Introduction to Modal Logic. Exercise class 1

September 15, 2016

(1) Let $W_1 = \{1,2,3,4\}$, $W_2 = \{1,2,3,4\}$, $R_1 = \{(1,2),(2,4),(3,1),(3,4)\}$, $R_2 = \{(1,2),(1,3),(2,4),(3,4)\}$.

(a) Draw the frames $F_1 = (W_1, R_1)$ and $F_2 = (W_2, R_2)$ and extend R_2 with $(1,1),(2,2),(3,3),(4,4)$. Do frame F_1 and the extended version of F_2 validate the same formulas? Explain your answer.

(b) Consider a valuation V_1 for F_1 and a valuation V_2 for F_2 such that $V_1(p) = V_2(p) = \{2,3,4\}$, $V_2(q) = \{3\}$ and $V_1(t) = V_2(t) = \emptyset$ for all the other propositional variables $t \neq p, q$.

i. Does the world 3 in M_1 satisfies the same formulas as the world 2 in M_2?

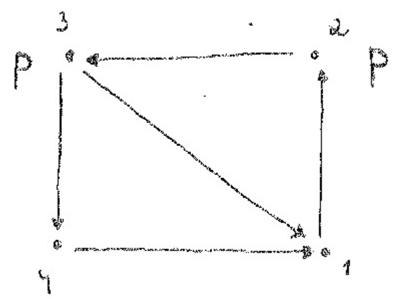
ii. How can you modify the frames (the minimum amount of modifications) so that the world $3 \in M_1$ satisfy the same formulas as the world $2 \in M_2$? Hint: Think of modifying one of the relations R_i.

iii. With the valuation V_2, the world 1 in M_2 and the world 4 in M_2 satisfy $\Box p$ simultaneously?

iv. What changes you would need to do in the model M_2 so that the worlds 1 and 4 in M_2 satisfy the same propositional variables, $\Diamond p$, $\Box p$ and $\Diamond q$? Evaluate if with your changes they satisfy the same formulas.

Explain your answers and, if necessary, make drawings.

(2) In the model below determine where the formula $\varphi := \Diamond\Box\Diamond p$ is true. Compute $\neg\varphi$ and do the same analysis.



(3) Consider the following formulas:

(a) $\Diamond p \to \Diamond\Diamond p$

(b) $(\Box p \wedge \Diamond q) \to \Diamond(p \wedge q)$

(c) $\Diamond(p \vee q) \to (\Diamond p \vee \Diamond q)$

(d) $\Box\Diamond p \to \Diamond\Box p$

i. Which of these formulas are valid/not valid in all frames? Provide a formal proof or a counterexample.

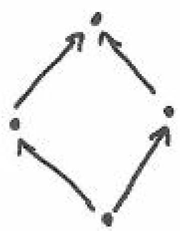
ii. Find a class of frames for which the formula $\Diamond p \to \Diamond\Diamond p$ is valid.

(4) Is it possible to express the following operator in the basic modal language?

$\mathfrak{M}, w \models D\varphi$ iff there is some $x \neq w : \mathfrak{M}, x \models \varphi$

(5) Consider the model $\mathfrak{M} = (W, R, V)$ such that $W = \{a, b, c, d\}$, $R = \{(a, b), (a, c), (b, d), (c, d), (d, d)\}$ and $V(p) = \{b, c, d\}$ and $V(q) = \emptyset$ for all the other propositional variables $q \in \Theta$.

Use this model to show that bounded morphisms and generated submodels (may) yield different models.



(6) Solve Exercise 2.2.1 (page 71) of Blackburn et al and answer in detail the following questions:

(a) Is the model in the left a generated submodel of the model in the right?

(b) Can you find a generated submodel for each of these models?

(7) Consider the model $(\mathbb{N}, R, V)$ such that nRm iff $n = m+1$, $V(p) = ODD$, $V(q) = EVEN$ (ODD is the set of all odd numbers and EVEN is the set of all even numbers) and the valuation is empty for all the other propositional variables. Find a bounded morphism for $(\mathbb{N}, R, V)$.