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1. Religion vs rewriting

Religion $=$ re $+$ legere $=$ read again

1. Religion vs rewriting

Religion = re + legere = read again

Religion: same text, new meaning

1. Religion vs rewriting

Religion = re + legere = read again

Religion: same text, new meaning

Rewriting: new text, same meaning

2. Rewriting

Two central questions to ask

- What are the objects?
- What are the rules?

3. High-school arithmetic

What are the objects?

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What are the objects?

Arithmetic expressions
$(3+5)\cdot(1+2)$, $8\cdot(1+2)$, $(3+5)\cdot 3$,...

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What are the rules?

3. High-school arithmetic

What are the objects?

Arithmetic expressions
$(3+5)\cdot(1+2)$, $8\cdot(1+2)$, $(3+5)\cdot 3$,...

What are the rules?

rules
$3+5\rightarrow 8$, $1+2\rightarrow 3$, $(x+y)\cdot z\rightarrow x\cdot z+y\cdot z$,...

3. High-school arithmetic

What are the objects?

Arithmetic expressions
$(3 + 5) \cdot (1 + 2)$, $8 \cdot (1 + 2)$, $(3 + 5) \cdot 3$,...

What are the rules?

rules
$3 + 5 \to 8$, $1 + 2 \to 3$, $(x + y) \cdot z \to x \cdot z + y \cdot z$,...

a reduction (rewrite sequence)
$(3 + 5) \cdot (1 + 2) \to 8 \cdot (1 + 2) \to 8 \cdot 3 \to 24$

4. Counting

What are the objects?

4. Counting

What are the objects?

Natural numbers

4. Counting

What are the objects?
Natural numbers

What are the rules?

4. Counting

What are the objects?
Natural numbers

What are the rules?
plus one (take successor)

4. Counting

What are the objects?
Natural numbers

What are the rules?
plus one (take successor)

a reduction
$2 \to 3 \to 4 \to 5 \to \cdots$

5. Syracuse problem

What are the objects?

5. Syracuse problem

What are the objects?
Natural numbers

5. Syracuse problem

What are the objects?
Natural numbers

What are the rules?

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What are the objects?
Natural numbers

What are the rules?

for $n > 1$, $n \rightarrow n'$ if

- $n' = n/2$ in case n is even
- $n' = 3n + 1$ in case n is odd

5. Syracuse problem

What are the objects?
Natural numbers

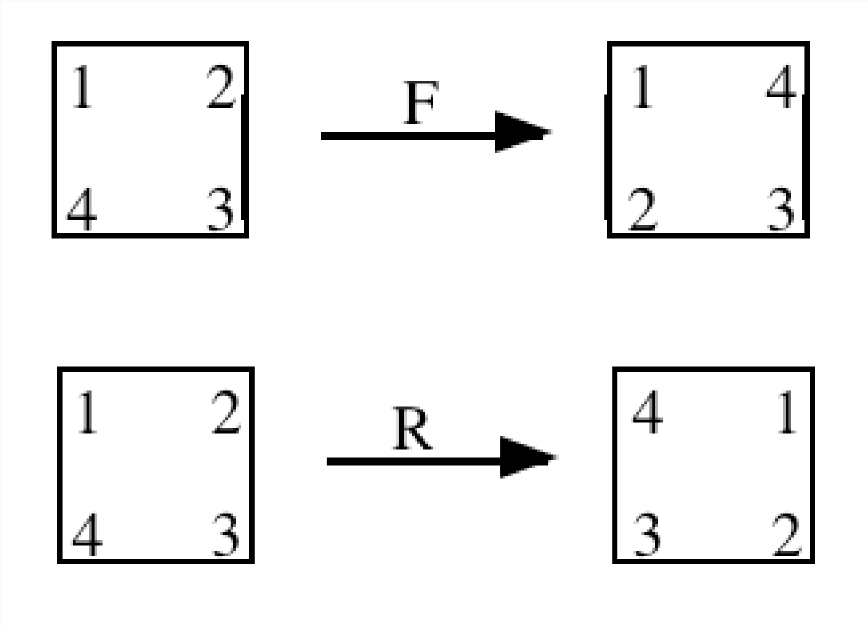
What are the rules?
for $n > 1$, $n \to n'$ if

