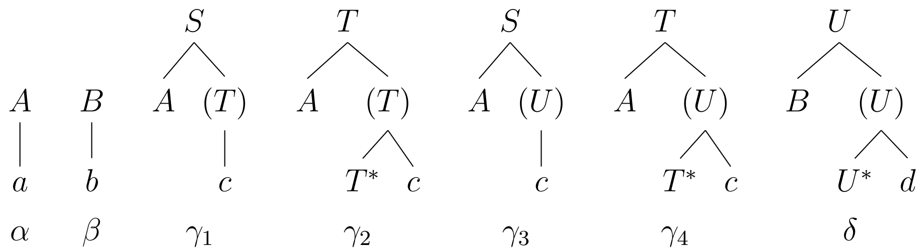
LMNLP 2012. Take-home assignment 4

These exercises go with Part 3 of the course. References are to the LIRa paper.

1

Below a TAG for crossed dependencies $a^n b^m c^n d^m$, and the corresponding **LG** type assignments. T' is a type such that $T' \vdash T$ but $T \not\vdash T'$, i.e. you can connect T' as a hypothesis to conclusion T, but not vice versa. Similarly for U', U.



$$
\begin{array}{rccl}
\alpha & a & :: & A \\
\beta & b & :: & B \\
\gamma_1 & c & :: & (T \oslash (A \backslash S)) \obslash T' \\
\gamma_2 & c & :: & T' \backslash ((T \oslash (A \backslash T)) \obslash T') \\
\gamma_3 & c & :: & (U \oslash (A \backslash S)) \obslash U' \\
\gamma_4 & c & :: & T' \backslash ((U \oslash (A \backslash T)) \obslash U') \\
\delta & d & :: & U' \backslash ((U \oslash (B \backslash U)) \obslash U')
\end{array}
$$

1. Using the links of Fig 3, draw the lexical unfolding for the γ_1 and δ type assignments.

2. Build a proof net for *abbcdd*: draw the abstract proof structure, and show how it can be rewritten to a tensor tree with the contraction of Figs 6–7 and the distributivity rules of Figs 8–9.

2

Below a focused proof for the sequent $a/b \cdot\otimes\cdot (a \oslash s^-) \obslash b \vdash \boxed{s^-}$, together with its proof term, according to the rules of Equations (7)–(11). Atomic type s is assigned negative polarity, atomic types a, b are positive.

$$\dfrac{\dfrac{\dfrac{\dfrac{a \overset{x_1}{\vdash} \boxed{a} \qquad \boxed{s^-} \overset{\alpha_0}{\vdash} s^-}{a \cdot \oslash \cdot s^- \vdash \boxed{a \oslash s^-}}\ \oslash R}{\boxed{a} \vdash a \oslash s^- \cdot \oplus \cdot s^-}\ \rightleftharpoons \qquad b \overset{z_0}{\vdash} \boxed{b}}{\boxed{a/b} \vdash (a \oslash s^- \cdot \oplus \cdot s^-) \cdot / \cdot b}\ /L}{a/b \cdot \otimes \cdot (a \oslash s^-) \obslash b \vdash \boxed{s^-}}\ \leftrightharpoons$$

$$(\dagger) \qquad \mu\alpha_0.(\frac{\beta_0\ z_0}{\mathsf{c2}}.\langle\ \mathsf{c1} \upharpoonleft (\tilde{\mu}x_1.\langle\ (x_1 \oslash \alpha_0) \upharpoonright \beta_0\ \rangle\ /\ z_0)\ \rangle)$$

1. In the table below, compute the target *types* for the CPS translation, according to the definition of $\lceil\cdot\rceil$ in Table 1, and the polarities of the atomic formulas.

