Practice mid-term exam
Recursion Theory

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October 17, 2006

1. Prove that a total map $f : \mathbb{N} \to \mathbb{N}$ is p.c. if and only if its graph is Δ^0_1.

2. A function φ_e is said to have a *computable completion* if there is a recursive function f such that

$$\varphi_e(x) = f(x) \qquad \text{whenever } \varphi_e(x)\downarrow .$$

   (a) Show that there is a p.c. function that is not total but has a computable completion.

   (b) Show that there is a p.c. function that is not total and has no computable completion.

3. Give the input/output convention for URM's. Next,write a URM that calculates $x \cdot y$.

4. Let g be a p.c. function, and let R be a computable predicate. Show that the function

$$\psi(x) = \begin{cases} g(x) & \text{if} \exists y R(x,y) \\ \uparrow & \text{otherwise} \end{cases}$$

   is partial computable.

5. Prove that there exists a TM that outputs 42 on all inputs except when its input is equal to it own code. In this case it will loop. (Hint: use the fixed point theorem. Moreover, make sure that the description is well-defined.)

6. A computable function f is *self-describing* if $e = (\mu x)[f(x) \neq 0]$ exists and $\varphi_e = f$. Show that self-describing functions exist.