The Logic of Provability

Suggestions for exercises. Week 1.

1. (B)

   Give a formula using only the logical connectives $\to, \bot$ that is classically equivalent to $(p \wedge q) \vee r$. Show that $\to, \bot$ is truth functionally complete for propositional (classical) logic. Show that we can express the existential quantifier using the universal quantifier and negation.

2. (B)

   Give a proof in natural deduction with identity of $\forall x, y, z\ (x \neq y \to x \neq z \vee y \neq z)$.

3. (B)

   Prove in predicate logic by induction on terms that for all terms t $\vdash \forall z(z = x \to z = y) \to t(y) = t(x)$. Plea that the argument carries over to more variables.

4. (C)

   Write down some different closed terms that all denote the number 10. What is the shortest one?

5. (C)

   Write $(x + S0) \cdot (x + S0) + SS0$ as a triple conform our convention on what terms are and how we represent them.

6. (C)

   Show that v_i is free in $\exists x((x + v_i) = x \cdot x)$ conform our definition on page 19.
   (Recall that $\exists$ is an abbreviation!)

7. (C) (A)

   Show that $(x + v_i)_x(v_i + 3) = ((v_i + 3) + v_i)$ and that $(x \cdot x)_x(v_i + 3) = ((v_i + 3) \cdot (v_i + 3))$ by applying our definition of substitution on page 19(footnote). Argue (informally) that we would like $(x + v_i = x \cdot x)_x(v_i + 3) = (v_i + 3 + v_i = (v_i + 3) \cdot (v_i + 3))$.

8. (C)

   Give the formal definition of $F_v(t)$ which is omitted on page 19.

9. (C)

   Provide proofs in PA of (3) and (4) of chapter 2.

10. (C) (A)

    Provide proofs in PA of $(5), (6)$ and (8) of chapter 2.

11. (C)

    Formulate precisely what is meant by unique readability on page 18.

12. (C)

    Prove (15) on page 23.

13. (D) (A)

    Why was Gödel's incompleteness theorem considerd to be a death-blow to Hilbert's programme?

14. (D**)

    Proof the deduction theorem for Hilbert style proofs for predicate logic, i.e. $\Gamma, A \vdash_H B \Leftrightarrow \Gamma \vdash_H A \to B$. Provide a proof in Hilbert style of $A \wedge B \to B \wedge A$. Study the transformation of proofs in natural deduction into Hilbert style proofs as is given in "Basic Proof Theory" of Troelstra and Schwichtenberg and provide an upperbound on the growth of the length of related proofs.