CONSTANT	SOURCE TYPE	IMAGE UNDER $\lceil\cdot\rceil$
c1	a/b	
c2	$(a \oslash s^-) \varobslash b$	

2. Compute $\lceil\dagger\rceil$, the CPS translation of the proof *term*, according to Eq (16).

3. Assume a mapping $a^\ell = a$, $b^\ell = b$, $s^\ell = \perp^\ell = o$. Give $\cdot^\ell$ translations for the constants c1, c2 so that the composed mapping $\lceil\dagger\rceil^\ell$ reduces to

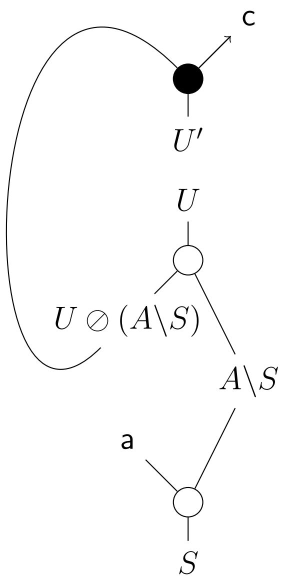
$$\lambda k.(k\ (\mathsf{f}\ (\mathsf{g}\ \mathsf{b}))$$

Use a constant $\mathsf{g}^{b\multimap a}$ in the translation of c1, and constants $\mathsf{f}^{a\multimap o}$ and b^b in the translation of c2.

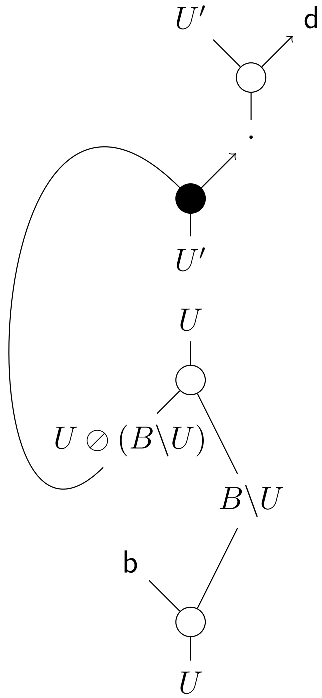
Solutions

1.1 On the left, the unfolding for γ_3 (a TAG initial tree), on the right the unfolding for δ (a TAG auxiliary tree), with the terminals a and b already substituted. For the other lexical entries, the geometry is the same — just change the labels.

The auxiliary δ is for the iteration of $b \ldots d$ pairs, γ_2 for iteration of $a \ldots c$, and γ_4 for switching from $a \ldots c$ to $b \ldots d$ iteration.



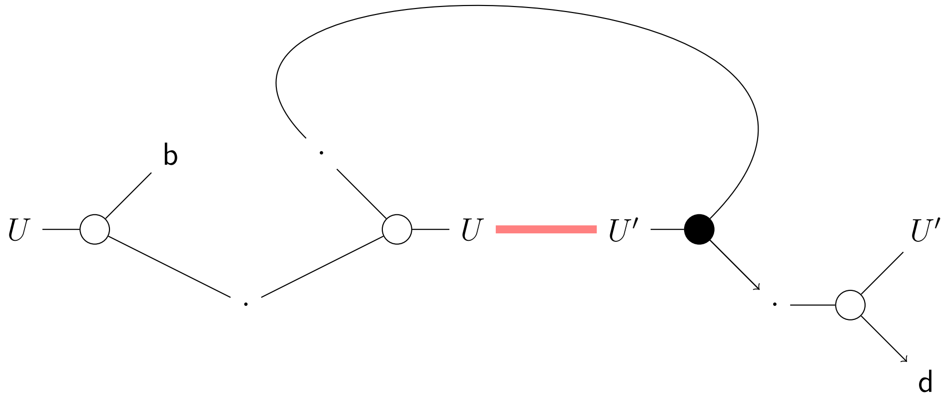
$\gamma_3 : (U \oslash (A \backslash S)) \odot U'$



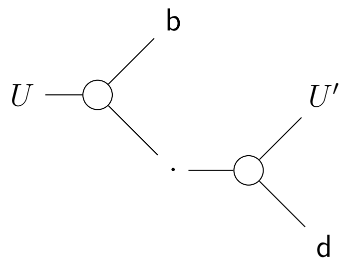
$\delta : U' \backslash ((U \oslash (B \backslash U)) \odot U')$

1.2 For the derivation of *abbcdd*, we use δ twice. Notice that δ can be internally connected (left) and then contracts to the tensor tree on the right (δ'). The reduced tree δ' can then be internally adjoined in a second copy of δ.

