Logical Methods in Natural Language Processing On some properties of the sequent calculus for **NL** and **NL**$\diamondsuit$

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Abstract

The familiar properties of a relation of consequence have been generalized to hold independently of the proof calculus of a logic. In this paper, we show that those properties hold for **NL** and **NL**$\diamondsuit$ and we point out some applications, as well as some future work

The properties of a relation of consequence that were postulated for the first time by Tarski [16, 17, 18] caused great controversy when new substructural or non-monotonic logics emerged. However, new work trying to answer the question What is a logic? have bring them back into scene. This re-born of the tarskian characterization of the consequence relation took place in the very complicated world of Category Theory.

Then, the plan of the paper will be to give a very general sketch of how those properties are used. However, for the interested reader, we provide the appropriate bibliography to get the whole formalization. This constitutes the Motivation section. We proceed then to briefly present the calculus with which we are concern. The core of the paper is the proof of the mentioned properties for **NL** and **NL**$\diamondsuit$. Finally we provide what can be an application of this proof.

1 Motivation

During the decade of the 80's (past century), a line of research looking for a definition of a logical system was particularly flourishing. [3, 5, 10, 11, 15] The main tool used in achieving this formalization was a couple of ideas: *institution*[5] and $\pi-institution$[3]. The first of them works with the semantics of a logic, while the second does it with its proof calculus. Later, both ideas were put together to constitute the formalism called *General Logic* [11, 10].

A General logic is based on the idea of giving a fix abstract formalization that can be suitable to model, translate or rewrite any logic just by giving its signature. A signature Σ is a tuple that determines the non-logical symbols of a system. For instance, for first order logic $\Sigma_{FOL} = (F, P)$ where F is a set of ranked function symbols and P a ranked alphabet of predicate symbols. For the logics that concern us here, a signature $\Sigma_{NL} = (\mathcal{A}, \mathcal{T})$ can be given by a set of atomic types and a set of composed types.

Everything is settle within a category theory framework, thus signatures form a category, whose objects are signatures, and whose morphisms are signature morphisms which preserves the relevant properties in question. For instance for the first example, preserving rank of symbols, for our second example, preserving types. Those morphisms induce a functor $sen(\Sigma)$ which gives the set of sentences of a given signature and its translation into a target signature.

These very general ideas are enough to sketch what a $\pi-$institution or entailment system is. An entailment system[11, 10] is a triple $\epsilon = (\mathbf{Sign}, sen, \vdash)$ with **Sign** a category of signatures, $sen : \mathbf{Sign} \to \mathbf{Set}$ a functor associating to each signature Σ in **Sign** its set of sentences, and $\vdash$, is taken to be a function associating to each signature Σ, a relation $\vdash_\Sigma \subseteq \wp(sen(\Sigma)) \times sen(\Sigma)$, the entailment relation of Σ satisfying:

1. Reflexivity: For all $\varphi \in SEN(\Sigma)$, $\{\varphi\} \vdash_\Sigma \varphi$.
2. Monotony: If $\Gamma \vdash_\Sigma \varphi \ y \ \Gamma \subseteq \Gamma',\ then\ \Gamma' \vdash_\Sigma \varphi$.
3. Transitivity: If $\Gamma \vdash_\Sigma \varphi_i, i \in I$, and $\Gamma \cup \{\varphi_i | i \in I\} \vdash_\Sigma \psi,\ then\ \Gamma \vdash_\Sigma \psi$.
4. $\vdash$-traductibility: If $\Gamma \vdash_\Sigma \varphi$ then for any $H \in Mor(\mathbf{Sign})$, $H : \Sigma \to \Sigma'$:
$$SEN(H(\Gamma)) \ \vdash_{\Sigma'} \ SEN(H(\varphi)).$$

