# MATHEMATICAL STRUCTURES IN LOGIC 2016
# HOMEWORK 1

- Deadline: February 9 — at the **beginning** of class.
- In exceptional cases homework can be submitted electronically (in a single pdf-file!) to Frederik Lauridsen (`f.m.lauridsen@uva.nl`)
- Grading is from 0 to 10 points.
- Success!

(1) (2pt) Let $(P, \leq)$ be a poset. Show that if $\sup(A)$ exists for each $A \subseteq P$, then $\inf(B)$ also exists for each $B \subseteq P$, and therefore $(P, \leq)$ is a complete lattice.

(2) (3pt) Give an example of a poset $(P, \leq)$ in which there are three elements $x, y, z$ such that

(a) $\{x, y, z\}$ is an antichain (a set $A \subseteq P$ is an *antichain* if $a \not\leq b$ for distinct $a, b \in A$),

(b) $x \vee y$, $y \vee z$ and $z \vee x$ fail to exist,

(c) $\bigvee\{x, y, z\}$ exists.

It is sufficient to just provide the Hasse diagram for this lattice. (Hint: $P$ will have more than three elements.)

(3) (3pt) Let $L$ be a lattice. We say that a non-zero element $a \in L$ is *join irreducible* if $a = b \vee c$ implies $a = b$ or $a = c$. Let $(P, \leq)$ be a poset. $A \subseteq P$ is an *up-set* if $x \in A$ and $x \leq y$ imply $y \in A$. Let $\mathrm{Up}(P)$ be the set of all up-sets of $P$.

(a) Show that $(\mathrm{Up}(P), \subseteq)$ is a distributive lattice

(b) Characterize join irreducible elements of $(\mathrm{Up}(P), \subseteq)$ for a finite $P$.

(4) (2pt) Let $L$ be a lattice. We say that a non-zero element $a \in L$ is *join prime* if $a \leq b \vee c$ implies $a \leq b$ or $a \leq c$.

(a) Show that in a distributive lattice the join irreducible elements coincide with the join prime elements.

(b) Give an example of a lattice having a join irreducible element which is not a join prime element.