- $n' = n/2$ in case n is even
- $n' = 3n + 1$ in case n is odd

a reduction

$$7 \to 22 \to 11 \to 34 \to 17 \to 52 \to 26 \to 13$$
$$\to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$$

6. Symmetries of the square



What are the objects?

What are the objects?

Sequences of F(lip), R(otate) operations

Examples $FRFRFRFRFR$

What are the objects?

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Wat are the rules?

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Examples $FRFRFRFRFR$

Wat are the rules?

$$\begin{aligned} FF &\rightarrow \lambda \\ RRRR &\rightarrow \lambda \\ FR &\rightarrow RRRF \end{aligned}$$

What are the objects?
Sequences of F(lip), R(otate) operations
Examples $FRFRFRFRFR$

Wat are the rules?

$$\begin{aligned} FF &\rightarrow \lambda \\ RRRR &\rightarrow \lambda \\ FR &\rightarrow RRRF \end{aligned}$$

Typical question:

$$FRFRFRFRFR \stackrel{?}{=} RRFRRFRRFRRF$$

7. Rewriting theory

Wittgenstein: don't ask for its meaning ask for its use

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Rewriting: don't ask for its meaning ask for its rules!

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Meaning $=$ reduction (to normalform)

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Wittgenstein: don't ask for its meaning ask for its use

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Meaning = reduction (to normalform)

Rewriting theory = theory of meaning via rules

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Wittgenstein: don't ask for its meaning ask for its use

Rewriting: don't ask for its meaning ask for its rules!

Meaning = reduction (to normalform)

Rewriting theory = theory of meaning via rules

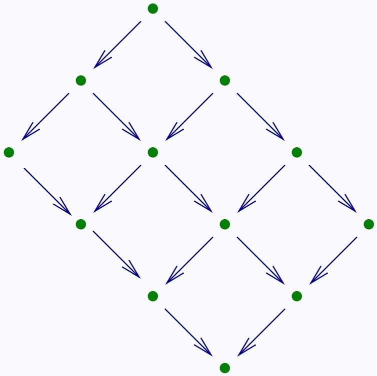
Unique meaning is guaranteed if rules are

- Terminating, and
- Confluent

8. Termination and Confluence

Termination (SN, Strong Normalisation): no infinite reductions

Confluence (CR): micro-parallelism, macro-determinism



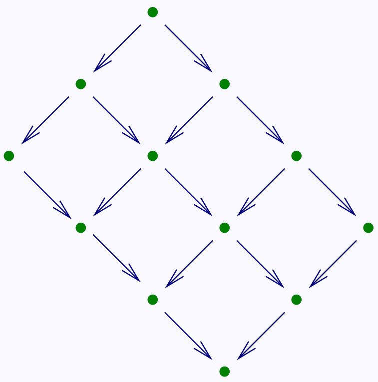
Which rivers join at Koblenz

8. Termination and Confluence

Termination (SN, Strong Normalisation): no infinite reductions

$$\bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \cdots$$

Confluence (CR): micro-parallelism, macro-determinism



Which rivers join at Koblenz
Rine and Moezel

9. SN and CR in highschool arithmetic

Is arithmetic SN?

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Is arithmetic SN?

Yes (here, the result is 24), but . . .

9. SN and CR in highschool arithmetic

Is arithmetic SN?
Yes (here, the result is 24), but ...
depends on the rules!

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Yes (24 is the unique result), but ...