Last condition can be seen as a generalization of the requirement that Łos and Susko [9] added to the tarskian characterization of a consequence relation. Thus, the only thing we want to focus on is the fact, that first three properties should hold for any $\vdash$ relation, independently of the proof calculus generating it. For logical systems like linear logic [4], in which we do not have weakeing nor contraction, it may look straightforward impossible to fulfill. However, given a signature of linear logic Σ_{LL} we have to take as objects of $sen(\Sigma_{LL})$ sequents, not formulas. [11, 10]. Then, $\vdash_{LL}$ should be identified with the horizontal bar of the closure of a derivation [10, p.284]

A bit more precisely, we can say that given a set $\Phi = \{(\Gamma \Rightarrow \Delta)_i\}_{i \in I}$ of sequents in $sen(\Sigma_{LL})$ and a sequent $\Gamma_0 := \Gamma' \Rightarrow \Delta' \ \in sen(\Sigma_{LL})$, $\Phi \vdash_{LL} \Gamma_0$ iff there exists a derivation δ such that its root is Γ_0 and its branches contain sequents of Φ

As Lambek calculus [7] as well some extensions of it used in typelogical grammars [13] are directly linked with linear logic, it is natural to ask whether the calculus for these logics satisfies the aforementioned properties. In the following, we prove they hold as well for **NL** [8] and **NL**$\Diamond$ [12]

2 NL

Let´s recall the Gentzen presentation for the **NL** calculus. Here we follow the presentation given in [13]

Given a set $\mathcal{A}$ of atomic types, the language $\mathcal{F}$ of a simple Lambek system is given by:

$$\mathcal{F} := \mathcal{A} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F} \bullet \mathcal{F} \mid \mathcal{F} \backslash \mathcal{F}$$

For the Gentzen presentation, derivability is given between a term $\mathcal{T}$ and a type formula, where

$$\mathcal{T} := \mathcal{F} \mid (\mathcal{T}, \mathcal{T})$$

Thus, a sequent is a pair (Γ, A) where $\Gamma \in \mathcal{T}$ and $A \in \mathcal{F}$, and is denoted $\Gamma \Rightarrow A$

The Gentzen presentation is given by:

$$\frac{}{A \Rightarrow A}\ Ax \qquad \frac{\Delta \Rightarrow A \quad \Gamma[A] \Rightarrow C}{\Gamma[\Delta] \Rightarrow C}\ Cut$$

$$\frac{(\Gamma, B) \Rightarrow A}{\Gamma \Rightarrow A/B}\ /R \qquad \frac{\Delta \Rightarrow B \quad \Gamma[A] \Rightarrow C}{\Gamma[(A/B, \Delta)] \Rightarrow C}\ /L$$

$$\frac{(B, \Gamma) \Rightarrow A}{\Gamma \Rightarrow B\backslash A}\ \backslash R \qquad \frac{\Delta \Rightarrow B \quad \Gamma[A] \Rightarrow C}{\Gamma[(\Delta, B\backslash A)] \Rightarrow C}\ \backslash L$$

$$\frac{\Gamma[(A, B)] \Rightarrow C}{\Gamma[A \bullet B] \Rightarrow C}\ \bullet L \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{(\Gamma, \Delta) \Rightarrow A \bullet B}\ \bullet R$$

Thus, let's fix some basic notation, already mentioned. Let $\Sigma_{NL} = (\mathcal{A}, \mathcal{T})$ be a **NL** signature. Let $sen(\Sigma_{NL})$ be the set of sequents over Σ_{NL}. We will denote by $\Gamma_i = (\Gamma, A)$ an arbitrary sequent in $sen(\Sigma_{NL})$, by δ_i, a derivation of Γ_i in **NL**, by Φ a set of sequents in $sen(\Sigma_{NL})$. Let $\Vdash_{NL}$ be the following relation:

$\Phi \Vdash_{NL} \Gamma_i$ iff there exists δ_i a derivation of Γ_i in **NL** such that for any sequent $\Delta_i \in \delta_i$, $\Delta_i \in \Phi$.
Then we want to show that

