Recursion Theory

Joost J. Joosten

Institute for Logic Language and Computation

University of Amsterdam

Plantage Muidergracht 24

1018 TV Amsterdam

Room P 3.26, +31 20 5256095

jjoosten@phil.uu.nl

www.phil.uu.nl/∼jjoosten

Incomputable and the real world

- Many problems reduce to K

Incomputable and the real world

- Many problems reduce to K E.g., K_0
- are essentially the same...

Incomputable and the real world

- Many problems reduce to K E.g., K_0
- are essentially the same...
- Can we characterize phenomena that cause incomputability?

Incomputable and the real world

- Many problems reduce to K E.g., K_0
- are essentially the same...
- Can we characterize phenomena that cause incomputability?
- How rich a variety is there of incomputable sets?

Incomputable and the real world

- Many problems reduce to K E.g., K_0
- are essentially the same...
- Can we characterize phenomena that cause incomputability?
- How rich a variety is there of incomputable sets?
- Does incomputability impinge on every day life?

Incomputable and the real world

- Many problems reduce to K E.g., K_0
- are essentially the same...
- Can we characterize phenomena that cause incomputability?
- How rich a variety is there of incomputable sets?
- Does incomputability impinge on every day life?
- What informational content does our study yield?

Incomputable and the real world

- Many problems reduce to K E.g., K_0
- are essentially the same...
- Can we characterize phenomena that cause incomputability?
- How rich a variety is there of incomputable sets?
- Does incomputability impinge on every day life?
- What informational content does our study yield?
- Is the mind like a Turing Machine?

Creative sets

- We call a set A creative (E. Post) if:

Creative sets

- We call a set A creative (E. Post) if:
  1. A is c.e.

Creative sets

- We call a set A creative (E. Post) if:
  1. A is c.e.
  2. A has a *creative function*

Creative sets

- We call a set A creative (E. Post) if:
  1. A is c.e.
  2. A has a *creative function*
- If A is creative, it is not computable

Creative sets

- We call a set A creative (E. Post) if:
  1. A is c.e.
  2. A has a *creative function*
- If A is creative, it is not computable
- K is a creative set

Creative sets

- We call a set A creative (E. Post) if:
  1. A is c.e.
  2. A has a *creative function*
- If A is creative, it is not computable
- K is a creative set
- and the identity is its creative function

Creative sets

- We call a set A creative (E. Post) if:
  1. A is c.e.
  2. A has a *creative function*
- If A is creative, it is not computable
- K is a creative set
- and the identity is its creative function
- Every complement of a creative set contains an infinite c.e. set

Simple sets

- Question: are there undecidable sets that are not creative?

Simple sets

- Question: are there undecidable sets that are not creative?
- Let's try to find a coinfinite c.e. set that has nonempty intersection with any other infinite c.e. set

Simple sets

- Question: are there undecidable sets that are not creative?
- Let's try to find a coinfinite c.e. set that has nonempty intersection with any other infinite c.e. set
- Definition: Simple (E. Post) set (as above)

Simple sets

- Question: are there undecidable sets that are not creative?
- Let's try to find a coinfinite c.e. set that has nonempty intersection with any other infinite c.e. set
- Definition: Simple (E. Post) set (as above)
- Simple sets are undecidable

Simple sets

- Question: are there undecidable sets that are not creative?
- Let's try to find a coinfinite c.e. set that has nonempty intersection with any other infinite c.e. set
- Definition: Simple (E. Post) set (as above)
- Simple sets are undecidable
- Simple sets do exist!

Simple sets

- Question: are there undecidable sets that are not creative?
- Let's try to find a coinfinite c.e. set that has nonempty intersection with any other infinite c.e. set
- Definition: Simple (E. Post) set (as above)
- Simple sets are undecidable
- Simple sets do exist!
- Proof is an example of a *priority argument*

Simple sets exist

- The proof is divided in a construction

Simple sets exist

- The proof is divided in a construction
- and a verification.

Simple sets exist

- The proof is divided in a construction
- and a verification.
- Very common in recursion theory!

