Mathematical Logic in the Netherlands 2010

27-28 May 2010

Utrecht University

Location

The meeting takes place on De Uithof in Utrecht.

Thursday 27 May: Buys Ballot Laboratorium (BBL) 061, Princetonplein 5.

Friday 28 May: The first lecture in Minnaertgebouw 202, the other lectures in Minnaertgebouw 208, Leuvenlaan 4.

Programme

Thursday 27 May

10:00 - 10:50	Wim Veldman (Radboud University Nijmegen) *The principle of Open Induction on the unit interval* $[0, 1]$ *and some of its equivalents*
10:50 - 11:10	Coffee, tea, cookies
11:10 - 11:40	Iris Loeb (VU University Amsterdam) *On Tarski's Foundations of the Geometry of Solids*
11:40 - 12:05	Charlotte Vlek (University of Amsterdam) K*-trivial sequences and degrees of randomness*
12:05 - 13:30	Lunch
13:30 - 14:20	Sonja Smets (University of Groningen) *Belief Dynamics under Iterated Revision: From Cycles of Upgrades to Doxastic Fixed Points*
14:20 - 14:45	Martijn Baartse (Ghent University) *The Phase Transition for Friedman's Long Finite Sequences*
14:45 - 15:05	Coffee, tea, cookies
15:05 - 15:55	Albert Visser (Utrecht University) *Sameness of theories*

Friday 28 May

10:00 - 10:50	Ieke Moerdijk (Utrecht University) *Introduction to the theory of classifying toposes*
10:50 - 11:10	Coffee, tea, cookies
11:10 - 11:40	Raul Andres Leal (University of Amsterdam) *Coalgebras for Monads: Towards a coalgebraic framework for dynamic computations*
11:40 - 12:05	Tom Sterkenburg (University of Amsterdam) *Splittings in randomness degrees*
12:05 - 13:30	Lunch
13:30 - 14:00	Sam Sanders (Ghent University) *A copy of several Reverse Mathematics*
14:00 - 14:30	Daisuke Ikegami (University of Amsterdam) *Gale-Stewart games and Blackwell games*
14:30 - 15:00	Dion Coumans (Radboud University Nijmegen) *Duality for classical first order logic*
15:00 - 15:20	Coffee, tea, cookies
15:20 - 15:50	Jacob Vosmaer (University of Amsterdam) *Applying coalgebraic modal logic in point-free topology*
15:50 - 16:40	Andreas Weiermann (Ghent University) *Recent trends in phase transitions for Gödel incompleteness*
18:30 -	Conference dinner at the Faculty Club - Achter de Dom 7 - Utrecht

Abstracts

Martijn Baartse

The Phase Transition for Friedman's Long Finite Sequences

In his paper "Long finite sequences" Friedman defines a function n as $n(k) = l$ where l is the maximum length a sequence $x_1, \ldots, x_{2n}$ over $1, ..., k$ with the property that for every $i < j \leq n$ the sequence $x_i, \ldots, x_{2i}$ is not a subsequence of $x_j, \ldots, x_{2j}$ can have. He proves that the totality of this function can be proved in $I\Sigma_3$, but not in $I\Sigma_2$. This property can be generalized to depend on some function f which gives rise to functions n_f. Now one can ask the question for which (not very slow growing) f the function n_f is not provably total in $I\Sigma_2$ and for which (very slow growing) f the function n_f is provably total in $I\Sigma_2$. Between provability and non-provability there is some sort of phase transition. If we define $f_\alpha(i) = \frac{1}{F_\alpha^{-1}(i)} log_2(i)$ then we get provability for $\alpha < \omega^\omega$ and non-provability for $\alpha = \omega^\omega$.

Dion Coumans

Duality for classical first order logic

Duality theory is a powerful tool to obtain information about a logic by studying the structures dual to their algebraic semantics. The algebraic semantics for Classical Propositional Logic (CPL) are given by Boolean algebras which are dually equivalent to Stone spaces. We will describe how this duality for CPL may be a extended to a duality for Classical First Order Logic (CFOL).

The algebraic semantics for CFOL are given by Boolean hyperdoctrines. We begin by explaining what these are by abstracting the essential properties of the collection of all first order formulas over a given signature. Thereafter we identify the dual notion of a Boolean hyperdoctrine and consequently describe a duality for CFOL.

