Proof Theory an Automated Theorem Proving 2013
Example of a derivation in Tait Calculus
Week 2

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1 Propositional logic in Tait calculus

I received a question on how to properly write down proofs in the one-sided Tait style calculus. Note that in the book there is no format specified directly to write proofs as trees. We have just given rules like "if $\vdash^n_T \Delta, A$, then $\vdash^m_T \Delta, A \vee B$ for all formulas B and all natural numbers $m > n$".

We will however, write our proofs just as trees very much the way we did in Natural Deduction. As an emaple, let us try to prove the following formula in Tait calculus.

$$(A \rightarrow C) \rightarrow (A \wedge B \rightarrow C)$$

Remember that we first has to translate the formula below to a formula in the Tait language. Just as in the book we shall refrain from distinguishing from $\neg$. Thus, we see that we need to prove the following formula

$$(A \wedge \neg C) \vee ((\neg A \vee \neg B) \vee C).$$

Now, here goes the format I would like us to present proofs (as was done in class):

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\dfrac{A, \neg A, \neg B, C \qquad \neg C, \neg A, \neg B, C}{A \wedge \neg C, \neg A, \neg B, C}\ (\wedge)
}{A \wedge \neg C, \neg A \vee \neg B, \neg B, C}\ (\vee)
}{A \wedge \neg C, \neg A \vee \neg B, C}\ (\vee)
}{A \wedge \neg C, (\neg A \vee \neg B) \vee C, C}\ (\vee)
}{A \wedge \neg C, (\neg A \vee \neg B) \vee C}\ (\vee)
}{(A \wedge \neg C) \vee ((\neg A \vee \neg B) \vee C), (\neg A \vee \neg B) \vee C}\ (\vee)
}{(A \wedge \neg C) \vee ((\neg A \vee \neg B) \vee C)}\ (\vee)
$$

Note that in Latex, it is actually very fast to make this tree, as starting from above you can do a lot of copy-paste. Note that all leaves in the proof tree are indeed axioms.