1. $\Phi, \Gamma_i \Vdash_{NL} \Gamma_i$
2. If $\Phi \Vdash_{NL} \Gamma_i$, and $\Phi, \Gamma_i \Vdash_{NL} \Delta_i$, then $\Phi \Vdash_{NL} \Delta_i$
3. If $\Phi \Vdash_{NL} \Gamma_i$, then $\Phi, \Psi \Vdash_{NL} \Gamma_i$, for some Ψ set of sequents

Then, for 1, given any $\Gamma_i = (\Gamma, A)$, such that there is a derivation δ_i in **NL** with set of sequents Φ,

$$\frac{\Gamma \Rightarrow A \quad A \Rightarrow A}{\Gamma \Rightarrow A}$$

is always a derivation of Γ_i such that $\Phi, \Gamma_i \Vdash_{NL} \Gamma_i$

Now, let´s assume that property 2 holds for the premises of the rules of the calculus given. Let´s show by induction on the derivation that it holds for the conclusion. Assume that $\Phi \Vdash_{NL} \Gamma_0$, with derivation δ_1 and $\Phi, \Gamma_0 \Vdash_{NL} \Delta_0$ with derivation δ_2 for $\Gamma_0 = (\Gamma', A')$, $\Delta_0 = (\Delta', B')$

Then we have two main cases: either the last rule used in δ_2 introduces the main connective of B' or not.

Case 1 .- The last rule R used in δ_2 does not introduce the main connective of B' Then the following rules should be considered:

- For any application of [Ax], it holds vacuously, as then $\Phi = \varnothing$, and Γ_0 is the empty sequent
- If $R = [Cut]$, then we have the following:

  $\Delta_0 = \Gamma[\Delta'] \Rightarrow C$ $\Gamma_0 = \Gamma[X] \rightarrow C$, where

$$\frac{\Delta' \Rightarrow X \quad \Gamma_0 = \Gamma[X] \rightarrow C}{\Delta_0 = \Gamma[\Delta'] \Rightarrow C}$$

  for some $X, A, B, C \in \mathcal{F}$, $\Gamma', \Delta' \in \mathcal{T}$

  as **NL** has cut-elimination (see [7]), we can find a cut $X \Rightarrow Y$, $\Gamma[Y] \Rightarrow C$ of lower degree by which we can obtain Γ_0. By inductive hypothesis non of those sequents will be Γ_0 Then, by transitiviy, we have $\Delta' \Rightarrow Y$ and applying cut with $\Gamma[Y] \Rightarrow C$ we obtain Δ_0, thus $\Phi \Vdash_{NL} \Delta_0$

  For the remaining cases recall that any other rule have the subformula property [14] i.e. either the types in the conclusion are the same than the types in the premises, or are the immediate subtypes of the active formula.

- Thus if $R = [\bullet L]$, then conclusion and premise have the same types and: $\Delta_0 = \Gamma'_1[(A \bullet B)] \Rightarrow C$, and $\Gamma_0 = \Gamma'_1[(A, B)] \Rightarrow C$ for some $A, B, C \in \mathcal{F}$, $\Gamma'_1 \in \mathcal{T}$

  Then we have the following subcases for δ_1: Γ_0 could have been obtained by $[\bullet R], [\backslash L]$ or $[/L]$

  - If $[\bullet R]$ was applied, then there exists $\Gamma'_0, \Gamma''_0 \in \Phi$ such that $\Gamma'_0 = (A, X)$, $\Gamma''_0 = (B, Y)$ and

$$\frac{A \Rightarrow X \quad B \Rightarrow Y}{(A, B) \Rightarrow X \bullet Y}$$

    was the last step in δ_1, with $\Gamma'_1 = \varnothing$, $C = X \bullet Y$
    Then, by monotonicity we can have $A \bullet B \Rightarrow X \bullet Y$ Thus $\Phi \Vdash_{NL} \Delta_0$
  - $[/L]$ was applied. Then there exists $\Gamma'_0, \Gamma''_0 \in \Phi$ such that
    $\Gamma'_0 = B \Rightarrow X$, $\Gamma''_0 = \Gamma'_1[Y] \Rightarrow C$ and such that

