who cheats on the exam then the professor will rejoice.

b. IS: For every professor there is a student of his such that if this student cheats on the exam the professor will rejoice.

(40) Every professor rewarded every student who read some book *he* had recommended. (Abusch)

Farkas, Ruys and Abusch conclude: *there are IS readings*!

Kratzer (1998): contrasts cases like (39) and (40) with the following:

(41) Every professor will rejoice if a student of *mine* cheats on the exam.

(42) Every professor rewarded every student who read some book *Mary* had recommended.

Kratzer (1998): The *main* evidence for the need to derive IS readings of indefinites is when they contain a pronoun that is bound by a quantifier that appears higher than the IS position. (Kratzer does not discuss Farkas' facts).

Reinhart (1997):

(43) Every linguist has looked at every analysis that solves some problem.

a. IS: For every linguist x there is a problem y such that x has looked at every analysis that solves y.

b. NS: Every linguist has looked at every analysis. (assuming that for every analysis it is true that it solves some problem)

c. WS: There is a problem x, such that every linguist has looked at every analysis of x.

More examples:

(44) Every movie director is happy to direct every film that features some actor.

a. IS: For every director x there is an actor y such that x is happy to direct every film featuring y.

(45) Every country's security will be threatened if some building is attacked by terrorists.

a. IS: For every country there is a (potentially different) building, an attack on it might threaten the country's security.

Reinhart's solution:

- Indefinites are ambiguous between existential quantifiers and CFs.
- CF variables can be "existentially closed" at any level.

Examples:

(46) IS: $\forall x[\mathbf{linguist}'(x) \rightarrow \exists f[CH(f) \wedge \forall y[[\mathbf{analysis}'(y) \wedge \mathbf{solve}'(f(\mathbf{problem}'))(y)] \rightarrow \mathbf{look_at}'(y)(x)]]]$

(47) IS: $\forall x[\mathbf{prof}'(x) \rightarrow \exists f[CH(f) \wedge [\mathbf{cheat}'(f(\mathbf{student_of}'(x))) \rightarrow \mathbf{rejoice}'(x)]]]$

Kratzer's solution:

- Indefinites are ambiguous between existential quantifiers and CFs.
- CF variables are not existentially closed at any level.
- CFs are parameterized with pronoun variables (=Skolemized).

Examples (informally):

(48) Pseudo-WS (only):
$\forall x[\mathbf{linguist}'(x) \rightarrow \forall y[[\mathbf{analysis}'(y) \wedge \mathbf{solve}'(f(\mathbf{problem}'))(y)] \rightarrow \mathbf{look_at}'(y)(x)]]$
(f contextually determined as a CF)

(49) Pseudo-IS:
$\forall x[\mathbf{prof}'(x) \rightarrow [\mathbf{cheat}'(f(\mathbf{student_of}', x)) \rightarrow \mathbf{rejoice}'(x)]]$
(f contextually determined as a parameterized CF, with one parameter)

3.2 What are parameterized CFs?

In fact, they are *Skolem functions* (SFs):

- A CF maps a set A to a member of A, provided that A is not empty.
- An SF of arity n (SF_n) maps an $(n+1)$-ary relation R, *and* n *entity parameters* $x_1, ..., x_n$, to a member of the set $\{x : R(x_1)...(x_n)(x)\}$, provided that this set is not empty.

Note: A CF is actually an SF of arity 0.

A simple example – is there a "WS" reading of the following sentence?

(50) Every teacher met a student of his.

With standard existential quantifiers and standard scope taking – of course not. But with CFs/SFs – there is:

(51) $\exists f[CH(f) \wedge \forall x[\mathbf{teacher}'(x) \rightarrow \mathbf{meet}'(f(\mathbf{student_of}'(x)), x)]]$
The CF function f maps the set (unary predicate) $\mathbf{student_of}'(x)$ to a member of this set, but this creates trouble, since x is bound by an operator lower than the existential quantifier over f: If two teachers happen to have the same students, then they must have met the same student!

(52) $\exists f[SF_1(f) \wedge \forall x[\mathbf{teacher}'(x) \rightarrow \mathbf{meet}'(f(\mathbf{student_of}', x), x)]]$
The Skolem Function f (or arity 1) maps the binary relation $\mathbf{student_of}'$ *and the teacher parameter* x to one of x's students. This is basically the same as the narrow scope construal of the indefinite as an existential quantifier.

Conclusion: Something like Kratzer's parameterized CFs is needed if we want to use CFs in the first place.

3.3 Taking stock of remaining problems

Kratzer:

- No IS is derived in the Farkas/Reinhart examples.
- Ambiguity of indefinites is inherent in Kratzer's theory, but not in Reinhart's (an issue taken up in Winter (1997)).

Reinhart:

- General SFs have independent motivation.

3.4 Chierchia's first observation: non-upward-monotone contexts

Negate Reinhart's example:

(53) Not every linguist has looked at every analysis that solves some problem (he studies).

Compare IS of CFs with WS of SFs:

(54) IS-CF: $\neg\forall x[\mathbf{linguist}'(x) \rightarrow \exists f[CH(f) \wedge \forall y[[\mathbf{analysis}'(y) \wedge \mathbf{solve}'(f(\mathbf{problem_study}'(x)))(y)] \rightarrow \mathbf{look_at}'(y)(x)]]]$

(55) WS-SF: $\exists f[SF_1(f) \wedge \neg\forall x[\mathbf{linguist}'(x) \rightarrow \forall y[[\mathbf{analysis}'(y) \wedge \mathbf{solve}'(f(\mathbf{problem_study}', x))(y)] \rightarrow \mathbf{look_at}'(y)(x)]]]$

$\mathbf{problem_study}'(l)(p)$ = "linguist l studies problem p"

Now suppose that every linguist x chooses a problem y with respect to which x is completely systematic: x looks at all of the analyses of y. But suppose that one linguist, John, is studies weak crossover but missed some of its analyses. Sentence (53) is intuitively false under its prominent interpretation. So is proposition (54). But (55) is true: choose an f that maps the relation $\mathbf{problem_study}'$ and John to the problem of weak crossover.
The same problem appears for SFs with any non-upward-monotone operator.
Chierchia's conclusion 1: We need existential closure, especially in IS positions.

A debate with Kratzer (2003): Kratzer claims that IS readings under the "contextualist" view of SFs can only appear if the context supplies a function that maps (in the above examples) linguists to problems. For reasons we don't understand, Kratzer claims, this has to be the "most salient" problem.

Be it as it may, Chierchia does not discuss the problem of how to block *wide scope* closure of CFs when the restriction set, as in (50) and (53), contains a bound variable.

Winter (2001:ch.3): The arity of an SF matches the number of free variables in the restriction of the indefinite.

