Misprints etc. in *Computability Theory* of S. Barry Cooper

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Chapter 2

Page 12; r4-10: It is explained what an inductive definition is. However, it is not said which functions are not in PRIM.

Page 12; In Definition 2.1.1, no mention is made of zero-ary functions. However, in further places in the book, e.g., Example 2.1.6., Example 2.2.15

Page 13; r-2: $U^1_1(n)$ should be $U^1_1(m)$

Page 16; r8: $\mathsf{sg}(|n - \mathsf{rm}(n,m)'|)$ should be $\mathsf{sg}(|m - \mathsf{rm}(n,m)'|)$

Page 16; Exercise 2.1.11: It has not yet been defined what it means for a relation to be in PRIM. It does make sense to talk about the **div** function.

Page 16; Exercise 2.1.14, one should say how $D(0)$ is defined.

Page 17; r13: "the fact that the above nested recursion gives the equivalent equations:" should be better: "the fact that the above nested recursion gives the equations:"

Page 17; r18; "$A(4,n)$" should be $A(m,4)$

Page 17; r20; $2^{2^{2^{65536}}}$ should be $2^{2^{2^{65536}}} - 3$

Page 17; r21: something went wrong with the brackets.

Page 21; Exercise 2.2.5 is essentially the same as Exercise 2.1.11

Page 21; In Theorem 2.2.6, 'each' should be 'for each'

Page 22; Exercise 2.2.12; many exercises get rather easy if invoking Church's Thesis is allowed in this way!

Page 25; Exercise 2.2.20 uses the fact that p_i is primitive recursive. However, we have in Example 2.2.15 it is only established that p_i is recursive. For example, using the fact that there is always a prime between n and $2n$ it is not hard to see that p_i is in fact in PRIM.

Page 26; r4: it is not unambiguous how the effect of the Transfer is described. Is r_n getting the value of r_m or the other way around. Or do they both get the same, but an entirely new value? We only learn the effect on Page 28; r2.

Page 27; r-1: $\nvdash$ should be $\mathbf{0}$

Page 33 on line -12 : $h(m)$ should be $h(m, n)$

Chapter 3

Page 46; r4: I thought that actually, the axioms of PA come from Grassmann (or was it Dedekind).

Page 47; r9: we want '0' to be in the tuple too.

Page 47; r 11: $x_0 \leq x_1$ should be $x_1 \leq x_2$. And on the same line, what is a wf? This is only introduced some lines later.

Page 48; r-9: there is an opening bracket missing.

Page 48; r-1: Gödel's completeness theorem does not tell you that the logical axioms are simple, e.g., computable.

Page 49; r-1: If we want to axiomatize arithmetic, we <u>need</u> to use a first order language. [maybe it is better to say 'prefer' in stead of 'need']; Moreover, shouldn't we write 'axiomatize' rather than 'axiomatise'?

Page 50; r3: is that $\vdash$ meant to be there?

Page 53; Theorem 3.1.15: Of course, the language of $\mathcal{T}$ is assumed to be countable.

Page 54; Solution of Example 3.1.19: 'By Exercise 3.1.17' should be 'By Exercise 3.1.18'. (r6)

Page 54; r9: $0 \cdot r_i(0)r_i(1)\ldots$ should be $0.r_i(0)r_i(1)\ldots$

Page 54; Solution of 3.1.19: In the current formulation the cunning reader might argue that indeed the defined number r is not on the list, but there might be a number on the list with a different representation than r which is actually equal to r just as $0.4999\ldots = 0.5$. Of course it is easy to block this way out.

Page 56; r18: $\mathsf{graph}(\vec{m}, n)$ should be $\mathsf{graph}(f)(\vec{m}, n)$?

Page 57; Solution to Example 3.2.7: various occurrences of x_0 and m_0 should be changed to x_1 and m_1.

Page 59; Definition 3.2.12: Some of the $\overline{m}$ should actually be $\vec{m}$; In Item (ii) there is a closing bracket missing.

Page 59 r20: 'f in ϕ' should be f in PA

Page 59; Exercise 3.2.14: the hint seems to be inadequate. Rather, we should replace $\neq$ by $\leq$.

Chapter 4

Page 67; Definition 4.5.2: $\varphi_{e,s}$ should be $\varphi_{e,s}(x)$

Chapter 5

Page 74; Example 5.2.6: $\varphi_{e,s}(x)\downarrow$ should be $\varphi_e(x)\downarrow$ and likewise in the left hand side of the equivalence in the solution.

Page 77; Exercise 5.2.16: ψ should be f.

Page 78; second line under Definition 5.2.21: $\langle x, e\rangle$ should be $\langle x, y\rangle$

Page 79; two lines below Definition 5.3.2: the $x \in W_y$ should be $x \in W_x$, and $[x \in W_x]$ should be $[x \in W_{x,s}]$

Page 79; Exercise 5.3.3: It is only in Definition 7.2.9 that the reader learns what it means to be a lattice

Page 80; r11 $\forall \in \mathbb{N}$ should be $\forall x \ \in \mathbb{N}$
Page 82; l-3: $4n(n^2 + 2n + 1)$ should be $4n(n^2 + n + 1)$

Page 83; l5/6: the difficulties arising by replacing $\geq$ with $=$ might occur because it would yield a wrong statement. A student (Erica Neutel) found a program that establishes $B(10) \geq 44$:
1. T(2,3)
2. T(1,2)
3. S(1)
4. S(1)
5. S(1)
6. S(1)
7. J(3,4,1)
8. J(2,5,11)
9. S(5)
10. J(1,1,3)

Chapter 6

Page 89; Definition 6.1.1; Item 2: the $\subset$ should really be a $\subseteq$.