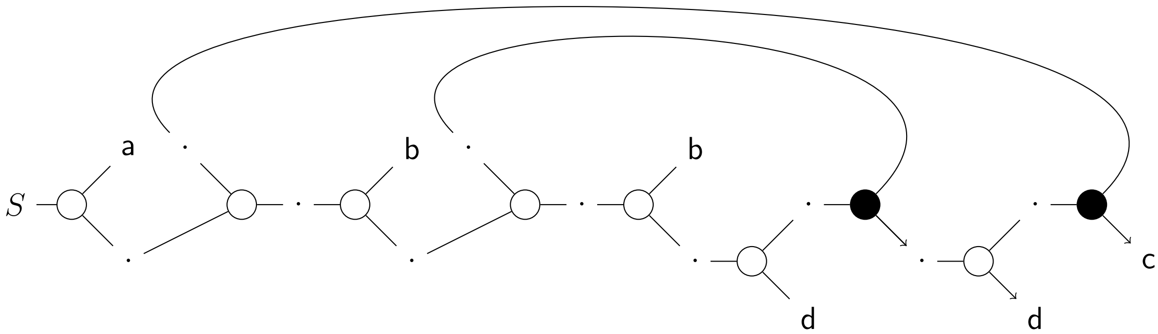
δ :

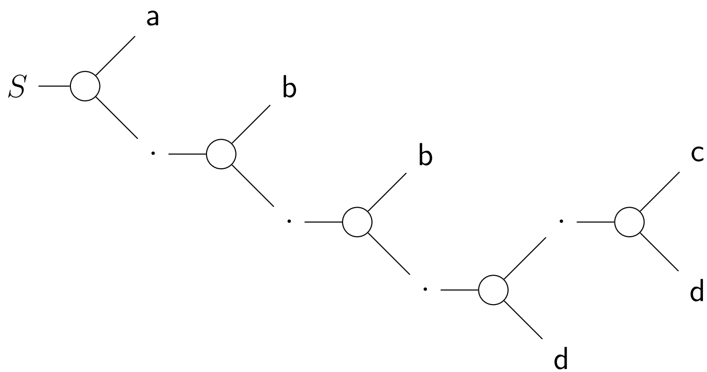


δ' :



On the left the abstract proof structure that results from adjoining $\delta + \delta'$ within γ_3. On the right the final tensor tree, after contractions and Grishin distribution.





If you like symbol manipulation, here is the corresponding focused proof.

$$\dfrac{s \overset{z_3}{\vdash} \boxed{s}}{\boxed{s} \vdash s^-}\ \rightleftharpoons$$

$$\dfrac{a \overset{\mathsf{a}}{\vdash} \boxed{a} \qquad \boxed{s} \vdash s^-}{\boxed{a\backslash s} \vdash a \cdot \backslash \cdot s^-}\ \backslash L$$

$$\dfrac{u \overset{y_3}{\vdash} \boxed{u} \qquad \boxed{a\backslash s} \vdash a \cdot \backslash \cdot s^-}{u \cdot \oslash \cdot (a \cdot \backslash \cdot s^-) \vdash \boxed{u \oslash (a\backslash s)}}\ \oslash R$$

$$\dfrac{u \cdot \oslash \cdot (a \cdot \backslash \cdot s^-) \vdash \boxed{u \oslash (a\backslash s)}}{\boxed{u} \vdash u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-)}\ \rightleftharpoons$$

$$\dfrac{b \overset{\mathsf{b}}{\vdash} \boxed{b} \qquad \boxed{u} \vdash u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-)}{\boxed{b\backslash u} \vdash b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))}\ \backslash L$$

$$\dfrac{u \overset{x_3}{\vdash} \boxed{u} \qquad \boxed{b\backslash u} \vdash b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))}{u \cdot \oslash \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))) \vdash \boxed{u \oslash (b\backslash u)}}\ \oslash R$$

$$\dfrac{u \cdot \oslash \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))) \vdash \boxed{u \oslash (b\backslash u)}}{\boxed{u} \vdash u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-)))}\ \rightleftharpoons$$

