INTRODUCTION TO MODAL LOGIC 2016
HOMEWORK 2

- Deadline: October 11 — at the **beginning** of class.
- Grading is from 0 to 100 points.
- Success!

(1) (30pt) Let Z_i be a bisimulation between models $\mathfrak{M}_1$ and $\mathfrak{M}_2$, for each $i \in I$. Are the following relations bisimulations between $\mathfrak{M}_1$ and $\mathfrak{M}_2$:

- the union $\bigcup_{i\in I} Z_i$?
- the intersection $\bigcap_{i\in I} Z_i$?

Give a proof or a counter-example.

(2) (20pt) Let $\mathfrak{M} = (\mathbb{N}, R, V)$ be a model such that nRm iff $m = n + 1$, $V(p) = \{3k : k \in \mathbb{N}\}$ and $V(q) = \{3k + 2 : k \in \mathbb{N}\}$. Let also $\Sigma = \{p, q, \Diamond q\}$. As usual we assume that $0 \in \mathbb{N}$.

Describe the models $\mathfrak{M}^s$ and $\mathfrak{M}^l$, where $\mathfrak{M}^s$ is the smallest and $\mathfrak{M}^l$ is the largest filtration of $\mathfrak{M}$ through Σ.

(3) (30pt) Prove that the filtrations $\mathfrak{M}^s$ and $\mathfrak{M}^l$ are indeed the smallest and largest filtrations, respectively.

(4) (20pt) Show that, given a transitive relation R, the relation R^t (the transitive (Lemmon) filtration from Lemma 2.42 in the Blackburn, de Rijke, Venema book) is indeed a filtration and that any filtration of a transitive model that makes use of R^t is guaranteed to be transitive.

(5) (**BONUS!**) (10pt) Show that any finite transitive filtration of a model based on the rationals with their usual ordering is a finite linear sequence of clusters, perhaps interspersed with singleton irreflexive points, no two of which can be adjecent.

Here a *cluster* on a transitive frame (W, R) is a subset $C \subseteq W$ that is a maximal equivalence relation under R. That is, the restriction of R to C is an equivalence relation, and this is not the case for any other $D \subseteq W$ such that $C \subsetneq D$.