Daisuke Ikegami

Gale-Stewart games and Blackwell games

Gale-Stewart games are infinite games with perfect information. The determinacy of Gale-Stewart games has been investigated for over 40 years and it is one of the main topics in set theory. Blackwell games are infinite games with imperfect information coming from game theory and it has not been researched so much. In 1998, Martin proved that the Axiom of Determinacy (AD) implies the Axiom of Blackwell determinacy ($Bl - AD$) and conjectured the converse, which is still not known to be true. In 2003, Martin, Neeman, and Vervoort proved that AD and $Bl - AD$ are equiconsistent. In this talk, we discuss the connection between the Axiom of Real Determinacy (AD_R) and the Axiom of Real Blackwell Determinacy ($Bl - AD_R$). This is joint work with W. Hugh Woodin.

Raul Andres Leal

Coalgebras for Monads: Towards a coalgebraic framework for dynamic computations

(Joint work with Helle Hvid Hansen)

In this talk, we will first illustrate how to impose an algebraic structure on coalgebraic modalities. The prime example of such modalities are those of Propositional Dynamic Logic (PDL). In PDL, modalities are labelled by programs $\alpha, \beta, \pi, \dots \in L$. A modal formula $[\alpha]\varphi$ has then the following intuitive reading: "after executing α, φ holds". One important feature of PDL programs is that they can be combined, using algebraic operations, to obtain new programs, e.g. sequential composition ($\alpha;\beta$: after executing α, execute β), choice ($\alpha \cup \beta$: execute either α or β), iteration (α^*: execute α some finite number of times). Our aim is to understand and develop such dynamic modalities in a general coalgebraic setting. Subsequently, we will show how axioms like

$$[\alpha;\beta]\varphi \Leftrightarrow [\alpha][\beta]\varphi; \ [\alpha \cup \beta]\varphi \Leftrightarrow [\alpha]\varphi \vee [\beta]\varphi.$$

naturally appear by making considerations on the type of coalgebras. For example, the axiom on the left rises when considering coalgebras for a monad whereas the one on the right when considering coalgebras such that the set of successors has a meet semilattice structure.

Our approach is based on the following double perspective

$L \to (GS)^S$	$S \to (GS)^L$
(algebraic view: structure, dynamics)	(coalgebraic view: behaviour, modalities)

This perspective also applies to other dynamic modal logics such as Coalition Logic, Game Logic, and Hoare Logic for Java.

Iris Loeb

On Tarski's Foundations of the Geometry of Solids

(Joint work with Arianna Betti)

Tarski sketched in his short paper "Foundations of the Geometry of Solids" (1929), which he translated and edited in 1956, a formal approach to solid geometry. I will discuss the tension between the use of Lesniewski's Mereology and Russell's type theory that both the original and the edited version exhibit. Furthermore, I will point out the little known distinction between the notions "universe of discourse" and "range of the quantifiers", and explain the importance of this for mathematics as well as philosophy by means of issues surrounding Tarski's paper.

Ieke Moerdijk

Introduction to the theory of classifying toposes

The notion of "classifying topos" is formally similar to that of "classifying space" in algebraic topology, and has close connections to first order logic. For example, the universal structures over classifying toposes, similar to the universal bundles in topology, are closely related to generic models in set theory. The theory of classifying toposes is a main tool to construct toposes and deduce structure theorems. It also has applications in logic. I aim to give a gentle introduction to some aspects of this beautiful theory.

Sam Sanders

A copy of several Reverse Mathematics

Reverse Mathematics is a program in foundations of Mathematics initiated by Friedman ([1, 2]) and developed extensively by Simpson ([6]). Its aim is to determine which minimal axioms prove theorems of ordinary mathematics. Nonstandard methods have played an important role in this program ([5, 7]). We are interested in Reverse Mathematics where equality is replaced by the nonstandard relation $\approx$, i.e. equality up to infinitesimals. We obtain a 'copy' of Reverse Mathematics for WKL_0 in a weak system of nonstandard arithmetic. Surprisingly, the same system is also a 'copy' of Constructive Reverse Mathematics ([3, 4]). We discuss applications of our results in Physics and the Philosophy of Science.