Thus, instead of (54), we get the following, equivalent interpretation of (53):

(56) IS-SF: $\neg\forall x[\mathbf{linguist}'(x) \rightarrow \exists f[SF_1(f) \wedge \forall y[[\mathbf{analysis}'(y) \wedge \mathbf{solve}'(f(\mathbf{problem_study}', x))(y)] \rightarrow \mathbf{look_at}'(y)(x)]]]$

This still does not rule out the interpretation in (55), though!

3.5 Chierchia's second observation: IS and weak crossover

Passivize Reinhart's example:

(57) Every linguist has looked at every analysis that solves some problem. (=(43)).

(58) Every analysis that solves some problem has been looked at by every linguist.

Why doesn't (58) seem to have the IS reading of (57)?
Chierchia's claim: under the SF WS-only analysis (with implicit parameters) of IS phenomena, this is accounted for. He contrasts the following sentences, and claims that only the second has a WS reading of the universal NP:

(59) A certain technician inspected every plane.

(60) A technician inspected every plane.

Chierchia assumes, with Kratzer, that *a certain*, unlike *a*, must involve a hidden parameter, so (59) with object WS must be a case of weak crossover, similar to the following sentence.

(61) Her mother admires every girl.

Is the contrast between (59) and (60) so strong?

(62) A certain woman is adored by every child. (?)It's his mother.

3.6 CFs vs. general SFs: summary

- Fodor and Sag: No IS – hence "referential" indefinites.
- Farkas, Ruys, Abusch: IS actually appears.
- Reinhart: And even without anaphora – hence existential closure of CFs.
- Kratzer: But "referential" SFs are quite powerful also without closure.
- Chierchia: Sometimes not powerful enough.

Conclusion: We need SFs and we probably need IS closure as well. Restrictions on the availability of readings are not always well-understood.

4 CFs and plurality

4.1 Plural individuals and distributivity

Plural individuals:

(63) a. The girls met.
b. The girls slept.

(64) a. $\mathbf{meet}'(G)$
b. $\mathbf{sleep}'(G)$

Why distributivity?

1. For getting a more realistic (?) analysis of (63b). But see Scha (1981), Roberts (1987), Dowty (1986), among others.
2. For allowing definites to have quantificational force:

   (65) The girls are wearing a dress.

   (66) The dogs like their masters.

Two approaches to distributivity:

- Ambiguity of determiners: Bennett (1974), Scha (1981), van der Does (1993), and others.
- A distributivity operator: Link (1983) and (many) others, including these lecture notes.

Simple definition of a distributivity operator:

(67) If P is a predicate and A is a plural individual, then the *distributive application* of P to A is denoted $(\mathcal{D}(A))(P)$, which is simply interpreted as universal quantification over the elements of A: $\forall x \in A[P(x)]$.

Some questions we will not discuss:

1. Is $\mathcal{D}$ indeed located on *arguments*?
2. Does $\mathcal{D}$ indeed distribute over *atomic* elements of A?
3. Is $\mathcal{D}$ indeed an *unary* operator?

For more on these questions see Gillon (1987), Schwarzschild (1996), Winter (2000).

An odd question: Are simple sentences like *the girls slept* or *the girls met* ambiguous?
Derivationally they may be, and following Dowty, we say that the non-distributive analysis of *the girls slept* may have *subentailed distributivity* – partial distributivity that results from the lexical properties of the predicate.

4.2 Plural indefinites: modificational vs. quantificational numerals

(68) a. Some/three girls met.
b. Some/three girls slept.

(69) Some/three girls are wearing a dress.

(70) Some/three dogs like their masters.

The two approaches to distributivity dictate different approaches to plural indefinites:

1. Ambiguity of determiners: *some, three*, etc. are ambiguous determiners.
2. Ambiguity of determiners: *some, three*, etc. are unambiguous. *Three* is a modificational element, and the existential force comes from outside the numeral element. (We leave *some* aside for a while)

The modificational analysis of numerals:

(71) [[three girls]] $= \{A \subseteq G : |A| = 3\}$

Examples:

(72) Three girls met.

a. $\exists A \in$ [[three girls]][$\mathbf{meet}'(A)$] $\Leftrightarrow$
$\exists A \subseteq G[|A| = 3 \wedge \mathbf{meet}'(A)]$

(73) Three girls are wearing a dress.

a. $\exists A \in$ [[three girls]][$\mathcal{D}(A)$([[are wearing a dress]])] $\Leftrightarrow$
$\exists A \subseteq G[|A| = 3 \wedge \forall x \in A \exists y \in \mathbf{dress}'[\mathbf{wear}'(y)(x)]]$

4.3 Plural indefinites: Ruys's observation and double scope

Ruys (1992,1995):

(74) If three relatives of mine die I will inherit a house.

Claim – *a* is a WS reading of (74), but not *b*:

a. I have three relatives such that if they (all) die I will inherit a house.
b. I have three relatives such that for *each* of them, if he dies I will inherit a house.

"Proof":

(75) #If three women gave birth to John then he has a nice mother.

Three *a priori* possible interpretations,

(76) a. Existential NS + subentailed distributivity (or NS Q-distributivity):
If (each member of) any set of three women gave birth to John, then he has a nice mother.
b. Existential WS + subentailed distributivity (or NS Q-distributivity):
There is a set A of three women such that if (each member of) A gave birth to John then he has a nice mother.
c. Existential WS + WS Q-distributivity:
There is a set A of three women such that for each member x of A, if x gave birth to John then he has a nice mother.

among which only *c* is coherent. But (75) is incoherent. Likewise:

(77) #Every artist who was born in three cities became famous.

a. There is a set A of three cities such that for each member x of A, every artist who was born in x became famous.

(78) #A baby was adopted by John or was born to three women.

a. There is a set A of three women such that for each member x of A, a baby was adopted by John or was born to x.

Conclusion –

The Ruys observation: While the *existential* scopal semantics of indefinites can violate island constraints, the scope of *Q-distributivity* is island-restricted.

Therefore – the following scope shifting assumption is implausible.

(79) [three relatives] [λx.[if x die I will inherit a house]]

Using choice functions:

(80) $\exists f[CH(f) \wedge [\mathbf{die}'(f([[\text{three relatives}]])) \rightarrow \mathbf{inherit_house}']]$
"There is a choice of a plural individual, A, consisting of three relatives of mine, such that if A dies I will inherit a house."

(81) #Some artist who was born in two small villages in Holland is a genius.

(82) $\exists g \exists f[CH(f) \wedge CH(g) \wedge$
$\mathbf{genius}'(f(\mathbf{artist}' \cap \lambda x.\mathbf{born_in}'((g([[\text{two villages}]]))(x)))]$

Remaining question: Where does distributivity go when the indefinite "escapes an island"?