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Yes (here, the result is 24), but . . .
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9. SN and CR in highschool arithmetic

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Yes (here, the result is 24), but ...
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is arithmetic CR?
Yes (24 is the unique result), but ...
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What are the normal forms?

9. SN and CR in highschool arithmetic

Is arithmetic SN?
Yes (here, the result is 24), but ...
depends on the rules!

is arithmetic CR?
Yes (24 is the unique result), but ...
depends on the rules

What are the normal forms?
the natural numbers
(if expression still contains, say, a $+$, some rule can be applied)

10. SN in CR in counting

Is counting SN?

10. SN in CR in counting

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No, of course not

10. SN in CR in counting

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Yes, at every moment exactly one step (deterministic)

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10. SN in CR in counting

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No, of course not
Is counting CR?
Yes, at every moment exactly one step (deterministic)

What are the normal forms?
No

11. SN and CR in Syracuse

Is the Syracuse process SN?

11. SN and CR in Syracuse

Is the Syracuse process SN?
Unknown!

11. SN and CR in Syracuse

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Unknown!

Is the Syracuse process CR?

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Unknown!

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Yes, at any moment exactly one step

What are the normal forms?

11. SN and CR in Syracuse

Is the Syracuse process SN?
Unknown!

Is the Syracuse process CR?
Yes, at any moment exactly one step

What are the normal forms?
Just one.

12. SN and CR for symmetries

Is the simplification process SN?

12. SN and CR for symmetries

Is the simplification process SN?
Yes (why?)

12. SN and CR for symmetries

Is the simplification process SN?
Yes (why?)

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Yes, all critical pairs are CR
$F \leftarrow FFF \rightarrow F$
$RRRFRRR \leftarrow FRRRR \rightarrow F$

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Is the simplification process SN?
Yes (why?)

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Yes, all critical pairs are CR
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12. SN and CR for symmetries

Is the simplification process SN?
Yes (why?)

Is the simplification process CR?
Yes, all critical pairs are CR
$F \leftarrow FFF \rightarrow F$
$RRRFRRR \leftarrow FRRRR \rightarrow F$

What are the normal forms?
λ, R, RR, RRR, F, RF, RRF, $RRRF$.

13. Knots

What are the objects?

13. Knots

What are the objects?
knots

13. Knots

What are the objects?
knots

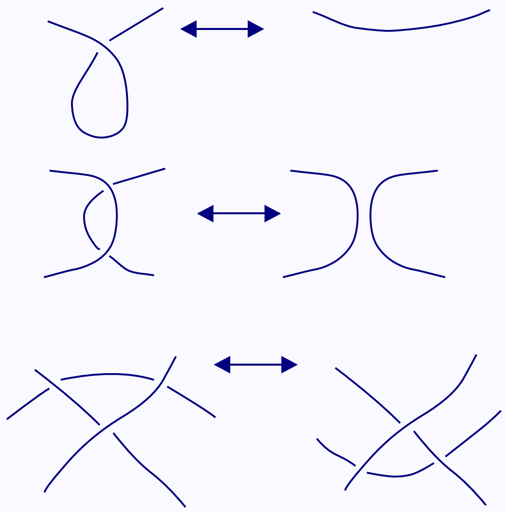
What are the rules?

13. Knots

What are the objects?
knots

What are the rules?
disentanglement

13.1. Reidemeister moves



13.2. Reidemeister moves good rules?

Termination: of course knot (bidirectional, cycle)
Confluence: of course (bidirectional, undo)

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Proof: try it
(for instance with a shoelaces)

