INTRODUCTION TO MODAL LOGIC 2016
HOMEWORK 6

- Deadline: December 13 — at the **beginning** of Q & A session.
- Grading is from 0 to 100 points.
- Results from the exercise class may be used in the proofs
- Success!

(1) (40pt) (From the 2014 Exam)

(a) Let Σ be a finite subformula closed set. Let $\mathfrak{M} = (W, R, V)$ be a model such that (W, R) is a rooted transitive reflexive frame with no branching to the right. Show that a transitive filtration of $\mathfrak{M}$ through Σ is a rooted reflexive transitive frame with no branching to the right. See Homework 5 for the definition of no branching to the right. (Hint: start by showing that if r is a root of $\mathfrak{M}$, then $[r]$ is a root of the filtrated model $\mathfrak{M}_\Sigma$.)

Recall that a reflexive and transitive frame (W, R) is *rooted* if there is $x \in W$ such that for each $y \in W$ we have Rxy.

(b) Deduce that **S4.3** has the finite model property. See Homework 5 for the definition of **S4.3**.

(c) Deduce that **S4.3** is decidable.

(2) (20pt) Let Σ be a set of formulas and A any element of $At(\Sigma)$. Show that for all $\langle \pi^* \rangle \varphi \in \neg\mathrm{FL}(\Sigma)$: $\langle \pi^* \rangle \varphi \in A$ iff $(\varphi \in A$ or $\langle \pi \rangle \langle \pi^* \rangle \varphi \in A)$.

(3) (20pt) (From the 2015 Exam) Let $\mathcal{M} = (W, \{R_\pi\}_{\pi \in \Pi}, V)$ and $\mathcal{M}' = (W', \{R'_\pi\}_{\pi \in \Pi}, V')$ be two regular models of **PDL**. Let $Z \subseteq W \times W'$ be a bisimulation for each R_a and R'_a, where a is a basic program. Show by induction on the complexity of programs that then Z is a bisimulation for each R_π and R'_π, where $\pi \in \Pi$ is any program.

(4) (20pt) Show that if $[\pi^*](p \to [\pi]p) \to (p \to [\pi^*]p)$ is valid on a frame (W, R_π, R_{π^*}), then $R_{\pi^*} \subseteq (R_\pi)^*$