Tree Adjoining Grammars

Logical Methods in Natural Language Processing
Lecturer: M. Moortgat

Cecilia Chávez Aguilera

May 3, 2012

Joshi’s Program

- Joshi, Aravind K., Leon S. Levy, and Masako Takahashi. 1975. ‘Tree Adjunct Grammars.’*Journal of Computer and System Sciences*, 10(2)

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- The string language generated by G is defined to be the set

$$L(G) = \{y(t)|t \in T(G)\}$$

where $T(G)$ is the set of all derived trees of G, and $y(t)$ is the unique string associated with t, (the yield of t) obtained by concatenating all terminal symbols labeling the frontier of t from left to right.

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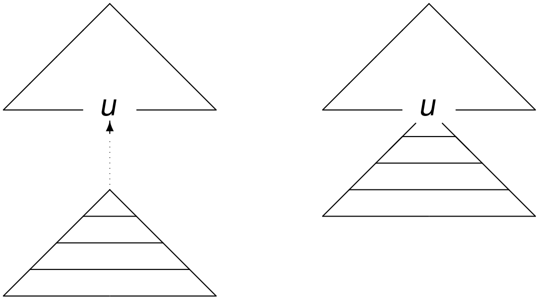
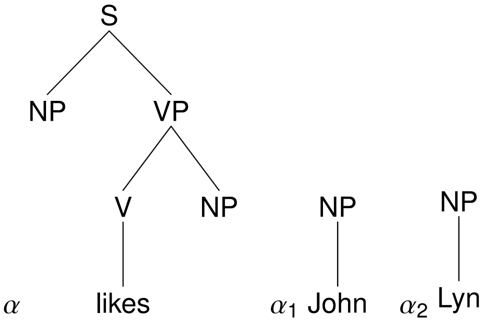
- This operation can be iterated as long as there is a substition node.

Substitution

- In a picture



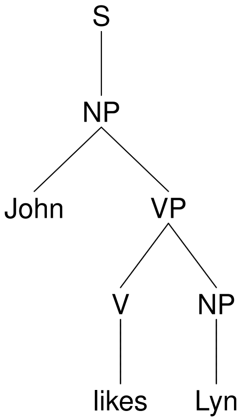
Example



Substituting

Derived tree

$$\alpha\,[NP, \alpha_1];\, \alpha\,[NP, \alpha_2]$$



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- This operation can always be iterated

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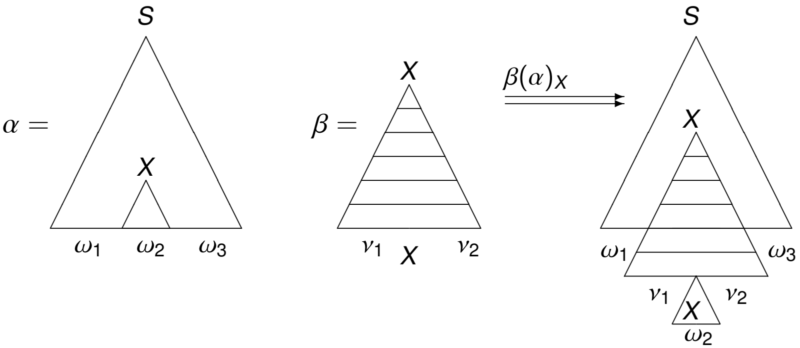
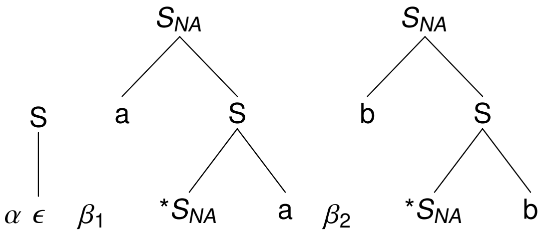


Example

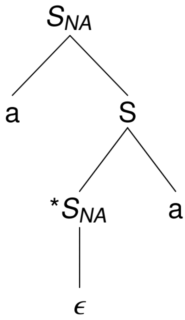
Copy Language



Adjunction

Derived tree

$\beta_1(\alpha)_S$



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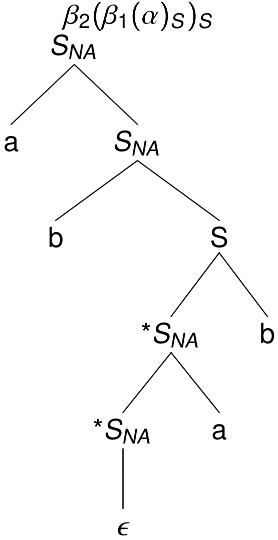

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- Constant growth property ?

