EXERCISE CLASS 2-12-2016: EXERCISES ON PDL

(1) Correspondence for **PDL**-formulas.
   (a) $\langle\pi_1;\pi_2\rangle p \leftrightarrow \langle\pi_1\rangle\langle\pi_2\rangle p$ is valid on a frame $(W, R_{\pi_1}, R_{\pi_2})$ iff $R_{\pi_1;\pi_2} = R_{\pi_1} \circ R_{\pi_2}$.[1]
   (b) $\langle\pi_1 \cup \pi_2\rangle p \leftrightarrow \langle\pi_1\rangle p \vee \langle\pi_2\rangle p$ is valid on a frame $(W, R_{\pi_1}, R_{\pi_2})$ iff $R_{\pi_1 \cup \pi_2} = R_{\pi_1} \cup R_{\pi_2}$.
   (c) $p \vee \langle\pi\rangle\langle\pi^*\rangle p \to \langle\pi^*\rangle p$ is valid on a frame (W, R_π, R_{π^*}) iff $(R_\pi)^* \subseteq R_{\pi^*}$.

(2) A logic L is *compact for the class $\mathcal{C}$ of Kripke frames* if the following condition is met: for every set of formulas Σ, if every finite subset of Σ is satisfiable in a model based on a frame in $\mathcal{C}$, then Σ itself can be satisfied in a model based on a frame in $\mathcal{C}$. Show that **PDL** is not compact for the class of regular frames.

(3) Show that the induction axiom $[\pi^*](p \to [\pi]p) \to (p \to [\pi^*]p)$ is not canonical.

(4) Let Σ be a non-empty finite set of formulas in the language of **PDL**. Prove that $At(\Sigma) = \{\Gamma \cap \neg FL(\Sigma) \colon \Gamma \text{ a } \mathbf{PDL}\text{-MCS}\}$.

(5) Prove the properties (i)–(iv) of atoms listed in Lemma 4.81.

(6) Show that the finite models defined in the **PDL** completeness proof can be obtain (up to isomorphism) via certain filtrations.

1. Additional exercises

(1) Explain why **PDL** is a decidable logic.

(2) Let Σ be a non-empty finite set of **PDL**-formulas. Show that $\vdash_{\mathbf{PDL}} \bigvee_{A \in At(\Sigma)} \widehat{A}$.

(3) Let $\Sigma = \{\varphi\}$ be a singleton set (of formulas in the language of **PDL**). Show that $\neg\mathrm{FL}(\Sigma)$ is finite. *Hint: This is not so easy.*

[1] In BdRV $R_{\pi_1} \circ R_{\pi_2}$ is also denoted $R_{\pi_1}; R_{\pi_2}$.