(83) If three workers in our staff have a baby soon we will have to face some hard organizational problems.

a. There are three workers such that if *each* of them has a baby soon we will have to face some hard organizational problems. (available)

b. There are three workers such that for *each* of them, if he/she has a baby soon we will have to face some hard organizational problems. (<u>un</u>available)

c. #There are three workers such that if there is a baby that they *all* have soon, we will have to face some hard organizational problems. (perhaps available, but incoherent)

The double scope observation: The existential and the Q-distributivity imports of numeral indefinites can have two distinct scopes.

Another example:

(84) Every teacher who gave a lecture to three excellent classes in our school was very satisfied afterwards.

a. There are three excellent classes in our school, such that every teacher who gave a (potentially different) lecture to them was very satisfied afterwards.

Double scope without islands:

(85) Exactly one teacher gave a lecture to three excellent classes in our school.

(86) Every teacher of three excellent classes in our school that were given a lecture was satisfied.

(87) $\exists A[\mathbf{classes}'(A) \wedge \mathbf{three}'(A) \wedge \exists! x[\mathbf{teacher}'(x) \wedge$
$\forall z{\in}A\ \exists y[\mathbf{lecture}'(y) \wedge \mathbf{gave}'(y)(z)(x)]]]$

(88) $\exists A[\mathbf{classes}'(A) \wedge \mathbf{three}'(A) \wedge$
$\forall z{\in}A\ \exists! x[\mathbf{teacher}'(x) \wedge \exists y[\mathbf{lecture}'(y) \wedge \mathbf{gave}'(y)(z)(x)]]]$

It is impossible to capture the two observations only using standard scope:

(89) [three workers] [λx.[[if x have a baby] we will face problems]]

With such a representation distribution would, under any plausible assumption on the conditions for its application, violate the Ruys observation. Without distribution there is no way known to capture the double scope observation.

Double scope with choice functions:

(90) $\exists f[\ CH(f) \wedge [\ \mathcal{D}(f([[\text{three workers}]]))([[\text{have a baby}]])\ \rightarrow \mathbf{problems}']]$

Standard scope within island boundaries:

(91) An American flag was hanging in front of two buildings.

a. Hanging in front of two buildings there was a (potentially different) American flag.

No way to get distributive WS using CFs. Hence - standard scope applies to indefinites as well as other NPs.

(92) Exactly one cat is sitting in five baskets.

(93) The mayor was disqualified in nine cities.

(94) Two guards are standing in front of twenty buildings in this town.

4.4 IS coming from indefinites and collective coordination

Winter (2001):

(95) John and some man met.

a. $\exists f[CH(f) \wedge \mathbf{C}(\mathrm{J} \sqcap \langle f\rangle(\mathbf{man}'))\mathbf{meet}']$
$\Leftrightarrow\ \exists x[\mathbf{man}'(x) \wedge \mathbf{meet}(\{\mathbf{j}', x\})]$

(96) Every piano was lifted by John and some man.

(97) $\forall x[\mathbf{piano}'(x) \rightarrow \exists f[CH(f) \wedge \mathbf{lifted_by}'(\{\mathbf{j}, f(\mathbf{man}')\})(x)]]$

5 Predicate-quantifier flexibility

Two paradigms on DPs:

1. A *syntactic* paradigm (Abney 1987): the structure of the category formerly known as "noun phrase" is richer than what X-bar theory allows for N projections. Hence the *DP hypothesis*.

2. A *semantic* paradigm (Partee 1987): the meanings available for DPs are richer than what was formerly assumed in generalized quantifier theory. Hence the *flexibility* (type shifting) *hypothesis*.

The proposal: Syntactic richness is manifested as semantic flexibility – different DP meanings are obtained at different levels within the DP. This is obtained as follows:

1. The DP level is unambiguously quantificational.

2. The NP level is unambiguously predicative.

3. The intermediate D' level is ambiguously quantificational/predicative, with the two *category shifting mechanisms* of the minimum operator and choice functions mediating between the two semantic categories.

5.1 The Partee Triangle

Partee (1987) – three *types* of DP meanings:

1. Referential (type e): *Mary*, *the student*, *some student* (in a DRT analysis).

2. Predicative (type et): *Mary is* <u>*a student*</u>.

3. Quantificational (type ett): *every student*, *no student*, *Mary or John*, *neither Mary nor John*.

There are (at least) six *type shifting principles* that allow us to move from one type to another. Examples:

(1) This man is <u>John</u>. ($e \overset{\mathbf{Q}}{\Rightarrow} et$)

(2) <u>Mary</u> or <u>John</u> arrived. ($e \overset{\mathbf{M}}{\Rightarrow} ett$)

(3) John is <u>no friend of mine</u>. ($ett \overset{\mathrm{BE}}{\Rightarrow} et$)

(4) <u>A man</u> arrived. ($et \overset{\mathbf{E}}{\Rightarrow} ett$)

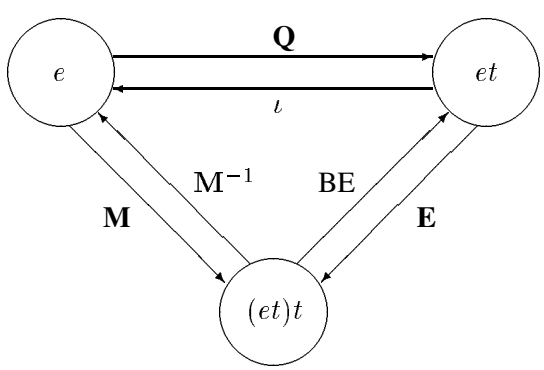


Figure 1: The Partee Triangle

$$\mathbf{M} = \lambda x.\lambda A.A(x)$$

$$\mathbf{M}^{-1} = \lambda Q.\begin{cases} x & \text{if } Q = \mathbf{M}(x) \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$\mathbf{Q} = \lambda x.\lambda y.x = y$$

$$\iota = \lambda A.\begin{cases} x & \text{if } A = \mathbf{Q}(x) \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$\mathbf{E} = \lambda A.\lambda B.A \cap B \neq \emptyset$$

$$\text{BE} = \lambda Q.\lambda x.Q(\{x\})$$

5.2 Alternative: *min* and CFs as category shifting principles

There are only two *semantic categories* of DP meanings – predicative (−Q) and quantificational (+Q). Accordingly there are only two *category shifting principles*:

1. A *choice function* mechanism from predicates to quantifiers (on the formalization of CFs using generalized quantifiers see section 7).
2. A *minimum* mechanism from quantifiers to predicates.

