# Inleveropgave 2

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1. This question can be divided into two parts:

   – The set $P(A \times B)$ has $2^{|A \times B|}$ elements. Because $|A \times B| = |A| \times |B|$, we can simplify this by saying that it has $2^{|A| \times |B|}$ elements.

   – If $a \in A$, it holds that $\{a\} \notin A \times B$. The set $A \times B$ is a set of relations, and $\{a\}$ is not a relation, thus it is not in $A \times B$.

2. This question can be divided into two parts:

   – The relation $A \times B$ is serial iff $A = \emptyset$ **or** $B \neq \emptyset$ (non-exclusive). The definition of seriality is $\forall a \in A, \exists b \in B(Rab)$. It follows from the definition of the cartesian product on pp9 that this always holds if A is empty or if B is non-empty.

   – If $A \neq B$, $A \times B$ can still be symmetrical. Suppose $A = \emptyset$ and $B \neq \emptyset$ (clearly $A \neq B$ holds). The cartesian product $A \times B = \emptyset$. But since the empty relation is also symmetrical, we found an example where $A \neq B$ holds, and also $A \times B$ is symmetrical.

5. All properties:

**Reflexive** The relation is reflexive iff $\forall z \in \mathbb{Z}(Rzz)$. But, for example, $\langle 1, 1 \rangle$ is not in the relation, because $1 - 1 \neq 2$. This is a counter example, so we must conclude that the relation is not reflexive.

**Transitive** The relation is transitive iff $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \forall z \in \mathbb{Z}(Rxy \wedge Ryz \rightarrow Rxz)$. But for example, $\langle 6, 4 \rangle$ and $\langle 4, 2 \rangle$ are in the relation (because $6 - 4 = 2$ and $4 - 2 = 2$), but $\langle 6, 2 \rangle$ is not (because $6 - 2 \neq 2$). This is a counter example, so we must conclude that the relation is not transitive.

**Symmetric** The relation is symmetric iff $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}(Rxy \rightarrow Ryx)$. But for example, $\langle 6, 4 \rangle$ is in the relation (because $6 - 4 = 2$), but $\langle 4, 6 \rangle$

is not (because $4 - 6 \neq 2$). This is a counter example, so we must conclude that the relation is not symmetric.

6. All properties:

**Reflexive** The relation is reflexive iff $\forall w \in W(Rww)$. This is true, because $l(w) = l(w)$.

**Transitive** The relation is transitive iff $\forall x \in W, \forall y \in W, \forall z \in W(Rxy \wedge Ryz \rightarrow Rxz)$. Suppose we have an arbitrary $a \in W$,$b \in W$ and $c \in W$ for which $Rab$ and $Rbc$ holds. We must conclude that $l(a) = l(b)$ and also $l(b) = l(c)$. It follows from this that $l(a) = l(c)$, and so also $Rac$ must be in the relation. Because $a$,$b$ and $c$ were chosen arbitrarily, we conclude that it follows for every element in $W$.

**Symmetric** The relation is symmetric iff $\forall x \in W, \forall y \in W(Rxy \rightarrow Ryz)$. Suppose we have an arbitrary $a \in W$ and $b \in W$ for which $Rab$ holds. We can conclude that $l(a) = l(b)$, but also $l(b) = l(a)$, and so also $Rba$ must be in the relation. Because $a$ and $b$ were chosen arbitrarily, we conclude that it follows for every element in $W$.

The equivalence class $[w]$ is a set containing all elements that are equivalent to $w$. Since every element in $W$ that has the same length as $w$ is equivalent to $w$, we can conclude that $[w] = \{w' \mid l(w') = n\}$. It can be seen that this set has exactly $2^n$ elements (all rows we can construct from zero's and one's of length $n$).