Simple sets exist

- The proof is divided in a construction
- and a verification.
- Very common in recursion theory!
- Simple sets are closed under intersections

Quite a few simple sets exist

- Every c.e. set X whose complement includes an infinite c.e. set can be extended to a simple set

Quite a few simple sets exist

- Every c.e. set X whose complement includes an infinite c.e. set can be extended to a simple set
- Proof: let the infinite c.e. subset of the complement be given by

$$\{x_1, x_2, \ldots, x_n, \ldots\}$$

Quite a few simple sets exist

- Every c.e. set X whose complement includes an infinite c.e. set can be extended to a simple set
- Proof: let the infinite c.e. subset of the complement be given by

$$\{x_1, x_2, \ldots, x_n, \ldots\}$$

- We are going to construct a set A meeting a couple of requirements:

Quite a few simple sets exist

- Every c.e. set X whose complement includes an infinite c.e. set can be extended to a simple set
- Proof: let the infinite c.e. subset of the complement be given by

$$\{x_1, x_2, \ldots, x_n, \ldots\}$$

- We are going to construct a set A meeting a couple of requirements:
- (1) $X \subseteq A$

Quite a few simple sets exist

- Every c.e. set X whose complement includes an infinite c.e. set can be extended to a simple set
- Proof: let the infinite c.e. subset of the complement be given by

$$\{x_1, x_2, \ldots, x_n, \ldots\}$$

- We are going to construct a set A meeting a couple of requirements:
- (1) $X \subseteq A$
- (2) $|[x_1, \ldots, x_{2n}] \cap A| \leq n$

Quite a few simple sets exist

- Every c.e. set X whose complement includes an infinite c.e. set can be extended to a simple set
- Proof: let the infinite c.e. subset of the complement be given by

$$\{x_1, x_2, \ldots, x_n, \ldots\}$$

- We are going to construct a set A meeting a couple of requirements:
- (1) $X \subseteq A$
- (2) $|[x_1, \ldots, x_{2n}] \cap A| \leq n$
- (3) A is creative

Quite a few simple sets exist

- Every c.e. set X whose complement includes an infinite c.e. set can be extended to a simple set
- Proof: let the infinite c.e. subset of the complement be given by
$$\{x_1, x_2, \ldots, x_n, \ldots\}$$
- We are going to construct a set A meeting a couple of requirements:
- (1) $X \subseteq A$
- (2) $|[x_1, \ldots, x_{2n}] \cap A| \leq n$
- (3) A is creative
- Describe construction and do the verification

Natural simple sets

- Randomness and Kolmogorov complexity (7.3 of reader Terwijn)

Natural simple sets

- Randomness and Kolmogorov complexity (7.3 of reader Terwijn)
- Fix a universal TM U

Natural simple sets

- Randomness and Kolmogorov complexity (7.3 of reader Terwijn)
- Fix a universal TM U
- Kolm. Compl. of a string σ is +- the length of the shortest TM program that on empty input outputs σ

Natural simple sets

- Randomness and Kolmogorov complexity (7.3 of reader Terwijn)
- Fix a universal TM U
- Kolm. Compl. of a string σ is +- the length of the shortest TM program that on empty input outputs σ
- Is dependent on U

Natural simple sets

- Randomness and Kolmogorov complexity (7.3 of reader Terwijn)
- Fix a universal TM U
- Kolm. Compl. of a string σ is +- the length of the shortest TM program that on empty input outputs σ
- Is dependent on U but only in a $\mathcal{O}(1)$ sense

Natural simple sets

- Randomness and Kolmogorov complexity (7.3 of reader Terwijn)
- Fix a universal TM U
- Kolm. Compl. of a string σ is +- the length of the shortest TM program that on empty input outputs σ
- Is dependent on U but only in a $\mathcal{O}(1)$ sense
- A string σ is k-random if $C(\sigma) \geq |\sigma| - k$

Natural simple sets

- Randomness and Kolmogorov complexity (7.3 of reader Terwijn)
- Fix a universal TM U
- Kolm. Compl. of a string σ is +- the length of the shortest TM program that on empty input outputs σ
- Is dependent on U but only in a $\mathcal{O}(1)$ sense
- A string σ is k-random if $C(\sigma) \geq |\sigma| - k$
- The set of non-k-random strings is simple