Introduction to Modal Logic
Exercise class 3

September 24, 2015

(1) Show that the filtration $\mathfrak{M}^f$ of a model $\mathfrak{M}$ need not be its bounded morphic image.

(2) Show that the temporal modal formula $p \wedge Hp \to FHp$ is valid in the frame $(\mathbb{Z}, <)$ where $\mathbb{Z}$ is the set of integer numbers ordered by the usual less-than relation.

(3) What is the standard translation of the following formulas?

    (a) $\Box p \to q$

    (b) $\Diamond(\Box p \to q)$

    (c) $\Diamond \Box p \to q$

    (d) $\Box(\Diamond p \wedge q)$

(4) Prove that the standard translation is correct:

    (a) For all models $\mathfrak{M}$, all states $w \in \mathfrak{M}$, and all modal formulas φ,

$$\mathfrak{M}, w \Vdash \varphi \quad \Leftrightarrow \quad \mathfrak{M} \models ST_x(\varphi)[w].$$

    (b) For all models $\mathfrak{M}$ and all modal formulas φ,

$$\mathfrak{M} \Vdash \varphi \quad \Leftrightarrow \quad \mathfrak{M} \models \forall x ST_x(\varphi).$$