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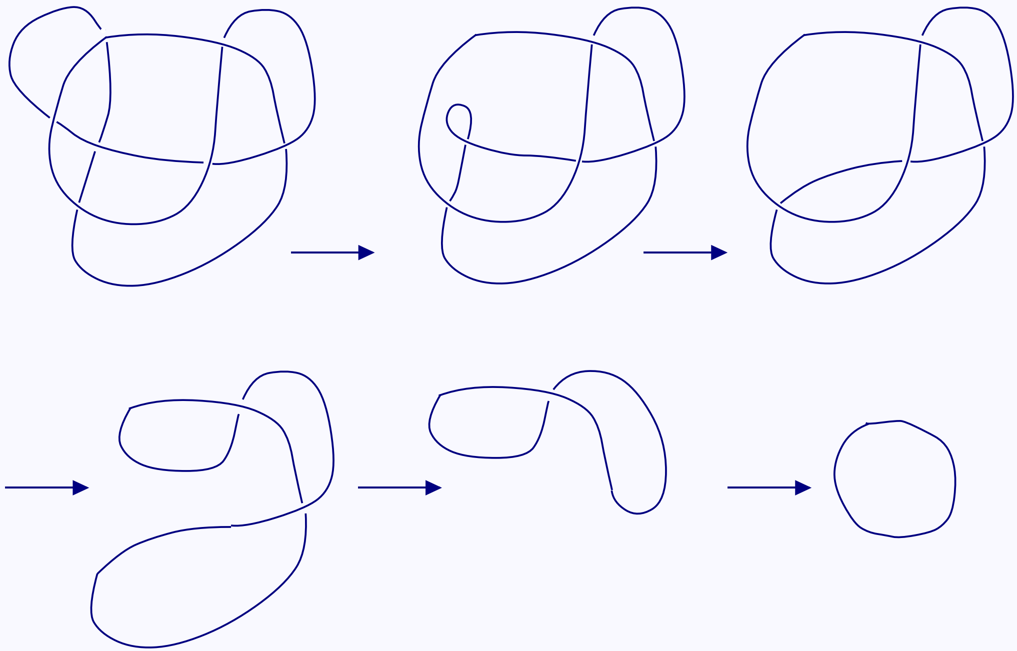
Termination: of course knot (bidirectional, cycle)
Confluence: of course (bidirectional, undo)

Rules are sound
Proof: try it
(for instance with a shoelaces)

Rules are complete
Reidemeister

Convertibility to the unknot?
Decidable (bound on number of Reidemeister moves)
In NP, not known whether it is in P.

13.3. A disentanglement



14. Tautology checking

What are the objects?

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What are the objects?
(propositional) formulas
$p \Rightarrow (q \Rightarrow p)$, $\neg p \vee p$, p ...

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What are the rules?

14. Tautology checking

What are the objects?
(propositional) formulas
$p \Rightarrow (q \Rightarrow p)$, $\neg p \vee p$, p ...

What are the rules?
equivalence preserving simplificiation (Herbrand moves)

14.1. Herbrand moves

$$\begin{array}{lcl} x \Rightarrow y & \rightarrow & x \cdot y + x + 1 \\ x \vee y & \rightarrow & x \cdot y + x + y \\ \neg x & \rightarrow & x + 1 \\ x + 0 & \rightarrow & x \\ x + x & \rightarrow & 0 \\ x \cdot 0 & \rightarrow & 0 \\ x \cdot 1 & \rightarrow & x \\ x \cdot x & \rightarrow & x \\ x \cdot (y + z) & \rightarrow & x \cdot y + x \cdot z \end{array}$$

$+$ is exclusive or (either or) en $\cdot$ is conjunction (and)

vb $p \vee \underline{\neg p} \rightarrow \underline{p \vee (p+1)} \rightarrow p \cdot (p+1) + \underline{p + p} + 1 \rightarrow \underline{p \cdot (p+1) + 0} + 1 \rightarrow \underline{p \cdot (p+1)} + 1 \rightarrow p \cdot p + p \cdot 1 + 1 \twoheadrightarrow 1$

14.2. Herbrand moves good rules?

Termination?

14.2. Herbrand moves good rules?

Termination?

Yes!

14.2. Herbrand moves good rules?

Termination?
Yes!

Confluent?

14.2. Herbrand moves good rules?

Termination?
Yes!