Page 91; The proof of Theorem 6.2.3 is not yet a proof. With very little more precision it will deserve that predicate.

Page 93; Exercise 6.2.7: why should we work at Item (i) under the assumption that a simple set exists as we have just proved that this is the case?

Page 93; Exercise 6.2.8: We have already seen in Example 6.1.4 that the complement of a creative set contains an infinite c.e. set.
Page 94; l6 the Hint is lacking a closing bracket "]".

Page 96; l7: There is a "}" missing.

Page 97; Maybe it is better to call the proof, a proof-sketch or proof-ingredient as not to sort a wrong standard of proofs with students.

Page 99; Which variables in the prime enumerating polynomial are supposed to be existentially quantified?

Chapter 7

Page 104; l-6: Notice that an index set of a set of c.e. sets is also an index set of p.c. functions.

Page 105; Proof of Theorem 7.1.11, Case 2, l2, Then a similar argument, with $e', \overline{A}$ in place of e, A, gives ...

Chapter 8

Page 117; l-4 assign Gödel numbers to sentences of PA

Page 117; l-3 — l-1: I always thought that Gödel set out to prove the incompleteness, that he first had the idea of formalizing a variant of the liar paradox and then started the programme of arithmetisation to implement that. Or does it follow from Gödel's Nachlass that he first attempted to prove consistency via arithmetisation?

The same applies to Page 123: l-2.

Page 119; Definition 8.1.5, l4: to disambiguate the phrase, it might be better to write "then for some $m \in \mathbb{N}$, not $\vdash \varphi(\overline{m})$." Moreover, the notion of ω-consistency can become less obscure if the notion of ω-inconsistent is first considered.

Page 121; l4: and MP gives $\vdash_{\mathsf{PA}} \exists x_1 \; \psi(\overline{m}, x_1)$.

Page 121; Exercise 8.1.12; l3: c.e.,

Page 123; l7: "flow" should be "follow", i guess

Page 124; Exercise 8.2.8: "compete" should be "complete".

Chapter 9

Page 129; Proof of Lemma 9.2.3: $\mathcal{T}'$ is defined as $\mathcal{T}' \cup \Sigma$, but this should be $\mathcal{T} \cup \Sigma$.

Page 130; Proof of Lemma 9.2.6, l-1 do we have $0 \notin T_{PC}$?

Chapter 10

Page 140; Definition 10.1.1, l5: It is nice to note here that the symbol S_k is completely irrelevant, in that it does not matter for the action which S_k is used. Moreover, we can do with another notion of consistency.

Page 141; l13: To see this — let $B \leq_m A$ via f ...

Page 142; l1-l6: the program does not calculate χ_A: if $n \in A$ then the program will output $n + 1$.

Page 146; l13: there is a bracket "}" missing.

Page 146; l17: if Φ_i^A is not total, $\mathsf{deg}(\Phi_i^A)$ does not make much sense. (If one does not refer to the halting set.)

Page 146; l21 (likewise in l-2): $\mathbb{N}$

Page 146; l-11: the proof only shows that the set of degrees is not countable, not that it equals the continuum. The notation $2^{\mathbb{N}} = \cup\{\mathbf{a} \mid \mathbf{a} \in \mathcal{D}\}$ suggests that you are claiming that the set of degrees equals the continuum. Which is not proven by the given proof.

Page 147; l5: $f \leq_T g$

Page 148; Definition 10.4.1 (2): Do you mean for all $A \in \mathbf{a}$ or for some $A \in \mathbf{a}$? In Lemma 10.4.15 we learn that it does not really matter, but at this point it should be made clear.

Page 149; Definition 10.4.5, l3: steps of the Turing program $\hat{P}_e$ with oracle A.

Page 149; Exercise 10.4.11 should come after Definition 10.4.13 and before Theorem 10.4.14.

Page 150; Definition 10.4.13 (2): we should say here that $A^{(0)} = A$.

Page 151; l8: to have the argument started we should replace $A \leq_T B$ by $B \leq_T A$.

Page 154; Definition 10.5.1, Items (2) and (3): shouldn't it be more appropriate to replace "of the form" by "definable by". If not, how can something be in Δ starting both with an existential and a universal at the same time...

Page 155, l5-7 and Page 157 l-9- -11: there is some redundancy, right?

Page 156; l1: $\emptyset^{(n)}$ should be $\emptyset^{(0)}$.

Page 156; l6: the $\Leftrightarrow$ is incorrect and confusing.

Page 156; l13: $(\Leftarrow)$

Various

The Index has running header "More Advanced Topics", which is probably not intended so.

There is no index of symbols