$$\frac{B \Rightarrow X, \quad \Gamma'_1[Y] \Rightarrow C}{\Gamma'_1[A, B] \Rightarrow C}$$

was the last step in δ_1

Then we can obtain:

$$\cfrac{\cfrac{\cfrac{Y \Rightarrow Y \quad B \Rightarrow X}{Y/X \Rightarrow Y/B} \qquad}{B \Rightarrow B}}{\cfrac{(Y/X) \bullet B \Rightarrow (Y/B) \bullet B}{(Y/X) \bullet B \Rightarrow Y}}$$

finally applying cut with $\Gamma_1'[Y] \Rightarrow C$ we have Δ_0 Thus, $\Phi \Vdash_{NL} \Delta_0$

- $[\backslash L]$ can be handled analogously
- $[Cut]$ follows by the case treated before.

If the types in the premises are the immediate subtypes of Δ_0, then either $[\backslash L], [/L]$ were applied. Then we have the following scenario:

$\Delta_0 = \Gamma_1'[(A/B), \Delta_1'] \Rightarrow C$

$\Gamma_0 = \Gamma_1'[A] \Rightarrow C$ and there exists Γ_0',

$\Gamma_0' = \Delta_1' \Rightarrow B$ for some $A, B, C \in \mathcal{F}$, $\Gamma_1', \Delta_1' \in \mathcal{T}$ and such that

$$\frac{\Gamma_1'[A] \Rightarrow C \quad \Delta_1' \Rightarrow B}{\Gamma_1'[(A/B), \Delta_1'] \Rightarrow C}$$

was the last step in δ_2. Then, by its configuration, we may assume that Γ_0 could have been obtained by $[Cut]$. Thus, by inductive hypothesis it can be handled with the previous case done by that rule and this is analogous in the case of $[/L]$

Case 2.- The rule applied introduces the main connective . Then we have to consider the following cases:

- $R = [/R]$ Then

  $\Delta_0 = \Gamma_1 \Rightarrow A/B$

  $\Gamma_0 = (\Gamma_1, B) \Rightarrow A$

  and the last step in δ_2 was

$$\frac{(\Gamma_1, B) \Rightarrow A}{\Gamma_1 \Rightarrow A/B}$$

By inductive hypothesis we may assume there exists Γ_0', Γ_0'' such that $\Gamma_0' = (\Gamma_1, B) \Rightarrow X$, $\Gamma_0'' = X \Rightarrow A$ and non of them is Γ_0

Thus we can have $(\Gamma_1 \bullet B) \Rightarrow X$ from $(\Gamma_1, B) \Rightarrow X$, by $[\bullet L]$ and $(\Gamma_1 \bullet B) \Rightarrow A$ by transitivity, and finally $\Gamma_1 \Rightarrow A/B$ Then $\Phi \Vdash_{NL} \Delta_0$

- The case for $[\backslash R]$ is analogous
- $[\bullet R]$ We have that $\Delta_0 = (\Gamma_1, \Delta_1) \Rightarrow A \bullet B$ $\Gamma_0 = (\Delta_1 \Rightarrow B)$ and there exists $\Gamma_0' = (\Gamma_1 \Rightarrow A)$ such that

$$\frac{\Gamma_1 \Rightarrow A \quad \Delta_1 \Rightarrow B}{(\Gamma_1, \Delta_1) \Rightarrow A \bullet B}$$

is the last step of δ_1

Then we may assume there exists Γ_0' and Γ_0'' with $\Gamma_0' = \Delta_1 \Rightarrow X$ $\Gamma_0'' = X \Rightarrow B$ for some X and different from Γ_0 from which Γ_0 was obtained then we can have:

$A \Rightarrow A$, $X \Rightarrow B$ by $[Ax]$ and Hypothesis

$A \bullet X \Rightarrow A \bullet B$ by monotonicity

$\Delta_1 \Rightarrow X \quad X \Rightarrow (A \bullet B)/A$ by hypothesis and property of $\bullet$

$\Gamma_1 \Rightarrow A \quad \Delta_1 \Rightarrow (A \bullet B)/A$ by hypothesis and transitivity

$(\Gamma_1, \Delta_1) \Rightarrow ((A \bullet B)/A) \bullet A$ by $[\bullet R]$

$(\Gamma_1, \Delta_1) \Rightarrow A \bullet B$ by application

Thus $\Phi \Vdash_{NL} \Delta_0$

Now, finally for weakening, we have that such a property holds. Strictly speaking nothing force me to use a sequent, as well we have some fool uses of a sequent. However, in practice we always avoid such derivations, and if we restrict ourselves to 'clean'derivations, there's no way for it to hold as the calculus keeps strict record of the rules used. Thus, theoretically speaking, this property holds and allows us to see this calculus within the class of calculus, in practice, any weakened proof will be avoided.

3 NL $\diamondsuit$

NL $\diamondsuit$ can be seen as the base case of a number of extensions of Lambek calculus made by Moortgat [13] In analogy with linear logic, new operators are introduced to bring back to the calculus structural properties in a controlled way. Here we present this extension following [13]

The class of type-formulas is extended with the following operators:

$$\mathcal{F} := \mathcal{A} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F} \bullet \mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \Diamond\mathcal{F} \mid \Box\mathcal{F}$$

then, the class of terms is also extended with unary operator:

$$\mathcal{T} := \mathcal{F} \mid (\mathcal{T}, \mathcal{T}) \mid (\mathcal{T})^{\diamond}$$

and the following rules are added to **NL**

$$\frac{\Gamma \Rightarrow A}{(\Gamma)^{\diamond} \Rightarrow \Diamond A}\,\Diamond R \qquad \frac{\Gamma[(A)^{\diamond}] \Rightarrow B}{\Gamma[(\Diamond A)^{\diamond}] \Rightarrow B}\,\Diamond L$$

$$\frac{(\Gamma)^{\diamond} \Rightarrow A}{\Gamma \Rightarrow \Box A}\,\Box R \qquad \frac{\Gamma[A] \Rightarrow B}{\Gamma[(\Box A)^{\diamond}] \Rightarrow B}\,\Box L$$

we have the residuation laws $\Diamond \Box A \Rightarrow A$ and $A \Rightarrow \Box\Diamond A$

Thus, we have half of the way done by the previous section, and now we need to prove the properties in question for the remaining rules. Note that here we will have reflexivity by the same reasoning as in the previous section. Also, as before, transitivity is the case that requieres more work. However, incidentally, we found a shortcut. We will make use of the following lemma due to Jäger See [6, p.52]

LEMMA Let $X[Y] \to A$ be a theorem of $\mathbf{NL}\Diamond$. Then there is a type B such that

1. $\mathbf{NL}\Diamond \vdash Y \Rightarrow B$
2. $\mathbf{NL}\Diamond \vdash X[B] \Rightarrow A$
3. There is a type occuring in $X[Y] \Rightarrow A$ which contains at least as many connectives as B

Thus assume

$\Phi \Vdash_{\mathbf{NL}\Diamond} \Gamma_0$ by a derivation δ_1 and
$\Phi, \Gamma_0 \Vdash_{\mathbf{NL}\Diamond} \Delta_0$ by a derivation δ_2
we want to show that $\Phi \Vdash_{\mathbf{NL}\Diamond} \Delta_0$

Note that as the new term operator $(\cdot)^{\diamond}$ will only be involved if the new rules are used, we may safely restrict to prove everything just for these rules.