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- Altough here and there it has been stated that TAG’s are closed under strong lexicalization, recently, it has been shown that TAG’s are NOT closed under strong lexicalization:
  Kuhlman and Satta ‘Tree-Adjoining Grammars are not closed under strong lexicalization’*Computational Linguistics* 2012

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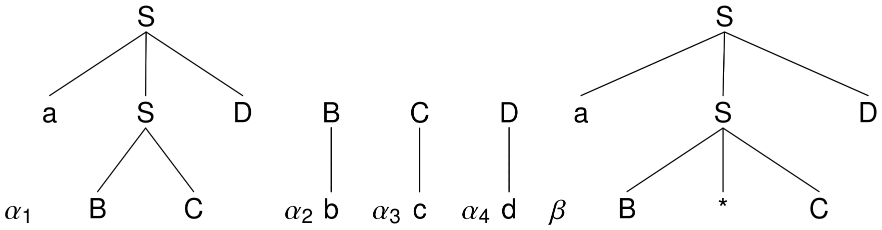
- Derivation Trees are terms over the signature of elementary trees.

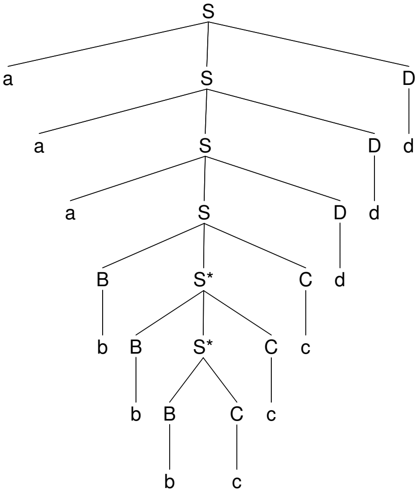
$\{a^n b^n c^n d^n | n \in \mathbb{N}\}$

Running Example



Derived Tree

aaabbbccc



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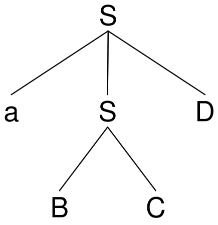
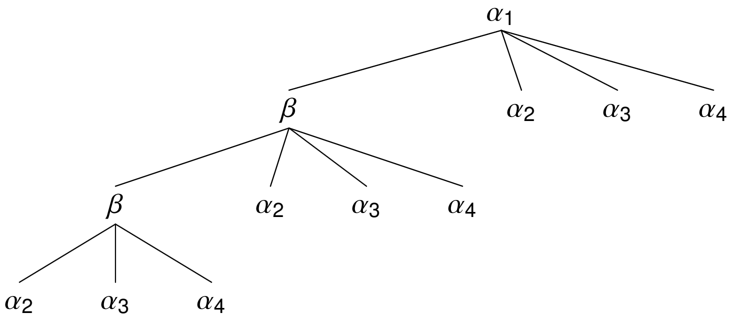
- α_1 

  aS_1BCS_2D

Term



- Recall that an order anotation ω is well-nested if it does not contain a string of the form *ijij* as a scattered substring, for $i \neq j \in \mathbb{N}$ Denote by Ω_{wn} the set of all well-nested order anotations.

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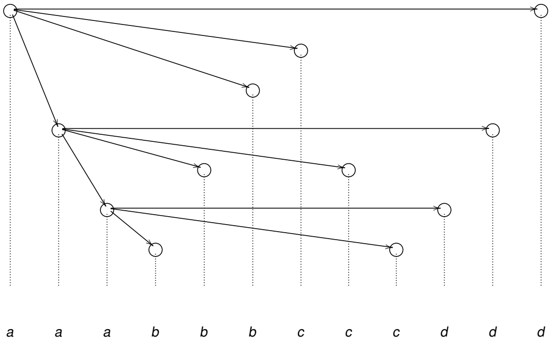
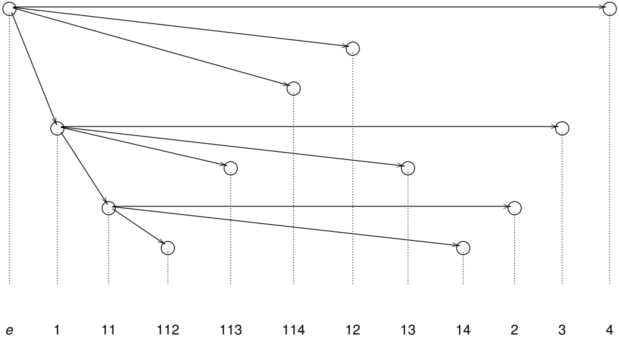
- **Theorem 5.2.1** A dependency structure $\mathcal{D}$ is well-nested if and only if $term(\mathcal{D}) \in T_{\Omega_{wn}}$


a a a b b b c c c d d d

Linearization



Block Degree

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- Thus the block degree of our dependency structure is two.

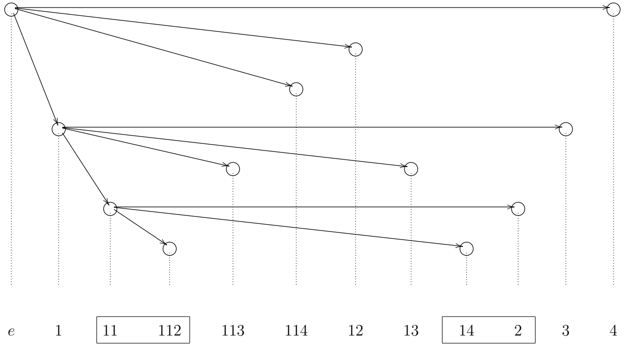

Figure 1: Blocks of node 11

Thanks