Confluent?
Yes!

14.2. Herbrand moves good rules?

Termination?
Yes!

Confluent?
Yes!

Wat are the normal forms?

14.2. Herbrand moves good rules?

Termination?

Yes!

Confluent?

Yes!

Wat are the normal forms?

0, 1, 'or-and polynomials' (e.g. $p \cdot q + q \cdot r$)

14.2. Herbrand moves good rules?

Termination?
Yes!

Confluent?
Yes!

Wat are the normal forms?
0, 1, 'or-and polynomials' (e.g. $p \cdot q + q \cdot r$)

Completeness (Herbrand):
Every tautology has 1 as normal form

14.3. A tautology checking

$$
\begin{aligned}
& p \Rightarrow (q \Rightarrow p) \\
& \quad \rightarrow \ p(q \Rightarrow p) + p + 1 \\
& \quad \rightarrow \ p(qp + q + 1) + p + 1 = p((qp + q) + 1) + p + 1 \\
& \quad \rightarrow \ p(qp + q) + p1 + p + 1 \\
& \quad \rightarrow \ pqp + pq + p1 + p + 1 \\
& \quad \rightarrow \ pqp + pq + p + p + 1 = pqp + pq + (p + p) + 1 \\
& \quad \rightarrow \ pqp + pq + 0 + 1 = pqp + (pq + 0) + 1 \\
& \quad \rightarrow \ pqp + pq + 1 = (pp)q + pq + 1 \\
& \quad \rightarrow \ pq + pq + 1 = (pq + pq) + 1 \\
& \quad \rightarrow \ 0 + 1 = 1 + 0 \\
& \quad \rightarrow \ 1
\end{aligned}
$$

'So' it works . . .

15. Functional Programming

What are the objects?

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What are the objects?
Expressions
e.g. arithmetic expression
but 'user-defined' in general

15. Functional Programming

What are the objects?
Expressions
e.g. arithmetic expression
but ‘user-defined’ in general

What are the rules?

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Expressions
e.g. arithmetic expression
but ‘user-defined’ in general

What are the rules?
e.g. rules of arithmetic
but ‘your program’ in general

15. Functional Programming

What are the objects?
Expressions
e.g. arithmetic expression
but ‘user-defined’ in general

What are the rules?
e.g. rules of arithmetic
but ‘your program’ in general

Slogan: declarative programming = user-defined computation
declarative: what not how: e.g. prolog and haskell

15.1. Sieve of Eratosthenes

Rules:

$$
\begin{aligned}
\mathit{filter}(x : y, 0, m) &\rightarrow 0 : \mathit{filter}(y, m, m) \\
\mathit{filter}(x : y, s(n), m) &\rightarrow x : \mathit{filter}(y, n, m) \\
\mathit{sieve}(0 : y) &\rightarrow \mathit{sieve}(y) \\
\mathit{sieve}(s(n) : y) &\rightarrow s(n) : \mathit{sieve}(\mathit{filter}(y, n, n)) \\
\mathit{nats}(n) &\rightarrow n : \mathit{nats}(s(n)) \\
\mathit{primes} &\rightarrow \mathit{sieve}(\mathit{nats}(s(s(0))))
\end{aligned}
$$

15.2. Eratosthenes moves good rules?

Terminating?

15.2. Eratosthenes moves good rules?

Terminating?
No, ‘of course’ not
The list of prime numbers is infinite

Confluent Yes (rules are orthogonal)

15.2. Eratosthenes moves good rules?

Terminating?
No, 'of course' not
The list of prime numbers is infinite

Confluent Yes (rules are orthogonal)

Normal forms?

15.2. Eratosthenes moves good rules?

Terminating?
No, 'of course' not
The list of prime numbers is infinite

Confluent Yes (rules are orthogonal)

Normal forms?
'Infinite normal form' of $nats(0)$ is the list of natural numbers
'Infinite normal form' of $primes$ is the list of prime numbers

16. Bowls and Beans

What are the objects?

