Computational Complexity, III

1. (*) Construct a p-reduction of CLIQUE to SAT. (Hint: consider boolean variables representing the adjacency matrix of a graph together with additional $n \cdot k$ variables v_{ij}, where n is the size of the graph and k is the size of the clique. Variable v_{ij} will be true iff i-th element of a clique is j-th vertex of a graph.)

2. (Huiswerk) Consider the following variant of CLIQUE problem: determine, if a given graph of size n has a clique of size $[n/2]$. Show that this problem is NP-complete.

3. Find good upper bounds on the space complexity of SAT and CLIQUE.

4. For any TM M such that $n = o(\mathsf{Space}_M(n))$ there is a TM N that decides the same language and $\mathsf{Space}_N(n) \leq 1/5\, \mathsf{Space}_M(n)$ for all but finitely many n. (Hint: use a suitably larger tape alphabet.)

5. Determine which of the following quantified boolean formulas is valid: $\exists p p$, $\forall p p$, $\exists p \forall q (p \rightarrow q)$, $\forall p((p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p))$.

6. Determine, if the following is a correct rule of inference in quantified boolean logic: If $\phi \rightarrow \psi$ is valid (under all free variable assignments), then $\phi \rightarrow \forall p \psi$ is valid. Under what condition it is?

7. Joost has proved a remarkable theorem: every language in PSPACE can actually be recognized in linear space. His proof goes as follows: It is easy to see that the validity of quantified boolean formulas (TQBF) can be recognized in linear space. Since TQBF is PSPACE-complete, any language $L \in$PSPACE is p-reducible to it. Hence, recognizing L also takes linear space. Joost is, of course, wrong. Find a flaw in his reasoning.