INTRODUCTION TO MODAL LOGIC 2017
HOMEWORK 4

- Deadline: November 14 — at the **beginning** of class.
- Grading is from 0 to 100 points.
- Success!

(1) (30pt)
  (a) Show that if a frame $\mathfrak{F}$ is a bounded morphic image of a frame $\mathfrak{G}$, then
  $$Log(\mathfrak{G}) \subseteq Log(\mathfrak{F}).$$
  (b) Show that if a frame $\mathfrak{F}$ is a generated subframe of a frame $\mathfrak{G}$, then
  $$Log(\mathfrak{G}) \subseteq Log(\mathfrak{F}).$$
  (c) Let $\mathcal{C}$ be a non-empty class of frames. Use (a) and (b) to show that $Log(\mathcal{C})$ is contained in the logic of a single reflexive point or $Log(\mathcal{C})$ is contained in the logic of a single irreflexive point.

(2) (30pt) Recall that $\mathbf{S5} = \mathbf{K} + (\Box p \to p) + (\Box p \to \Box\Box p) + (p \to \Box\Diamond p)$. Show:
  (a) $\vdash_{\mathbf{S5}} \Diamond p \to \Box\Diamond p$
  (b) Show that $\mathbf{S5}$ is sound and complete with respect to the class of frames (W, R), where R is an equivalence relation.
  (c) Use (b) to show that $\nvdash_{\mathbf{S5}} \Diamond p \to \Box p$.

(3) (40pt) Recall that $\mathbf{S4.2} = \mathbf{S4} + (\Diamond\Box p \to \Box\Diamond p)$. Find a class $\mathcal{C}$ of Kripke frames which $\mathbf{S4.2}$ is sound and complete for.

  You may use the fact that $\mathbf{S4}$ is sound and complete with respect to reflexive and transitive frames.

You are not allowed to use Sahlqvist's completeness theorem in any of these exercises.