Main motivations:

(5) The scope of indefinites:

a. Mary will be surprised if a student in the syntax class cheats on the exam.

b. $\exists f[f$ is a CF and Mary will be surprised if f(student in the syntax class) cheats on the exam]

(6) Collectivity of NP conjunctions:

a. Mary and John met.

b. $\exists f[f$ is a CF and f(*min*(Mary and John)) met]

Consequences:

- No such thing as "referential" DPs:
  - Even proper names must be analyzed as quantifiers when they are in argument positions (elementary GQ theory).
  - CFs map nominal denotations to quantifiers, not to "referential" entities (less elementary GQ theory).
- Partee's constructions:

  (7) This man is <u>John</u>. (+Q ⇒ −Q)

  (8) <u>Mary</u> or <u>John</u> arrived. (+Q with no shifting)

  (9) John is no friend of mine. (unexplained)

  (10) <u>A man</u> arrived. (−Q ⇒ +Q)
- Restrictions on category shifting principles – some DPs can never leave the quantifier domain:

  (11) John is some teacher I know/*every teacher I know.

  (12) Mary and John/*both Mary and John/*all the students are a good team.

  (13) a. Mary will be surprised if every student in the syntax class cheats on the exam.

b. *$\exists/\forall f$[f is a CF and Mary will be surprised if f((every) student in the syntax class) cheats on the exam]

(14) a. Mary will be surprised if exactly one student in the syntax class cheats on the exam.

b. *$\exists/\exists! f$[f is a CF and Mary will be surprised if f(student in the syntax class) cheats on the exam]

Two classification problems:

1. Why is the meaning of some DPs flexible and the meaning of other DPs not?
2. Among the flexible DPs, which ones are basically predicative and which ones are basically quantificational?

Proposal:

1. Flexibility and DP structure:
   (a) NP is rigidly predicative.
   (b) DP is rigidly quantificational.
   (c) D' is flexible between the two.
2. Explanation:
   (a) NPs are predicative because the noun denotes a predicate.
   (b) DPs are quantificational because the SPEC denotes a determiner: a function from predicates to GQs.
   (c)
   - D's with a filled D position are basically quantificational because D (like SPEC) denotes a determiner.
   - Other D's are basically predicative as they dominate NP with no semantic operation.

   In any case – category shifting principles can change the category of D' denotations from predicate to quantifier or vice versa.
3. Needed criterions:
   (a) For deciding which DPs are D's (don't have SPEC).
   (b) For deciding which D's are NPs (don't have D).

DP/D' criterion I – collectivity:

(15) a. Mary and John are a nice couple.

b. *Both Mary and John/Mary as well as John/Mary and John too are a nice couple.

(16) a. These women are a good team.

b. *All these women are a good team.

(17) a. Five women I know are a good team.

b. *Less than/more than/exactly five women I know are a good team.

(18) a. Mary and four men I know are a good basketball team.

b. #Mary and at least four men I know are a good basketball team.

Note: Only certain collective predicates are useful for this criterion. Other predicates show collectivity with all plurals. For instance:

(19) a. All these women met/gathered.

b. Less than/more than/exactly five women I know met/gathered.

c. Mary and at least four men I know met/gathered.

The distinctions between collective predicates that lead to these contrasts are explained in Winter (2001:ch.5, 2002).

DP/D' criterion II – wide scope:

(20) a. If *some* woman I know gave birth to John then he has a nice mother.

b. #If *exactly one* woman I know gave birth to John then he has a nice mother.

(21) a. If *two* people I know are John's parents then he is lucky.

b. #If *exactly two* people I know are John's parents then he is lucky.

DP/D' criterion III – X-bar theory: Complex numerals like *more than three, between two and four, fewer than five* must be in SPEC.

Conclusion:

(22) D':

a. $[_{DP}[_{D'}[_{D'}$ Mary] and $[_{D'}$ John]]]

b. $[_{DP}[_{D'}$ these women]]

c. $[_{DP}[_{D'}$ five women]]

d. $[_{DP}[_{D'}[_{D'}$ Mary] and $[_{D'}$ three men]]]
$[_{DP}[_{DP}[_{D'}$ Mary]] and $[_{DP}[_{D'}$ three women]]]

(23) DP:

a. both $[_{DP}[_{D'}$ Mary] and $[_{DP}[_{D'}$ John]]]

b. $[_{DP}$ $[_{SPEC}$ all] $[_{D'}$ these women]]

c. $[_{DP}$ $[_{SPEC}$ less than five] $[_{D'}$ women]]

d. $[_{DP}[_{DP}[_{D'}$ Mary]] and $[_{DP}[_{SPEC}$ less than three] $[_{D'}$ women]]]
Note: a D' coordinated with a DP must be a DP!

Restrictions on both...and/either...or

1. Do not apply at the X' level (see Neijt 1979):

(24) every (*both) man and woman, three/most (*either) men or women

(25) very (*both) tall and thin, ten meters (*both) above the house and below the cloud

2. In Hebrew – require separate ACC marker on each conjunct:

(26) i. dan makir et rina ve/o sara
Dan knows ACC Rina and/or Sara
"Dan knows Rina and/or Sara"

ii. dan makir et rina ve/o et sara
Dan knows ACC Rina and/or ACC Sara
"Dan knows Rina and/or Sara"

(27) a. i. * dan makir et gam rina ve gam sara
Dan knows ACC too Rina and too Sara

ii. dan makir gam et rina ve gam et sara
Dan knows too ACC Rina and too ACC Sara
"Dan knows both Rina and Sara"

b. i. * dan makir et o rina o sara
Dan knows ACC or Rina or Sara

ii. dan makir o et rina o et sara
Dan knows or ACC Rina or ACC Sara
"Dan knows either Rina or Sara"

3. Accordingly – no collectivity with separate ACC marker in a conjunction:

(28) dilan avar be-mispar ha-širim še katav et simon ve
Dylan exceeded in-number the-songs that wrote ACC Simon and
garfunkel
Garfunkel
"Dylan wrote more songs than Simon and Garfunkel" (ambiguous)

(29) dilan avar be-mispar ha-širim še katav et simon
Dylan exceeded in-number the-songs that wrote ACC Simon
ve et garfunkel
and ACC Garfunkel
"Dylan wrote more songs than both Simon and Garfunkel" (unambiguous)

D'/NP criterion I – obligatory/optional copula (see Doron 1983):

(30) a. I found John my strongest supporter.
b. I found John to be my strongest supporter.

(31) a. *I found my strongest supporter John.
b. I found my strongest supporter to be John.

Optional copula in Hebrew:

(32) ha-xavera haxi tova šeli (hi) mora.
the-friend most good of-I (is) teacher
"My best friend is a teacher"

(33) dana (hi) ha-mora, lo at!
Dana (is) the-teacher, not you
"Dana is the teacher, not you!"

