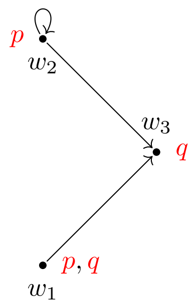
Introduction to Modal Logic. Exercise class 4

25 September 2017

Exercise 1. Compute both the largest and the smallest filtration of the model below through each of the following sets

$$\Sigma_1 \coloneqq \{p, q\} \quad \Sigma_2 \coloneqq \{p, q, \Diamond p\} \quad \Sigma_3 \coloneqq \{q\}.$$



Exercise 2 (BdRV, Ex. 2.3.5)**.** This exercise is about the so-called transitive (or Lemmon) filtration.

(1) Show that not every filtration of a transitive model is transitive.

(2) Proof Lemma 2. 24 of [BdRV]. That is, show that the relation R^t defined there is indeed a filtration (of transitive models), and that any filtration of a transitive model that makes use of the relation R^t is guranteed to be transitive.

Exercise 3 (BdRV 2.3.1)**.** Find two models $\mathbb{M}$ and $\mathbb{M}'$ and states w and w' in these models which are not bisimilar but $\mathbb{M}, w \leftrightarroweq_n \mathbb{M}', w'$ for all $n \in \omega$.

Exercise 4. Let $\mathbb{M}$ and $\mathbb{M}'$ be regular PDL-models, so that we may represent them as $\mathbb{M} = (W, (R_a)_{a \in A}, V)$ and $\mathbb{M}' = (W', (R'_a)_{a \in A}, V')$, where A is the set of atomic programs. Show that if Z is a bisimulation between (W, R_a, V) and (W', R'_a, V') for every atomic program $a \in A$, then Z is a bisimulation between (W, R_π, V) and (W', R'_π, V') for *every* PDL-program π.

Exercise 5. Which of the following properties of frames are preserved by taking suitable filtrations of Kripke models?

(1) reflexivity;

(2) symmetry;

(3) seriality;

(4) directedness: $\forall x_0 x_1 x_2 \,((Rx_0x_1 \wedge Rx_1x_2) \to \exists y(Rx_1y \wedge Rx_2y))$;

(5) density: $\forall x_1 x_2 (x_1 R x_2 \to \exists y\,(Rx_1 y \wedge Ryx_2))$;

(6) every world sees a reflexive world: $\forall x \exists y\,(Rxy \wedge Ryy)$.

Exercise 6. We say that a class of models K has the property $(\star)$ if any formula in the langugage of basic modal logic which is satisfiable in some K-model is also satisfiable in a finite K-model. Show that the following classes of models has the $(\star)$-property.

(1) The class of reflexive models;

(2) The class of transitive models;

(3) The class of directed models;

(4) The class of models with no branching to the right.

Exercise 7. Compute the standard translations of the following formulas

(1) $\Diamond p \to \Diamond\Diamond p$;

(2) $\Diamond\Box p \vee \Box p$;

(3) $\Box(\Box p \to q) \vee \Box(\Box q \to p)$;

(4) $(p \wedge \Box(\Diamond p \to \Box q)) \to \Diamond\Box\Box q$;

(5) $\Box(\Box p \to p) \to \Box p$;

(6) $[\pi^*](p \to [\pi]p) \to (p \to [\pi^*]p)$.

Exercise 8. $(*)$ Is it possible to find a pair of pointed Kripke models $(\mathbb{M}, s)$ and $(\mathbb{M}', s')$ one of which is finitely branching such that $\mathbb{M}, s \leftrightsquigarrow \mathbb{M}', s'$ but $\mathbb{M}, s \not\Leftrightarrow \mathbb{M}', s'$?