

Eerste opgaven (week 1 & 2)¹

Opgave 1

$$\frac{\frac{\frac{[\varphi]_2 \quad [\psi]_1}{\varphi \wedge \psi} \wedge I \quad [\varphi \wedge \psi \rightarrow \sigma]_3}{\frac{\frac{\sigma}{\psi \rightarrow \sigma} \rightarrow I_1}{\varphi \rightarrow (\psi \rightarrow \sigma)} \rightarrow I_2} \rightarrow E}{(\varphi \wedge \psi \rightarrow \sigma) \rightarrow (\varphi \rightarrow (\psi \rightarrow \sigma))} \rightarrow I_3$$

Opgave 2

$$\frac{\frac{\frac{[\varphi]_1 \quad [\psi]_2}{\varphi \wedge \psi} \wedge I \quad [\varphi \wedge \psi \rightarrow \sigma]_3}{\frac{\frac{\sigma}{\varphi \rightarrow \sigma} \rightarrow I_1}{\psi \rightarrow (\varphi \rightarrow \sigma)} \rightarrow I_2} \rightarrow E}{(\varphi \wedge \psi \rightarrow \sigma) \rightarrow (\psi \rightarrow (\varphi \rightarrow \sigma))} \rightarrow I_3$$

Opgave 3

$$\frac{\frac{\frac{[\varphi \wedge \chi]_2}{\varphi} \wedge E,l \quad [\varphi \rightarrow \psi]_1}{\psi} \rightarrow E}{\frac{\frac{\psi}{\varphi \wedge \chi \rightarrow \psi} \rightarrow I_2}{(\varphi \rightarrow \psi) \rightarrow (\varphi \wedge \chi \rightarrow \psi)} \rightarrow I_1}$$

Opgave 4

$$\frac{\frac{[\varphi \vee \psi]_1}{\varphi \vee (\varphi \vee \psi)} \vee I,r}{\varphi \vee \psi \rightarrow \varphi \vee (\varphi \vee \psi)} \rightarrow I_1$$

Opgave 5

$$\frac{\frac{[\varphi]_1}{\varphi \vee \psi} \vee I,l \quad [\varphi \vee \psi]_2}{\varphi \vee \psi} \vee E_{1,2}}{\varphi \vee (\varphi \vee \psi) \rightarrow \varphi \vee \psi} \rightarrow I_3$$

Opgave 6

$$\frac{\frac{\frac{[\varphi]_2 \quad [\psi]_1}{\varphi \wedge \psi} \wedge I}{\psi \rightarrow \varphi \wedge \psi} \rightarrow I_1}{\varphi \rightarrow (\psi \rightarrow \varphi \wedge \psi)} \rightarrow I_2$$

¹geLaTeXed door Ulrich

Opgave 7

$$\frac{\frac{\frac{[\varphi \wedge \psi]_1}{\psi} \wedge E,r \quad \frac{[\varphi \wedge \psi]_1}{\varphi} \wedge E,l}{\psi \wedge \varphi} \wedge I}{\varphi \wedge \psi \rightarrow \psi \wedge \varphi} \rightarrow I_1$$

Opgave 8

$$\frac{\frac{[(\varphi \wedge \psi) \vee \varphi]_3 \quad \frac{[\varphi \wedge \psi]_1}{\varphi} \wedge E,l \quad [\varphi]_2}{\varphi} \vee E_{1,2}}{(\varphi \wedge \psi) \vee \varphi \rightarrow \varphi} \rightarrow I_3$$

Opgave 9

$$\frac{\frac{\frac{[\varphi]_1 \quad [\psi]_2}{\varphi \wedge \psi} \wedge I}{(\varphi \wedge \psi) \vee \varphi} \vee I_2}{\varphi \rightarrow (\varphi \wedge \psi) \vee \varphi} \rightarrow I_1$$

Opgave 10

$$\frac{\frac{\frac{[\varphi]_2 \quad [\varphi \rightarrow (\psi \wedge \chi)]_3}{\psi \wedge \chi} \rightarrow E \quad \frac{[\varphi]_1 \quad [\varphi \rightarrow (\psi \wedge \chi)]_3}{\psi} \rightarrow E}{\frac{\psi \wedge \chi}{\psi} \wedge E,l \quad \frac{\psi \wedge \chi}{\chi} \wedge E,r}{\varphi \rightarrow \psi} \rightarrow I_2 \quad \frac{\psi \wedge \chi}{\varphi \rightarrow \chi} \rightarrow I_1}{\frac{(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \chi)}{\varphi \rightarrow \psi} \wedge I} \rightarrow I_3$$

Opgave 11

$$\frac{\frac{[\varphi]_1 \quad \frac{[(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \chi)]_2}{\varphi \rightarrow \psi} \wedge E,l}{\psi} \rightarrow E \quad \frac{[\varphi]_1 \quad \frac{[(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \chi)]_2}{\varphi \rightarrow \chi} \wedge E,r}{\chi} \rightarrow E}{\frac{\psi \wedge \chi}{\varphi \rightarrow (\psi \wedge \chi)} \wedge I} \rightarrow I_1$$

$$\frac{\varphi \rightarrow (\psi \wedge \chi)}{(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \chi) \rightarrow (\varphi \rightarrow (\psi \wedge \chi))} \rightarrow I_2$$

Opgave 12

$$\frac{\frac{[\varphi]_1}{\varphi \vee \psi} \vee I,l \quad \frac{[\varphi \vee \psi \rightarrow \chi]_3}{\chi} \rightarrow E}{\frac{\chi}{\varphi \rightarrow \chi} \rightarrow I_1} \rightarrow E \quad \frac{\frac{[\psi]_2}{\varphi \vee \psi} \vee I,r \quad \frac{[\varphi \vee \psi \rightarrow \chi]_3}{\chi} \rightarrow E}{\frac{\chi}{\psi \rightarrow \chi} \rightarrow I_2} \rightarrow E$$

$$\frac{(\varphi \rightarrow \chi) \wedge (\psi \rightarrow \chi)}{(\varphi \vee \psi \rightarrow \chi) \rightarrow ((\varphi \rightarrow \chi) \wedge (\psi \rightarrow \chi))} \rightarrow I_3$$

Opgave 13

$$\frac{\frac{\frac{[(\varphi \vee \psi) \wedge (\psi \rightarrow \varphi)]_1}{\varphi \vee \psi} \wedge E, l \quad [\varphi]_2 \quad \frac{\frac{[\psi]_3 \quad \frac{[(\varphi \vee \psi) \wedge (\psi \rightarrow \varphi)]_1}{\psi \rightarrow \varphi} \wedge E, r}{\varphi} \rightarrow E}{\varphi} \vee E_{2,3}}{\frac{\varphi}{(\varphi \vee \psi) \wedge (\psi \rightarrow \varphi) \rightarrow \varphi} \rightarrow I_1}}{\varphi} \rightarrow I_1$$

Opgave 14

$$\frac{[\varphi \vee \psi]_3 \quad \frac{[\varphi]_1 \quad \frac{[(\varphi \rightarrow \chi) \wedge (\psi \rightarrow \chi)]_4}{\varphi \rightarrow \chi} \wedge E, l \quad \rightarrow E \quad [\psi]_2 \quad \frac{[(\varphi \rightarrow \chi) \wedge (\psi \rightarrow \chi)]_4}{\psi \rightarrow \chi} \wedge E, r \quad \rightarrow E}{\chi} \vee E_{1,2}}{\frac{\chi}{\varphi \vee \psi \rightarrow \chi} \rightarrow I_3} \rightarrow I_4$$