Obligatory copula in Hebrew:

(34) a. *ha-xavera haxi tova šeli dana.
the-friend most good of-I Dana
b. ha-xavera haxi tova šeli <u>hi</u> dana.
the-friend most good of-I is Dana
"My best friend is Dana"

(35) a. *ha-mora at, lo dana!
the-teacher you, not Dana
b. ha-mora hi at, lo dana!
the-teacher is you, not Dana!
"The teacher is you, not Dana!"

(36) a. *dana eizo mora še-ani makir.
Dana some teacher that-I know

b. dana hi eizo mora še-ani makir.
Dana is some teacher that-I know
"Dana is some teacher I know"

Relations with wide scope indefinites (due to Doron):

(37) a. rina ša'ala im dani psantran še-šaxaxti et šmo
Rina asked if Dani pianist that-forgot-I ACC name-his
"Rina asked whether Dani was a pianist whose name I had forgotten"

b. rina ša'ala im dani <u>hu</u> psantran še-šaxaxti et šmo
Rina asked if Dani <u>is</u> pianist that-forgot-I ACC name-his
"Rina asked whether Dani was a pianist whose name I had forgotten" or: "There is a pianist whose name I forgot and Rina asked whether Dani was that pianist"

To summarize:

(38) Optional copula (NP):

a. definites

b. *a*/bare indefinites interpreted narrowest scope

(39) Obligatory copula (D'):

a. proper names and pronouns

b. *some*/*eize* indefinites (all scopes)

c. bare indefinites interpreted wide scope

Further prediction: collective conjunctions in predicative positions are possible only with an overt copula, because no category shifting can apply at the NP level.

(40) To my delight, I found my new student (to be) a first-rate pianist and a professional singer.

(41) a. *To my delight, I found my two new students a first-rate pianist and a professional singer.

b. To my delight, I found my two new students <u>to be</u> a first-rate pianist and a professional singer.

(42) dana (hi) soferet ve-mora.
Dana (is) author and-teacher
"Dana is an author and a teacher"

(43) a. *(shtey) ha-našim halalu soferet ve-mora.
(two) the-women these author and-teacher

b. (shtey) ha-našim halalu <u>hen</u> soferet ve-mora.
(two) the-women these <u>are</u> author and-teacher
"These (two) women are an author and a teacher"

D'/NP criterion II – appositional conjunction:

(44) a. A famous logician and a great author is the person who wrote "Alice's Adventures in Wonderland".

b. The famous logician and the great author is the person who wrote "Alice's Adventures in Wonderland".

(45) a. A famous logician and a great author have/*has met.

b. The famous logician and the great author have/*has met.

(46) a. *Charles Lutwidge Dodgson and Lewis Carroll is the person who wrote ”Alice’s Adventures in Wonderland”.

b. *Some famous logician and some great author is the person who wrote ”Alice’s Adventures in Wonderland”.

(47) a. Charles Lutwidge Dodgson and Lewis Carroll were/*was the same person.

b. Some famous logician and some great author were/*was the same person.

Explanation: Assume that [D’ and D’] is plural, but [NP and NP] is singular.

(48) NP conjunction (only analysis):
$[_{\text{D}'}\, f\, [_{\text{NP}}\ \text{famous logician}\ \cap\ \text{great author}\ ]]$

$\exists x$[famous logician(x) and great author(x) such that ... x ...]

(49) D’ conjunction:
$g(min[_{\text{D}'}\ [_{\text{D}'}\ f_1\, [_{\text{NP}}\ \text{famous logician}\ ]]\ \cap\ [_{\text{D}'}\ f_2\, [_{\text{NP}}\ \text{great author}\ ]]])$

$\exists x \exists y$[famous logician(x) and great author(y) such that ... $\{x, y\}$ ...]

Summary:

	DP (+Q)	D’ (±Q)	NP (−Q)
collectivity	−	+	−
wide scope	−	+	−
predicate position	−	+copula	−copula
conjunction number	plural	plural	singular

6 Wide scope indefinites and functional interpretation

Functional Readings:

(50) Who does every man love? His mother.

(51) The woman that every man loves is his mother.

Wide-scope indefinites:

(52) Every man loves some (a certain) woman I know.

(53) Every man who loves some (a certain) woman I know is quite normal.

(54) Every man will get crazy if some (a certain) woman I know comes in.

Plan of talk:

- Distributional and intuitive similarities between functional readings and WS indefinites.
- A unified Skolem-based variable-free mechanism using a novel type-shifting operator (similar to Jacobson 2002).
- Theorem: Characterization of closure properties under the proposed type-shifting rule in terms of generalized quantifiers.
- Hypothesis: the distribution of generalized quantifiers with functional readings is follows from this theorem.

6.1 Similarities between the two phenomena

An intuitive similarity:

(55) Every man loves a (certain) woman – his mother.
(Groenendijk and Stokhof (1984:ch.3))

(56) According to Freud, every man unconsciously wants to marry a certain woman – his mother. (Hintikka (1986))

A distributional similarity:

1. *WH elements*:

   (57) *Which philosopher* does every philosophy student admire? Her advisor.

   (58) Who will be offended if we invite *which philosopher*? John will be offended if we invite Putnam.

2. *Simple singular indefinites*:

   (59) A (certain)/some woman that every man would be happy to see again is his childhood sweetheart.

   (60) John will be happy if he meets a (certain)/some woman from his past.

3. *Simple numerals*:

   (61) Two women that every Frenchman admires are his mother and Brigitte Bardot.

   (62) John will be happy if he meets two women from his past.

4. *Definites*: see (51) and Winter (2001:ch.4).

5. *Unacceptable functional/WS interpretations*:

(63) a. ??*At most one/at least one woman* that every man loves is his mother.
b. ??*No/every woman* that every man would be happy to see again is his childhood sweetheart.
c. ??*Between two and three women* that every Frenchman admires are his mother, Brigitte Bardot and possibly Isabelle Adjani.

(64) John will be happy if *at most one/at least one/no/every/between two and three woman (women)* come(s) to the party. – no WS interpretation

6.2 Jacobson's analysis and Skolem functions

6.2.1 The problem in a nutshell

(65) a. A (certain) woman that every man loves is his mother.
b. Every man loves a (certain) woman.

(66) a. Some function in the set $\{f : f \textit{ maps every man to a woman he loves}\}$ is the mother function.
b. There is a choice function f such that every man loves $f(\{x : x \textit{ is a woman}\})$.

The problem: In (66a) functions are in the *restricting predicate* of the indefinite; in (66b) they *apply to* an ordinary restricting predicate (a set of e-type entities).