16. Bowls and Beans

What are the objects?
Situations: distributions of finite numbers of beans over an (two sided) infinite array of bowls

16. Bowls and Beans

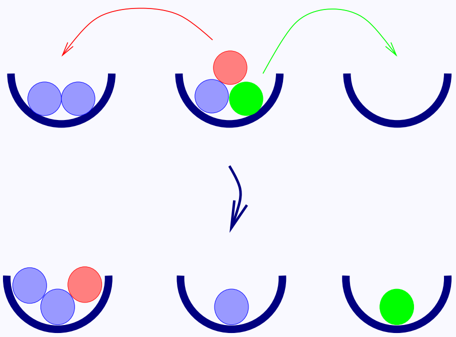
What are the objects?
Situations: distributions of finite numbers of beans over an (two sided) infinite array of bowls

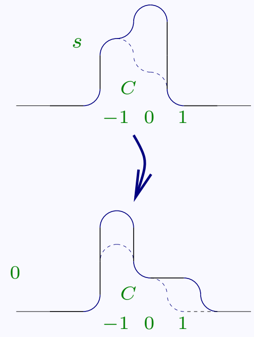
What are the rules?

16. Bowls and Beans

What are the objects?
Situations: distributions of finite numbers of beans over an (two sided) infinite array of bowls

What are the rules?
If a bowl contains at least two beans
move one bean to the bowl on its left and right





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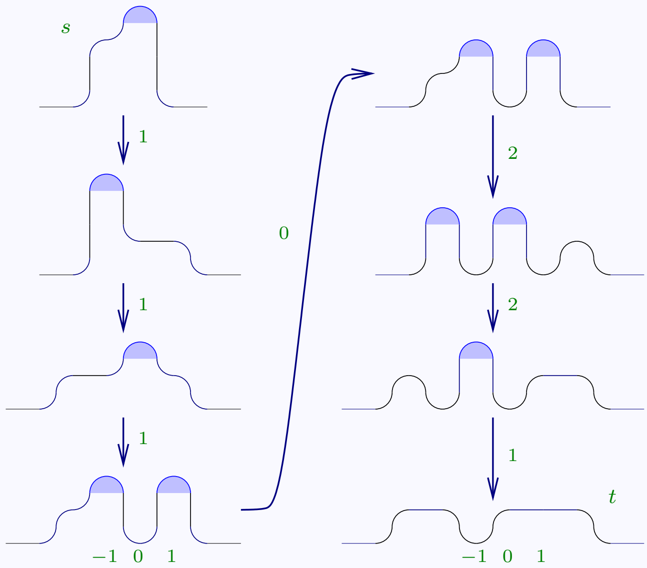
16.1. A reduction sequence



16.2. The puzzle

Show that

1. the process stops, for an arbitrary situation

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Show that

1. the process stops, for an arbitrary situation

   Termination!

2. a given situation always terminates in the same final situation, independent of the moves

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Show that

1. the process stops, for an arbitrary situation

   Termination!

2. a given situation always terminates in the same final situation, independent of the moves

   Confluence!

3. Moreover, the number of steps to reach the final situation only depends on the initial situation

16.2. The puzzle

Show that

1. the process stops, for an arbitrary situation

   Termination!

2. a given situation always terminates in the same final situation, independent of the moves

   Confluence!

