Introduction to Modal Logic
Exercise class 5

October 20, 2016

(1) Our primitive connectives are $\vee$, $\neg$, $\top$, $\bot$ and $\Diamond$. Let p be a propositional letter that occurs in φ. Define by induction on φ: The occurrence of p is positive (negative).

(2) A formula φ is called positive (negative) in p if all occurrences of p are positive (negative).

- Show that if φ is positive in p then it is upward monotone in p, and if it is negative in p then it is downward monotone in p.
- What about the converse? If φ upward (downward) monotone in p does it follow that φ is positive (negative) in p?

(3) Can you give first-order correspondents for the following formulas?

- $\Box p \wedge p \to \Diamond\Diamond p$
- $\Diamond\Box p \to \Box\Diamond\Diamond p$