6.2.2 Jacobson's variable-free semantics of functional readings

The Z **operator**:

(67) $\mathrm{Z}_{(e(et))((ee)(et))} \stackrel{def}{=} \lambda R_{e(et)}.\lambda f_{ee}.\lambda x_e.R(f(x))(x)$

In words: the Z operator maps a binary relation R to the relation $\mathrm{Z}(R)$ that holds exactly between those ee-type functions f and entities x that satisfy $R(f(x))(x)$.
For instance:

(68) $\mathrm{Z}(\mathbf{love}')$ = the pairs $\langle f, x\rangle$ such that x loves $f(x)$.

(69) a. Every man loves his mother.
b. $\mathbf{every}'_{(et)((et)t)}(\mathbf{man}'_{et})(\mathrm{Z}(\mathbf{love}'_{e(et)})(\mathbf{his_mother}'_{ee}))$
$\Leftrightarrow \mathbf{man}' \subseteq \{x : \mathbf{love}'(\mathbf{his_mother}'(x))(x)\}$

The Z^0 **operator** –
A stipulative simplification of the compositional mechanism:

(70) $\mathrm{Z}^0{}_{(e(et))(((et)t)((ee)t))} \stackrel{def}{=} \lambda R_{e(et)}.\lambda Q_{(et)t}.\lambda f_{ee}.Q(\mathrm{Z}(R)(f))$

In words: the Z^0 function maps a binary relation R and a quantifier Q to the $\mathrm{Z}(R)$ relation saturated by Q.
For instance:

(71) a. (the woman that) *every man loves*
b. $\mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}'))$ = the set of functions f such that every man x loves $f(x)$.

The N **operator**:

(72) $\mathrm{N}_{(et)((ee)t)} \stackrel{def}{=} \lambda P_{et}.\lambda f_{ee}.\forall x_e[P(f(x))]$

For instance:

(73) $\mathrm{N}(\mathbf{woman}')$ = the set of ee functions f such that for every x, $f(x)$ is a woman.

$$
\begin{array}{c}
\underline{\iota(\mathrm{N}(\mathbf{woman}') \cap \mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))} \quad \underline{\lambda g_{ee}.g = \mathbf{his_mother}'} \\
\underline{\iota(\mathrm{N}(\mathbf{woman}') \cap \mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))} \\
\underline{\iota_{(\tau t)\tau}} \quad \underline{\mathrm{N}(\mathbf{woman}') \cap \mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}'))} \quad \underline{id_{\tau(\tau t)}} \quad \underline{\mathbf{his_mother}'_{ee}} \\
\underline{\mathrm{N}(\mathbf{woman}')} \quad \underline{\cap_{(\tau t)((\tau t)(\tau t))}} \quad \underline{\mathbf{every}'(\mathbf{man}')} \quad \underline{\mathrm{Z}^0(\mathbf{love}')} \\
\underline{\mathbf{woman}'} \quad \underline{\mathbf{that}} \quad \underline{\mathbf{every\ man}} \quad \underline{\mathbf{loves}} \quad \underline{\mathbf{his\ mother}} \quad \underline{\mathbf{is}} \\
\mathbf{the}
\end{array}
$$

Figure 2: Jacobson's derivation of meaning for sentence (51)

6.2.3 Skolem functions and functional readings

Definition 2 (Skolem functions of arity ≥ 1) *Let E and A be non-empty sets. A function f from $(\wp(E))^A$ to E^A is a* Skolem function *iff for every function $g \in (\wp(E))^A$, for every $x \in A$: if $g(x) \neq \emptyset$ then $(f(g))(x) \in g(x)$. If A is a Cartesian product of arity $n \geq 1$ then we say that f is a Skolem function of arity n.*

Using Currying, SFs of arity $n \geq 0$ are functions of type:

$$(\tau_1(...(\tau_n(\tau t))...))(\tau_1(...(\tau_n\tau)...)).$$

The set SK^n of general Skolem functions of this type is defined by:

(74) $SK^n \stackrel{def}{=} \lambda f.\forall g_{\tau_1(...(\tau_n(\tau t))...)}\forall x_1...\forall x_n$
$[g(x_1)...(x_n) \neq \emptyset \rightarrow (g(x_1)...(x_n))((f(g))(x_1)...(x_n))]$

SFs of arity 0 are also called **choice functions**.

How do we apply SFs to wide-scope indefinites?

(75) Every man loves a certain woman – Brigitte Bardot.

(Almost) no problem – a choice function can apply to the unary predicate $\mathbf{woman}'$:

(76) $\exists f[SK^0(f) \wedge \forall x[\mathbf{man}'(x) \rightarrow \mathbf{love}'(f(\mathbf{woman}'))(x)]]$
$\Leftrightarrow \exists y[\mathbf{woman}'(y) \wedge \forall x[\mathbf{man}'(x) \rightarrow \mathbf{love}'(y)(x)]]$
(provided the set of women is not empty)

But how about the following?

(77) Every man loves a certain woman – his mother.

Proposed solution:

- Apply Jacobson's N operator to $\mathbf{woman}'$ and get a set of functions: an object of type $(ee)t$.
- Apply a new operator from type $(ee)t$ to type $e(et)$ of **binary predicates**.
- Apply a Skolem function of arity 1.

Formally – the "Range" operator:

(78) $\mathrm{RG} \stackrel{def}{=} \lambda F_{(ee)t}.\lambda x_e.\lambda y_e.\exists f \in F[f(x) = y]$

In words: the RG operator maps a set of functions F to the relation R that holds between x and y if there is $f \in F$ s.t. $f(x) = y$.
In other words: $\mathrm{RG}(F)$ is the **union** of the functions in F under their relational presentation.

(79) $\mathrm{RG}(\mathrm{N}(\mathbf{woman}'))$ = the binary relation that maps every entity to the set of women

(80) $\exists f_{(e(et))(ee)}$
$[SK^1(f) \wedge \mathbf{every}'(\mathbf{man}')(\mathrm{Z}(\mathbf{love}')(f(\mathrm{RG}(\mathrm{N}(\mathbf{woman}')))))]$
$\Leftrightarrow\ \forall x[\mathbf{man}'(x) \rightarrow \exists y[\mathbf{woman}'(y) \wedge \mathbf{love}'(y)(x)]]$
(provided the set of women is not empty)

6.2.4 A unified account

An alternative to Jacobson's approach:

- No quantification <u>over</u> functions.
- General SFs <u>apply to</u> a standard restricting predicate when the DP is "non-quantificational" (=*flexible* in Winter's (2001) sense).

⇒ No functional/wide-scope readings with "quantificational" (=*rigid*) DPs – cf. (63)-(64).