$$\dfrac{b \overset{\mathsf{b}}{\vdash} \boxed{b} \qquad \boxed{u} \vdash u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-)))}{\boxed{b\backslash u} \vdash b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))))}\ \backslash L$$

$$\dfrac{u \overset{z_2}{\vdash} \boxed{u} \qquad \boxed{b\backslash u} \vdash b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))))}{u \cdot \oslash \cdot (b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))))) \vdash \boxed{u \oslash (b\backslash u)}}\ \oslash R$$

$$\dfrac{u \cdot \oslash \cdot (b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))))) \vdash \boxed{u \oslash (b\backslash u)}}{\boxed{(u \oslash (b\backslash u)) \obslash u} \vdash b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))))}\ \rightleftharpoons$$

$$\dfrac{u \overset{z_1}{\vdash} \boxed{u} \qquad \boxed{(u \oslash (b\backslash u)) \obslash u} \vdash b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-))))}{\boxed{u\backslash((u \oslash (b\backslash u)) \obslash u)} \vdash u \cdot \backslash \cdot (b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-)))))}\ \backslash L$$

$$\dfrac{\boxed{u\backslash((u \oslash (b\backslash u)) \obslash u)} \vdash u \cdot \backslash \cdot (b \cdot \backslash \cdot (u \oslash (b\backslash u) \cdot \oplus \cdot (b \cdot \backslash \cdot (u \oslash (a\backslash s) \cdot \oplus \cdot (a \cdot \backslash \cdot s^-)))))}{\boxed{(u \oslash (b\backslash u)) \obslash u} \vdash (u \oslash (a\backslash s) \cdot \oplus \cdot (b \cdot \backslash \cdot (b \cdot \backslash \cdot (a \cdot \backslash \cdot s^-)))) \cdot / \cdot u\backslash((u \oslash (b\backslash u)) \obslash u)}\ \Leftarrow$$

$$\dfrac{u \overset{z_0}{\vdash} \boxed{u} \qquad \boxed{(u \oslash (b\backslash u)) \obslash u} \vdash (u \oslash (a\backslash s) \cdot \oplus \cdot (b \cdot \backslash \cdot (b \cdot \backslash \cdot (a \cdot \backslash \cdot s^-)))) \cdot / \cdot u\backslash((u \oslash (b\backslash u)) \obslash u)}{\boxed{u\backslash((u \oslash (b\backslash u)) \obslash u)} \vdash u \cdot \backslash \cdot ((u \oslash (a\backslash s) \cdot \oplus \cdot (b \cdot \backslash \cdot (b \cdot \backslash \cdot (a \cdot \backslash \cdot s^-)))) \cdot / \cdot u\backslash((u \oslash (b\backslash u)) \obslash u))}\ \backslash L$$

$$\dfrac{\boxed{u\backslash((u \oslash (b\backslash u)) \obslash u)} \vdash u \cdot \backslash \cdot ((u \oslash (a\backslash s) \cdot \oplus \cdot (b \cdot \backslash \cdot (b \cdot \backslash \cdot (a \cdot \backslash \cdot s^-)))) \cdot / \cdot u\backslash((u \oslash (b\backslash u)) \obslash u))}{a \cdot \otimes \cdot (b \cdot \otimes \cdot (b \cdot \otimes \cdot (((u \oslash (a\backslash s)) \obslash u \cdot \otimes \cdot u\backslash((u \oslash (b\backslash u)) \obslash u)) \cdot \otimes \cdot u\backslash((u \oslash (b\backslash u)) \obslash u)))) \vdash \boxed{s^-}}\ \leftrightharpoons$$

It may be helpful to compare the **LG** grammar with the ACG construction for this TAG. For a string interpretation, S is mapped to the string type σ $(* \multimap *)$, the adjunction nodes U, T to $\sigma \multimap \sigma$. The extra constants $\varepsilon_U, \varepsilon_T$ are there to terminate adjunction. We use the familiar abbreviation $+$ for concatenation/composition.

