# Introduction to Modal Logic
## Exercise class 4

October 13, 2016

(1) Show that $\Diamond p \to \Diamond\Diamond p$ characterizes dense frames. In other words, show that

$$F \models \Diamond p \to \Diamond\Diamond p \text{ iff } F \text{ has the density property.}$$

**Density:** $\forall x, y(xRy \to \exists z(xRz \wedge zRy))$.

(2) Prove that the validity of formulas is preserved under taking bounded morphic images and disjoint unions.

(3) Show that the following properties of frames are not modally definable

    (a) $|W| > 23$,

    (b) every state has at most one predecessor,

(4) Consider the language with three diamonds, $\Diamond_1$, $\Diamond_2$ and $\Diamond_3$. For each of the frame conditions on the corresponding accessibility relations below, find out whether it is modally definable or not:

    (a) $R_1$ is the union of $R_2$ and $R_3$,

    (b) $R_1$ is the intersection of $R_2$ and $R_3$,

    (c) $R_1$ is the complement of $R_2$.

(5) Let
$$\varphi := \Box\Diamond\Box\Diamond p \leftrightarrow \Box\Diamond p$$

    (a) Show that $\varphi$ is not valid on all frames.

    (b) Show that $\varphi$ is valid on all transitive and reflexive frames.