⇒ A unified treatment of "functional" DPs and "wide-scope" DPs.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{a}}{f_{(e(et))(ee)}} \quad \dfrac{\dfrac{\text{woman}}{\mathrm{RG}(\mathrm{N}(\mathbf{woman}'))} \quad \dfrac{\text{that}}{\cap_{(\tau t)((\tau t)(\tau t))}} \quad \dfrac{\dfrac{\text{every man}}{\mathbf{every}'(\mathbf{man}')} \quad \dfrac{\text{loves}}{\mathrm{Z}^0(\mathbf{love}')}}{\mathrm{RG}(\mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))}}{\mathrm{RG}(\mathrm{N}(\mathbf{woman}')) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))} \quad \dfrac{\dfrac{\text{is}}{id_{\tau(\tau t)}} \quad \dfrac{\text{his mother}}{\mathbf{his_mother}'_{ee}}}{\lambda g_{ee}.g = \mathbf{his_mother}'}}{f(\mathrm{RG}(\mathrm{N}(\mathbf{woman}')) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))) = \mathbf{his_mother}'}}{\exists f_{(e(et))(ee)}[SK^1(f) \wedge f(\mathrm{RG}(\mathrm{N}(\mathbf{woman}')) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))) = \mathbf{his_mother}']}\ EC$$

Figure 3: Functional readings using SFs

6.3 Generalized quantifiers and functional readings

Potential problem: The RG operator is not a one-to-one mapping from $(ee)t$ to $e(et)$ – in general there is no such mapping on finite non-trivial domains. Thus, a SK^1 function f that applies to $\mathrm{RG}(F)$ does not necessarily give a member of F!
In other words: $f \circ \mathrm{RG}$ is not a choice function from sets of functions.

Hypothesis: The RG operator is applicable only to a subdomain of the $(ee)t$ domain, over which it is one-to-one. This is the subdomain of *closed sets of functions*.
If in a sentence S the derived set of functions is not closed, then RG is not applicable, and S does not have a functional reading.

(81) The/A woman that { (almost) every/no man (but John) / *at most one man / ?exactly/at least one man } loves is his mother.

(82) Which woman does { every/no man (but John) / *at most one man / ?exactly/at least one man } love? His mother.

		h_1	h_2	h_3	h_4
a	$\mapsto$	c	c	c	c
b	$\mapsto$	a	a	b	b
c	$\mapsto$	a	b	a	b

Table 1: some ee functions

Definition 3 (closed sets of functions) *Let $F \subseteq B^A$ be a set of functions from A to B. The* closure of *F is the set of functions $\overline{F} \subseteq B^A$ that is defined by:*

$$\overline{F} \stackrel{def}{=} \{f \in B^A : \textit{for every } x \in A \textit{ there is } g \in F \textit{ s.t. } f(x) = g(x)\}.$$

We call F a closed set of functions *if $F = \overline{F}$.*

Example: In table 1 –

$\{h_1, h_2\}$ is closed;

$\{h_1, h_4\}$ is not closed: its closure is $\{h_1, h_2, h_3, h_4\}$.

Query: Which sentences lead to closed sets of functions?

Answer (theorem): The set of functions $\mathrm{Z}^0(Q)(R)$ is guaranteed to be closed for any binary relation R iff the quantifier Q is **bounded**.

Definition 4 (bounded quantifiers) *A quantifier $Q \subseteq \wp(E)$ is* bounded by the sets X and $Y \subseteq E$ *iff* $Q = \{A \subseteq E : X \subseteq A \subseteq Y\}$.

Noun Phrase:	Quantifier bounded by:	
every student	$\mathbf{student}'$	E
no student	$\emptyset$	$E \setminus \mathbf{student}'$
every student but no teacher	$\mathbf{student}'$	$E \setminus \mathbf{teacher}'$
every student but Mary	$\mathbf{student}' \setminus \{\mathbf{m}'\}$	$E \setminus \{\mathbf{m}'\}$
no student but Mary	$\{\mathbf{m}'\}$	$E \setminus (\mathbf{student}' \setminus \{\mathbf{m}'\})$
almost every student	$\mathbf{student}' \setminus A_0$	$E \setminus A_0$
	specific A_0: $A_0 \cap \mathbf{student}' \neq \emptyset$	

Contra Sharvit (1999), we argue that cases like the following involve discourse anaphora.

(83) The woman that most of the students invited was their mother.

Viz.:

(84) Most of the students admire their mother. They invited her.

By contrast:

(85) No/at most one student hates his mother. *He invited her.

Conclusion: in general, functional readings are not always reducible to discourse anaphora, though some may be.

7 Choice functions and generalized quantifiers

(86) Some cow is red.

(87) $\exists f[CH(f) \wedge \mathbf{red}'(f(\mathbf{cow}'))]$

Problem: when there are no cows (87) turns out <u>true</u>, unlike (86).
Question: can we make (88) equivalent to standard existential quantification?
Answer: Without non-standard logics - no!

(88) $\exists f[C(f) \wedge B(f(A))]$

(89) $D^C \stackrel{def}{=} \lambda A.\lambda B.\exists f[C(f) \wedge B(f(A))]$

Proposition 1 *For every $C_{((et)e)t}$: if D^C is conservative then $D^C = 0$.*

Corollary 2 *For every $C_{((et)e)t}$: (88) is <u>not</u> equivalent to $A \cap B \neq \emptyset$*

Back to Montagovian individuals:

1. When the restriction set A is empty, the indefinite denotes the *empty quantifier* ($0_{(et)t}$). This guarantees falsity because B is trivially not in the empty quantifier.

2. When A is not empty, the indefinite denotes the standard Montagovian individual corresponding to $f(A)$. Namely: the principal ultrafilter $\lambda X.X(f(A))$.

(90) $\langle\,\rangle_{((et)e)((et)((et)t))} \stackrel{def}{=} \lambda g_{(et)e}.\lambda X_{et}.\lambda Y_{et}.X \neq \emptyset \wedge Y(g(X))$

Notational "sugaring": we write $\langle f \rangle$ instead of $\langle\,\rangle(f)$.

(91) $\exists f[CH(f) \wedge \langle f \rangle(A)(B)] \Leftrightarrow A \cap B \neq \emptyset$

More concretely, sentence (86) is now modeled as follows.

(92) $\exists f[CH(f) \wedge \langle f \rangle(\mathbf{cow}')(\mathbf{red}')]$
$\Leftrightarrow \exists f[CH(f) \wedge \mathbf{cow}' \neq \emptyset \wedge \mathbf{red}'(f(\mathbf{cow}'))]$
$\Leftrightarrow \exists x[\mathbf{cow}'(x) \wedge \mathbf{red}'(x)]$

References

Abney, S. P. (1987). *The English Noun Phrase in its Sentential Aspect*. PhD thesis, Massachusetts Institute of Technology.

Abusch, D. (1994). The scope of indefinites. *Natural Language Semantics*, 3:88–135.

Beghelli, F. (1995). *The Phrase Structure of Quantifier Scope*. PhD thesis, University of California Los Angeles.