SOURCE		TARGET	
γ_1	$T \multimap S$	$\lambda f.(\mathtt{a} + (f\ \mathtt{c}))$	$(\sigma \multimap \sigma) \multimap \sigma$
γ_2	$T \multimap T$	$\lambda f \lambda x.(\mathtt{a} + (f\ (x + \mathtt{c})))$	$(\sigma \multimap \sigma) \multimap \sigma \multimap \sigma$
γ_3	$U \multimap S$	$\lambda f.(\mathtt{a} + (f\ \mathtt{c}))$	$(\sigma \multimap \sigma) \multimap \sigma$
γ_4	$U \multimap T$	$\lambda f \lambda x.(\mathtt{a} + (f\ (x + \mathtt{c})))$	$(\sigma \multimap \sigma) \multimap \sigma \multimap \sigma$
δ	$U \multimap U$	$\lambda f \lambda x.(\mathtt{b} + (f\ (x + \mathtt{d})))$	$(\sigma \multimap \sigma) \multimap \sigma \multimap \sigma$
ε_U	U	$\lambda x.x$	$\sigma \multimap \sigma$
ε_T	T	$\lambda x.x$	$\sigma \multimap \sigma$

Below the abstract proof term for the derivation of *abbcdd*.

$$\frac{\dfrac{\gamma_3}{s/u} \quad \dfrac{\dfrac{\delta}{u/u} \quad \dfrac{\dfrac{\delta}{u/u} \quad \dfrac{\varepsilon_u}{u}}{\delta \cdot \varepsilon_u \vdash u}[/E]}{\delta \cdot (\delta \cdot \varepsilon_u) \vdash u}[/E]}{\gamma_3 \cdot (\delta \cdot (\delta \cdot \varepsilon_u)) \vdash s}[/E]$$

$$\lambda i.(\mathtt{a}\ (\mathtt{b}\ (\mathtt{b}\ (\mathtt{c}\ (\mathtt{d}\ (\mathtt{d}\ i))))))$$

2.1 The effect of $\lceil\cdot\rceil$ on the types:

CONSTANT	SOURCE TYPE	IMAGE UNDER $\lceil\cdot\rceil$
c1	a/b	$(a^{\perp} \otimes b)^{\perp}$
c2	$(a \oslash s^{-}) \obslash b$	$(a \otimes s^{\perp})^{\perp} \otimes b$

2.2 The $\lceil\cdot\rceil$ translation of the proof term:

$$\lceil \mu\alpha_0.(\frac{\beta_0 \ z_0}{\mathsf{c2}}.\langle\ \mathsf{c1} \upharpoonleft (\tilde{\mu}x_1.\langle\ (x_1 \oslash \alpha_0) \upharpoonright \beta_0\ \rangle \ / \ z_0)\ \rangle)\rceil =$$

$$\lambda\widetilde{\alpha}_0.(\mathsf{case}\ \mathsf{c2}^{\ell}\ \mathsf{of}\ \langle\widetilde{\beta}_0, \widetilde{z}_0\rangle.(\mathsf{c1}^{\ell}\ \langle\lambda\widetilde{x}_1.(\widetilde{\beta}_0\ \langle\widetilde{x}_1, \widetilde{\alpha}_0\rangle), \widetilde{z}_0\rangle))$$

2.3 $\lceil\cdot\rceil^{\ell}$ translation of the constants yielding interpretation $\lambda k^{o \multimap o}.(k\ (\mathsf{f}^{a \multimap o}\ (\mathsf{g}^{b \multimap a}\ \mathsf{b}^{b})))$:

CONSTANT		
c1	$\lambda\langle c, y\rangle.(c\ (\mathsf{g}\ y))$	$((a \multimap o) \otimes b) \multimap o$
c2	$\langle\lambda\langle x, k\rangle.(k\ (\mathsf{f}\ x)), \mathsf{b}\rangle$	$(a \otimes (o \multimap o)) \multimap o) \otimes b$

This interpretation gives the $a \oslash s$ component of the constant c2 wide scope: the coimplication $a \oslash s$ behaves the same as an implication $a\backslash s$.