References

[1] Harvey Friedman, *Some systems of second order arithmetic and their use*, Proceedings of the International Congress of Mathematicians (Vancouver, B. C., 1974), Vol. 1, Canad. Math. Congress, Montreal, Que., 1975, pp. 235–242.

[2] ———, *Systems of second order arithmetic with restricted induction, I & II (Abstracts)*, Journal of Symbolic Logic **41** (1976), 557–559.

[3] Hajime Ishihara, *Reverse mathematics in Bishop's constructive mathematics*, Philosophia Scientiae (Cahier Spécial) **6** (2006), 43-59.

[4] ———, *Constructive reverse mathematics: compactness properties*, From sets and types to topology and analysis, Oxford Logic Guides, vol. 48, Oxford Univ. Press, Oxford, 2005, pp. 245–267.

[5] H. Jerome Keisler, *Nonstandard arithmetic and reverse mathematics*, Bull. Symbolic Logic **12** (2006), no. 1, 100–125.

[6] Stephen G. Simpson, *Subsystems of second order arithmetic*, 2nd ed., Perspectives in Logic, Cambridge University Press, Cambridge, 2009.

[7] Kazuyuki Tanaka, *The self-embedding theorem of* WKL_0 *and a non-standard method*, Annals of Pure and Applied Logic **84** (1997), 41-49.

Sonja Smets

Belief Dynamics under Iterated Revision: From Cycles of Upgrades to Doxastic Fixed Points

What happens in the long term with an agent's beliefs, knowledge and "epistemic state" (fully describable by her conditional beliefs), when receiving a sequence of public announcements of truthful but uncertain information? Do the agent's beliefs (or knowledge, or conditional beliefs) reach a fixed point? Or do they exhibit instead a cyclic behavior, oscillating forever? And in case that the beliefs do stabilize on a fixed point, what conditions ensure that they stabilize on the truth? I will present a formal setting to investigate and provide some answers to these questions. On the one hand I will show that on an initial finite Kripke model, a truthful belief upgrade (with the same true sentence) may be repeated "ad infinitum", without ever reaching a fixed point of the belief-revision process. On the other hand, I prove some positive convergence results: the agent's simple beliefs (and knowledge) will eventually stabilize when iterating updates or truthful radical upgrades - where updates and radical upgrades are types of model transformations. My presentation is based on joint work with Alexandru Baltag.

Tom Sterkenburg

Splittings in randomness degrees

The field of algorithmic randomness attempts to give an exact characterization of randomness for binary strings. A central notion is that of prefix-free Kolmogorov complexity, capturing the intuition that a random set, interpreted as an infinite binary string, cannot be compressed.

We can use this notion to define alternative ways of comparing the computational complexity of sets. For example, a set A is LK-reducible to another set B if A, used as an oracle, will not find significantly better compressions than B. Similarly, A is K-reducible to B if up to a constant the prefix-free Kolmogorov complexity of A is less than that of B. These weak reducibilities give rise to the structures of the LK- and the K-degrees.

For both of these degree structures I will present a splitting theorem, that divides a given computably enumerable nontrivial set into two sets of strictly lower complexity. Hence the c.e. LK- and K-degrees are downward dense. The construction resembles that of the classical Sacks Splitting Theorem in computability theory, which splits a given noncomputable c.e. set into two sets that are not in the cone above a third set.

Wim Veldman

The principle of Open Induction on the unit interval $[0, 1]$ and some of its equivalents

A long list can be made of statements that are equivalent, in basic intuitionistic analysis, to Brouwer's Fan Theorem, see [1]. One of these equivalents is the

> **Heine-Borel Theorem:** *Let $(a_0, b_0), (a_1, b_1), \ldots$ be an infinite sequence of pairs of rational numbers with the property that, for each x in $[0, 1]$, there exists n such that $a_n < x < b_n$.*
>
> *Then there exists N such that, for each x in $[0, 1]$, there exists $n \leq N$ such that $a_n < x < b_n$.*

Brouwer introduced and proved the famous Bar Theorem in order to derive the Fan Theorem. Another consequence of the Bar Theorem is the following

> **Principle of Open Induction on [0,1]:** *Let A be an open subset of $[0, 1]$. If, for every x in $[0, 1]$, x belongs to A if every $y < x$ belongs to A, then A coincides with $[0, 1]$.*

