Recursion Theory

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Computeability and provability

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Hilbert’s two themes
  - Computability
  - Provability
Hibert’s programme
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- We shall study some notions but the emphasis will not be on the computational models themselves
Primitive recursive functions

Basic definition
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- Recursive difference, absolute difference and, very useful, a sign function $sg(n)$
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- Recursive difference, absolute difference and, very useful, a sign function $sg(n)$ (homework)
The remainder function: $rm(m, n)$ is the remainder of dividing $n$ by $m$
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Recursive step:
\[
\text{rm}(m, n + 1) = \text{rm}(m, n)' \times \text{sg}(|m - \text{rm}(m, n)'|)
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More in PRIM

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- Bounded Sums
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Recursive step:
$$rm(m, n + 1) = rm(m, n)' \times sg(|m - rm(m, n)'|)$$

Bounded Sums

Bounded Products
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Bounded Sums
Bounded Products
We are building up a repertoire...