Degrees of interpretability of finitely axiomatized sequential theories
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Finitely axiomatized sequential theories are something like a natural kind of theories. They share a lot of salient and important properties. Moreover, many familiar theories belong to this kind. Examples of finitely axiomatized sequential theories are the basic theory $\mathsf{PA}^-$, Buss’s theory $\mathsf{S}^1_2$, Elementary Arithmetic $\mathsf{EA}$, $\mathsf{I\Sigma}_1$, $\mathsf{ACA}_0$ and the Gödel-Bernays theory of sets and classes $\mathsf{GB}$.

An interpretation of a theory $U$ into a theory $V$ is, very roughly, a translation of $U$ in $V$ that commutes with the propositional connectives and, modulo relativization, with the quantifiers. The theory $U$ is interpretable in $V$ iff there is an interpretation of $U$ in $V$. Interpretability gives rise to a degree structure on theories in the evident way. The study of interpretability degrees of a class of theories is important since it provides a notion of the strength for these theories. Such degree structures are the natural home for Reverse Mathematics. For example, the Observed Linearity of Reverse Mathematics only comes into focus against the background of the result that the surrounding degree structure contains infinite anti-chains.

In this talk we give an introduction to the interpretability degrees of finitely axiomatized sequential theories. We are especially interested in the question: the comparison of two relations between theories, the first being extension-in-the-same-language, the second interpretability. We will show a convexity result: for each $B$ that interprets $A$ there is an extension-in-the-same-language $A'$ of $A$ such that $A'$ is mutually interpretable with $B$.

We will briefly look at the case of a non-sequential theory, to wit: Robinson’s Arithmetic $\mathsf{Q}$. This case shows interesting similarities and differences with the sequential one.

As we will see arithmetical theories play a central role in the study of the interpretability degrees of finitely axiomatized sequential theories. This is already visible in the classical result that each such degree contains an arithmetical theory.