INTRODUCTION TO MODAL LOGIC 2016
HOMEWORK 5

- Deadline: November 29 — at the **beginning** of class.
- Grading is from 0 to 100 points.
- Results from the exercise class may be used in the proofs
- Success!

(1) (30pt)

(a) Prove that $\mathbf{S5} = \mathbf{K4} + (\Box p \to p, p \to \Box\Diamond p)$ is sound and complete with respect to the class of frames (W, R), where R is an equivalence relation (i.e., a reflexive, transitive and symmetric relation).

(b) Use (a) to show that **S5** is sound and complete with respect to the class of frames (W, R) where for each $w, v \in W$ we have Rwv. (Hint: use point-generated submodels, Prop 2.6 from the book.)

(2) (30pt) (From the 2014 Exam) In the following exercise you can use that the canonical model for **S4.3** is reflexive and transitive.

(a) Show that the canonical model for the modal logic

$$\mathbf{S4.3} = \mathbf{S4} + \Box(\Box p \to q) \vee \Box(\Box q \to p)$$

has no branching to the right. Recall that a reflexive Kripke frame has no branching to the right if

$$\forall x \forall y \forall z((Rxy \wedge Rxz) \to (Ryz \vee Rzy)).$$

You are not allowed to use Sahlqvist completeness theorem.

(b) Deduce that **S4.3** is sound and complete with respect to reflexive transitive frames with no branching to the right.

(3) (20pt) (Item (a) is from the 2014 Exam)

(a) Prove that for any modal formulas φ and ψ we have

$$\vdash_{\mathbf{K}} \Box\varphi \vee \Box\psi \text{ implies } \vdash_{\mathbf{K}} \varphi \text{ or } \vdash_{\mathbf{K}} \psi.$$

(Hint: use completeness of **K** with respect to Kripke frames.)

(b) Show that the above property does not hold for all normal modal logics. That is, give an example of a normal modal logic L which does not satisfy it.

(4) (20pt) Given a frame class $\mathcal{C}$, let $\Theta(\mathcal{C}) = Log(\mathcal{C})$ and given normal modal logic L let $Fr(L)$ be the class of frames where L is valid.

(a) What does it mean for a logic L if $L = \Theta(Fr(L))$? Give an example of a logic (modal or temporal) for which it does not hold.

(b) What does it mean for a frame class $\mathcal{C}$ if $\mathcal{C} = Fr(\Theta(\mathcal{C}))$? Give an example of a frame class $\mathcal{C}$ for which it does not hold.