If $[\Diamond R]$ was used in the last step of δ_2 then we have that

$\Delta_0 = (\Gamma)^\diamond \Rightarrow \Diamond A$

$\Gamma_0 = \Gamma \Rightarrow A$ for some $\Gamma \in \mathcal{T}, A \in \mathcal{F}$

by the lemma stated we know that there exists sequents Γ_0', Γ_0'' such that $\Gamma_0' = \Gamma \Rightarrow X$ and $\Gamma_0'' = X \Rightarrow A$. By inductive hy‘pothesis we know that non of those is Γ_0 Then form $\Gamma \Rightarrow X$ we obtain $(\Gamma)^\diamond \Rightarrow \Diamond X$ by $[\Diamond R]$ form $X \Rightarrow A$ we obtain $\Diamond X \Rightarrow \Diamond A$ by isotonicity of $\Diamond$ and finally by transitivity we obtain $(\Gamma)^\diamond \Rightarrow \Diamond A$ $\Phi \Vdash_{\mathbf{NL}\Diamond} \Delta_0$

If $[\Box R]$ was applied then we have that

$$\frac{\Gamma_0 = (\Gamma)^\diamond \Rightarrow A}{\Delta_0 = \Gamma \Rightarrow \Box A}$$

was the last step done in δ_2. By the lemma we know that there exists sequents $\Gamma \Rightarrow X$, $(\Gamma)^\diamond \Rightarrow \Diamond X$, and $\Diamond X \Rightarrow A$ from which Γ_0 was obtained by cut. By inductive hypothesis we may assume that non of those is equal to Γ_0 then $\Diamond X \Rightarrow A$ implies $X \Rightarrow \Box A$ by the residuation law. By transitivity with $\Gamma \Rightarrow X$ we obtain $\Gamma \Rightarrow \Box A$ which is what we wanted

For the following two cases, we can say that were proved by Jäger [6]. Of course, his lemma and proof were used for very different purposes and in a very different context, however, it happens that he actually proved what we wanted to prove for these cases:

$[\Diamond L]$ was used, then

$$\frac{\Gamma[(A)^\diamond] \Rightarrow B = \Gamma_0}{\Gamma[\Diamond A] \Rightarrow B = \Delta_0}$$

was the last step of δ_2

by the lemma we can assume that $\Gamma[(A)^\diamond] \Rightarrow B$ was obtained by cut from the sequents $(A)^\diamond \Rightarrow X$ and $\Gamma[X] \Rightarrow B$ for some $X \in \mathcal{F}$ different from Γ_0 by inductive hypothesis. Thus we can have $\Diamond A \Rightarrow X$ form $(A)^\diamond \Rightarrow X$ by $[\Diamond L]$ and by cut with $\Gamma[X] \Rightarrow B$ we obtain Δ_0 from Φ

$[\Box R]$ is applied. Then we have:

$$\frac{\Gamma[A] \Rightarrow B = \Gamma_0}{\Gamma[(\Box A)^\diamond] \Rightarrow B = \Delta_0}$$

was the last step in δ_2

Again we may assume that there exists sequents $A \Rightarrow X$ and $\Gamma[X] \Rightarrow B$ form which Γ_0 was obtained by cut. The same strategy works here, from $A \Rightarrow X$ we can obtain $(\Box A)^{\diamond} \Rightarrow X$ by $[\Box R]$; and by cut with $\Gamma[X] \Rightarrow B$ we obtain again Δ_0 from Φ

Finally for weakening we are again in the same situation as before, it holds though in practice we will avoid such kind of derivations.

4 Application

As we mentioned at the beginning of this work, those properties are required for a logic to be formalized as an institution. Morphisms required for translation are much every day work of typelogical grammarian. Thus we can say that as a matter of fact typelogical grammars are already formalized as a General Logic. As an example, of a more direct work on this vein see the work of Paiva[2], whose dialectica category have been found to be a more suitable model for linear logic than the one suggested by Seely[1]. Thus it will be an interesting exercise to actually enjoy of the fruits of this formalism.

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