Compare:

$$\dfrac{\dfrac{\dfrac{\dfrac{a \overset{x_1}{\vdash} \boxed{a} \qquad \boxed{s^-} \overset{\alpha_0}{\vdash} s^-}{a \cdot \oslash \cdot s^- \vdash \boxed{a \oslash s^-}}\ \oslash R}{\boxed{a} \vdash a \oslash s^- \cdot \oplus \cdot s^-}\ \rightleftharpoons \qquad b \overset{z_0}{\vdash} \boxed{b}}{\boxed{a/b} \vdash (a \oslash s^- \cdot \oplus \cdot s^-) \cdot / \cdot b}\ /L}{a/b \cdot \otimes \cdot (a \oslash s^-) \oslash b \vdash \boxed{s^-}}\ \leftrightharpoons$$

$$\mu\alpha_0.(\tfrac{\beta_0\ z_0}{\mathsf{c2}}.\langle\ \mathsf{c1} \upharpoonleft (\widetilde{\mu}x_1.\langle\ (x_1 \oslash \alpha_0) \upharpoonright \beta_0\ \rangle \ /\ z_0)\ \rangle)$$

$$\lambda\widetilde{\alpha}_0.(\mathsf{case}\ \mathsf{c2}^\ell\ \mathsf{of}\ \langle\widetilde{\beta}_0, \widetilde{z}_0\rangle.(\mathsf{c1}^\ell\ \langle\lambda\widetilde{x}_1.(\widetilde{\beta}_0\ \langle\widetilde{x}_1, \widetilde{\alpha}_0\rangle), \widetilde{z}_0\rangle))$$

$$\lambda k_0.(k_0\ (\mathsf{f}\ (\mathsf{g}\ \mathsf{b})))$$

$$\dfrac{\dfrac{\dfrac{\dfrac{a \overset{y_0}{\vdash} \boxed{a} \qquad \boxed{s^-} \overset{\alpha_0}{\vdash} s^-}{\boxed{a\backslash s^-} \vdash a \cdot \backslash \cdot s^-}\ \backslash L}{\boxed{a} \vdash s^- \cdot / \cdot a\backslash s^-}\ \Leftarrow \qquad b \overset{\mathsf{c6}}{\vdash} \boxed{b}}{\boxed{a/b} \vdash (s^- \cdot / \cdot a\backslash s^-) \cdot / \cdot b}\ /L}{(a/b \cdot \otimes \cdot b) \cdot \otimes \cdot a\backslash s^- \vdash \boxed{s^-}}\ \leftrightharpoons$$

$$\mu\alpha_0.\langle\ \mathsf{c1} \upharpoonleft (\widetilde{\mu}y_0.\langle\ \mathsf{c5} \upharpoonleft (y_0 \setminus \alpha_0)\ \rangle \ /\ \mathsf{c6})\ \rangle$$

$$\lambda\widetilde{\alpha}_0.(\mathsf{c1}^\ell\ \langle\lambda\widetilde{y}_0.(\mathsf{c5}^\ell\ \langle\widetilde{y}_0, \widetilde{\alpha}_0\rangle), \mathsf{c6}^\ell\rangle)$$

$$\lambda k_0.(k_0\ (\mathsf{f}\ (\mathsf{g}\ \mathsf{b})))$$

For a *string* interpretation, one can consider an alternative ACG-style $\cdot^\ell$ mapping, sending all atomic source types (including $\bot$) to the string type σ. In the translations we now use constants `f`,`g`,`b` of type σ.

CONSTANT		
c1	$\lambda\langle c, y\rangle.(c\ \lambda i.(\mathtt{g}\ (y\ i)))$	$((\sigma \multimap \sigma) \otimes \sigma) \multimap \sigma$
c2	$\langle\lambda\langle x, k\rangle.(k\ \lambda i.(x\ (\mathtt{f}\ (\mathtt{b}\ i)))), \lambda i.i\rangle$	$(\sigma \otimes (\sigma \multimap \sigma)) \multimap \sigma) \otimes \sigma$

The $\lceil\cdot\rceil^\ell$ translation of the proof term (†) then is

$$\lambda k.(k\ \lambda i.(\mathtt{g}\ (\mathtt{f}\ (\mathtt{b}\ i))))$$