The Principle of Open Induction on $[0, 1]$ implies the Fan Theorem. It turns out to be equivalent to the

> **Bolzano-Weierstrass Theorem:** *Let $x_0, x_1, x_2, \ldots$ be an infinite sequence of real numbers such that, for every strictly increasing sequence γ of natural numbers there exists n such that $|x_{\gamma(n+1)} - x_{\gamma(n)}| > \frac{1}{2^n}$. Then, for each M in $\mathbb{R}$ there exists n such that $|x_n| > M$.*

It also turns out to be equivalent to the following principle that may be compared to the $\mathbf{\Sigma}^0_1$- *comprehension principle* in (classical) Reverse Mathematics:

> *Let B be an enumerable subset of $\mathbb{N}$. Suppose that, for every decidable subset A of $\mathbb{N}$, if $A \subseteq B$ and $\exists n[n \notin A]$, then $\exists n[n \notin A$ and $n \in B]$. Then $B = \mathbb{N}$.*

It is impossible, in basic intuitionistic analysis, to prove the principle of Open Induction on [0,1] from the Fan Theorem.

We intend to sketch proofs of these results and to discuss their significance for the project of Intuitionistic Reverse Mathematics.

References

[1] W. Veldman, *Brouwer's Fan Theorem as an axiom and as a contrast to Kleene's Alternative*, Report No. 0509, Department of Mathematics, Radboud University Nijmegen, July 2005.

Albert Visser

Sameness of theories

Sameness of theories is an important notion both from the philosophical point of view and from the technical point of view. We review various notions of sameness of theories and provide some examples of samenesses and non-samenesses.

We sketch the proof of a theorem based on an idea of Harvey Friedman that tells us how to improve bi-interpretations to synonymies, for an important class of cases. This theorem employs a miniaturized version of the Schroeder-Bernstein Theorem that is interesting in its own right.

Charlotte Vlek

***K*-trivial sequences and degrees of randomness**

In order to capture the 'degree' of randomness of a certain set (represented as an infinite binary sequence), one can look at the prefix-free complexity K of the initial segments. When all initial segments $A \upharpoonright n$ require a long description, as long as their length modulo a constant ($K(A \upharpoonright n) \geq^{+} n$), the set is said to be random.

In this talk, we are interested in the structure of the K-degrees: the different degrees of randomness obtained by comparing sets with respect to the prefix-free complexity of their initial segments. Specifically we discuss minimal pairs for the K-degrees.

We can show that there is a Σ^0_2 set that forms a minimal pair with any c.e. set. Also we can construct a minimal pair through a gap function for K-triviality, where a set is defined to be K-trivial if the complexity of each initial segment is less than the complexity of the length of the segment, up to a constant. We show that there cannot be a Δ^0_2 non-decreasing unbounded gap function.

Jacob Vosmaer

Applying coalgebraic modal logic in point-free topology

(Joint work with Yde Venema and Steve Vickers)

The Vietoris hyperspace construction (which, given a compact Hausdorff space X, gives us a new topological space the points of which are the closed subsets of X) is intimately connected with modal logic. The algebraic (point-free) description of the Vietoris function essentially uses an axiomatization of geometric positive modal logic using box and diamond. We will argue however, that in many ways it is much more natural to study the (point-free version of) Vietoris functor using coalgebraic modal logic, namely using the cover modality. We will introduce a generalized version of the point-fee Vietoris construction, together with several preservation results, using relation lifting, a technique from coalgebraic modal logic.

Andreas Weiermann

Recent trends in phase transitions for Gödel incompleteness

We start with a survey about phase transitions for concrete Gödel incompleteness. Here we treat refinements of results by H. Friedman and by J. Paris and L. Harrington. (To obtain best possible bounds related to H. Friedman's miniaturization of Kruskal's theorem we apply recent results from Schlage Puchta on analytic combinatorics.)

In the second part we will cover recent developments on phase transitions which result from Kristiansen style subrecursive degree theory. The idea is study degrees of PA provably recursive functions and to investigate an induced degree structure on related threshold functions. (The second part of the talk is based on joint work with with S. Friedman and M. Rathjen and the members of the UGent logic and analysis team.)

If time is left we will discuss some truly amazing phase transition results related to the Ackermann function. (Joint work with J.C. Schlage Puchta.)