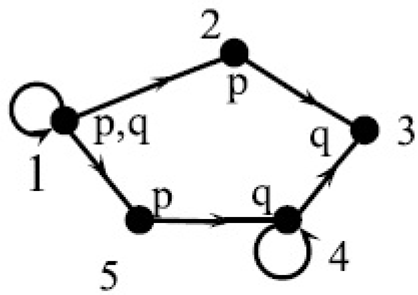
INTRODUCTION TO MODAL LOGIC 2016
HOMEWORK 1

- Deadline: September 27 — at the **beginning** of class.
- Electronic submissions can be sent to ana.varsa@gmail.com
- Grading is from 0 to 100 points.
- Success!

(1) (20pt) Consider the following model $\mathcal{M}$:



Check whether the following is true:

(a) $\mathcal{M}, 1 \models \Diamond q$

(b) $\mathcal{M}, 5 \models \Box q \to q$

(c) $\mathcal{M}, 5 \models \Box\Box(q \to \Box q)$

Evaluate r so that $\Box(p \vee \Diamond r)$ is satisfied only at 1, 3, 5.

(2) (30pt) Show that each of the following formulas is *not* valid by constructing a frame $\mathfrak{F} = (W, R)$ that refutes is.

(a) $\Box\bot$,

(b) $\Diamond p \to \Box p$,

(c) $\Diamond\Box p \to \Box\Diamond p$.

Find, for each of these formulas, a condition on frames that is sufficient to make the formula valid. Justify your solution.

(3) (30pt) This question is about temporal models with flow of time the real numbers, $\mathbb{R}$. Let $M = (\mathbb{R}, <, V)$ be a Kripke model.

(a) Suppose there are t, u in $\mathbb{R}$ with $M, u \models Gq$ and $M, t \models \neg q$.

(i) What is the order relation between t and u?

(ii) Show that there is a least number $b \in \mathbb{R}$ with $M, b \models Gq$. (It may help to remember that in $\mathbb{R}$, any non-empty set with a lower bound has a greatest lower bound.)

(b) Conclude that the 'Prior axiom', $FGq \wedge F\neg q \to F(Gq \wedge \neg PGq)$, is valid in the frame $(\mathbb{R}, <)$.

(c) Show that Prior's axiom is not valid in $(\mathbb{Q}, <)$.

(4) (20pt) Consider the binary until operator U. In a model $\mathfrak{M} = (W, R, V)$ its truth definition reads:

$\mathfrak{M}, t \models U(\varphi, \psi)$ iff there is a v such that tRv and $v \models \varphi$, and for all u such that tRu and uRv: $u \models \psi$.

Prove that U is not definable in the basic modal language. Hint: think about the following models, draw them with all the arrows to make sure that the relations are transitive:

- $\mathfrak{M}_1 = (W_1, R_1^t, V_1)$ such that $W_1 = \{s_0, s_1, t_0, t_1, v_0, v_1, u\}$, $R_1 = \{(s_0, t_0), (s_0, u), (t_0, v_0), (u, v_0), (s_1, t_1), (s_1, u), (t_1, v_1), (u, v_1)\}$ such that R_1^t is the transitive closure of R_1.
- $\mathfrak{M}_2 = (W_2, R_2^t, V_2)$ such that $W_1 = \{s', u', t', v'\}$, $R_1 = \{(s', t'), (s', u'), (t', v'), (u', v')\}$ such that R_1^t is the transitive closure of R_1.