Bennett, M. (1974). *Some Extensions of a Montague Fragment of English*. PhD thesis, University of California Los Angeles.

Chierchia, G. (2001). A puzzle about indefinites. In C. Cecchetto, G. C. and Guasti, M. T., editors, *Semantic Interfaces: reference, anaphora and aspect*. CSLI Publications, Stanford.

Cooper, R. (1975). *Montague's Semantic Theory and Transformational Syntax*. PhD thesis, University of Massachusetts at Amherst.

Corblin, F. (1997). Les indéfinis: variables et quantificateurs. *Langue Française*, 116:8–32.

Doron, E. (1983). *Verbless Predicates in Hebrew*. PhD thesis, The University of Texas at Austin.

Dowty, D. (1986). Collective predicates, distributive predicates and *all*. In *Proceedings of the Eastern States Conference on Linguistics, ESCOL3*. Cascadilla Press.

Farkas, D. (1981). Quantifier scope and syntactic islands. In *Papers from the 17th regional meeting of the Chicago Linguistic Society, CLS17*.

Fodor, J. D. and Sag, I. (1982). Referential and quantificational indefinites. *Linguistics and Philosophy*, 5:355–398.

Geurts, B. (2000). Indefinites and choice functions. *Linguistic Inquiry*, 31:731–739. Downloadable at `http://www.kun.nl/phil/tfl/bart/`.

Geurts, B. (2004). Specific indefinites, presuppositions and scope. In Bauerle, R., Reyle, U., and Zimmermann, T. E., editors, *Presuppositions and Discourse*. Elsevier, Amsterdam. To appear. Downloadable at `http://www.kun.nl/phil/tfl/bart/`.

Gillon, B. (1987). The readings of plural noun phrases in English. *Linguistics and Philosophy*, 10:199–219.

Groenendijk, J. and Stokhof, M. (1984). *Studies on the Semantics of Questions and the Pragmatics of Answers*. PhD thesis, University of Amsterdam.

Hendriks, H. (1993). *Studied Flexibility: categories and types in syntax and semantics*. PhD thesis, University of Amsterdam.

Hilbert, D. and Bernays, P. (1939). *Die Grundlagen der Mathematik II*. Springer, Berlin. Second edition, reprint 1970.

Hintikka, J. (1986). The semantics of *a certain*. *Linguistic Inquiry*, 17:331–336.

Jacobson, P. (1994). Binding connectivity in copular sentences. In *Proceedings of Semantics and Linguistic Theory, SALT4*. Downloadable from `http://www.cog.brown.edu/~pj/SALT_4-Jacobson.pdf`.

Kamp, H. and Reyle, U. (1993). *From Discourse to Logic: Introduction to Modeltheoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory*. Kluwer, Dordrecht.

Kratzer, A. (1998). Scope or pseudoscope? Are there wide scope indefinites? In Rothstein, S., editor, *Events and Grammar*. Kluwer, Dordrecht.

Kratzer, A. (2003). A note on choice functions in context. Downloadable from `http://semanticsarchive.net/Archive/zIyNTMxZ/Choice%20Functions%20in%20%Context.pdf`.

Link, G. (1983). The logical analysis of plurals and mass terms: a lattice theoretical approach. In Bauerle, R., Schwarze, C., and von Stechow, A., editors, *Meaning, Use and Interpretation of Language*. De Gruyter, Berlin.

Liu, F. (1990). *Scope Dependency in English and Chinese*. PhD thesis, University of California Los Angeles.

Montague, R. (1973). The proper treatment of quantification in ordinary English. In Hintikka, J., Moravcsik, J., and Suppes, P., editors, *Approaches to Natural Languages: proceedings of the 1970 Stanford workshop on grammar and semantics*. D. Reidel, Dordrecht. Reprinted in R. Thomason, editor (1974), *Formal Philosophy: selected papers of Richard Montague*, Yale, New Haven.

Neijt, A. H. (1979). *Gapping: a contribution to sentence grammar*. PhD thesis, Utrecht University.

Partee, B. (1987). Noun phrase interpretation and type shifting principles. In Groenendijk, J., de Jong, D., and Stokhof, M., editors, *Studies in Discourse Representation Theories and the Theory of Generalized Quantifiers*. Foris, Dordrecht.

Reinhart, T. (1992). Wh-in-situ: an apparent paradox. In *Proceedings of the 8th Amsterdam Colloquium*. University of Amsterdam, Institute for Logic, Language and Computation.

Reinhart, T. (1997). Quantifier scope: how labor is divided between QR and choice functions. *Linguistics and Philosophy*, 20:335–397.

Roberts, C. (1987). *Modal Subordination, Anaphora, and Distributivity*. PhD thesis, University of Massachusetts at Amherst.

Russell, B. (1919). *Introduction to Mathematical Philosophy*. George Allen and Unwin Ltd., London. Reprint 1953.

Ruys, E. G. (1992). *The Scope of Indefinites*. PhD thesis, Utrecht University.

Ruys, E. G. (1995). Weak crossover as a scope phenomenon. Unpublished ms., Utrecht University.

Scha, R. (1981). Distributive, collective and cumulative quantification. In Groenendijk, J., Stokhof, M., and Janssen, T. M. V., editors, *Formal Methods in the Study of Language*. Mathematisch Centrum, Amsterdam.

Schwarzschild, R. (1996). *Pluralities*. Kluwer, Dordrecht.

Schwarzschild, R. (2002). Singleton indefinites. *Journal of Semantics*, 19:289–314. Downloadable from `http://www3.oup.co.uk/semant/hdb/Volume_19/Issue_03/pdf/190289.pdf`.

Sharvit, Y. (1999). Functional relative clauses. *Linguistics and Philosophy*, 22:447–478.

Szabolcsi, A. (1997). Strategies for scope taking. In Szabolcsi, A., editor, *Ways of Scope Taking*. Kluwer, Dordrecht.

van der Does, J. (1993). Sums and quantifiers. *Linguistics and Philosophy*, 16:509–550.

Winter, Y. (1997). Choice functions and the scopal semantics of indefinites. *Linguistics and Philosophy*, 20:399–467.

Winter, Y. (2000). Distributivity and dependency. *Natural Language Semantics*, 8:27–69.

Winter, Y. (2001). *Flexibility Principles in Boolean Semantics: coordination, plurality and scope in natural language*. MIT Press, Cambridge, Massachusetts.

Winter, Y. (2002). Atoms and sets: a characterization of semantic number. *Linguistic Inquiry*, 33:493–505.

Winter, Y. (2003). Functional quantification. To appear in *Research on Language and Computation*. Downloadable from `http://www.cs.technion.ac.il/~winter/papers/fq.pdf`.