

**EXERCISE CLASS 25-11-2016:
GENERAL FRAMES, FMP AND DECIDABILITY**

(1) Show that if \mathcal{C} is a class of general (temporal) frames then $Log(\mathcal{C})$ is a normal modal (temporal) logic.

(2) General frames and a consistent logic without Kripke frames

(a) Show that $(\mathbb{N}, <, A)$, where $A = \text{FinCofin}(\mathbb{N})$ (the collection of finite and cofinite subsets of \mathbb{N}) is a general frame.

(b) Show that $GFp \rightarrow FGp$ is valid on this general frame.

(c) Prove that the logic:

$$\mathbf{K}_t\mathbf{ThoM} = \mathbf{K}_t\mathbf{Tho} + GFp \rightarrow FGp$$

is consistent.

(3) Let φ be your favourite formula in the language of basic modal logic. Construct a general frame $\mathfrak{f} = (\mathfrak{F}, A)$ such that $\mathfrak{F} \not\models \varphi$ but $\mathfrak{f} \models \varphi$.

(4) Let $\mathfrak{M} = (\mathfrak{F}, V)$ be model and let $A_{\mathfrak{M}} := \{V(\varphi) : \varphi \in \text{Form}(\tau, \Phi)\}$. Show that $\mathfrak{f}_{\mathfrak{M}} := (\mathfrak{F}, A_{\mathfrak{M}})$ is a general frame.

(5) Recall that **Den** is the normal modal logic $\mathbf{K} + (\Box\Box p \rightarrow \Box p)$

(a) Prove that **Den** is sound and complete with respect to the class of dense frames.

(b) Show that **Den** has the finite model property and is decidable.

1. ADDITIONAL EXERCISES

(1) Use Exercise 4 to conclude that every normal modal logic is sound and complete with respect to some class of general frames. *Hint: Show that $\mathfrak{f}_{\mathfrak{M}^L} \models L$ for every consistent normal modal logic L*