3. Moreover, the number of steps to reach the final situation only depends on the initial situation

   balanced confluence

16.3. Oplossing

Theorem: all this holds if

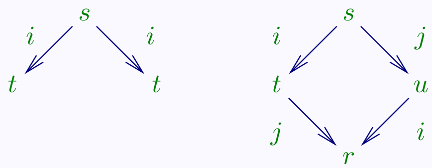
- Steps can be balanced
- a final state can be reached (WN)

16.3. Oplossing

Theorem: all this holds if

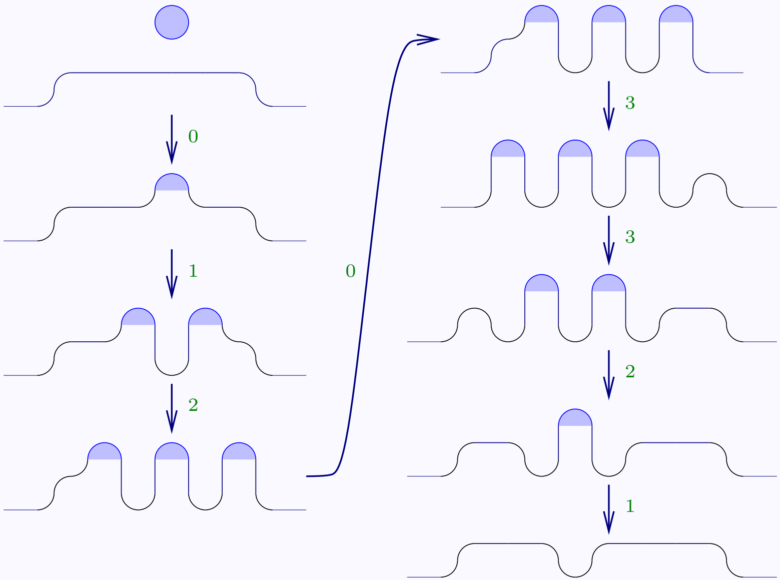
- Steps can be balanced
- a final state can be reached (WN)

Steps can be balanced



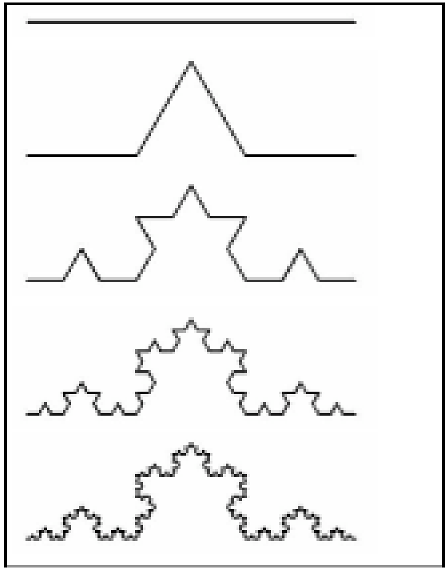
16.4. WN

Adjoin beans one at the time



Wave-front moving inside-out, then outside-in, then stop

17. Snowflakes



18. Solution cola-gene

```
operators
  t,a,g,c : unary
  x : variable

axioms
  t(c(a(t(x)))) = t(x);
  g(a(g(x))) = a(g(x));
  c(t(c(x))) = t(c(x));
  a(g(t(a(x)))) = a(x);
  t(a(t(x))) = c(t(x));

Result:
{ [3] t(a(t(x))) -> t(x),
```

```
[6] c(t(x)) -> t(x),
[7] g(a(x)) -> a(x),
[8] a(g(t(x))) -> a(t(x)),
[9] a(t(a(x))) -> a(x),
[10] t(c(a(x))) -> t(a(x)) } (6 rules)
```

confluent and terminating!

Hence: convertible $\iff$ same normalvorm

19. Milk-gene

```
TAGCTAGCTAGCT ->6
TAGCTAGCTAGT ->8
TAGCTAGCTAT ->3
TAGCTAGCT ->6
TAGCTAGT ->8
TAGCTAT ->3
TAGCT ->6
TAGT ->8
TAT ->3
T
```

20. Cola-gene

```
CTGACTGACT ->6
CTGACTGAT ->7
CTGACTAT ->3
CTGACT ->6
CTGAT ->7
CTAT ->3
CT ->6
T
```

Hence: Cola-gene convertible to milk-gene

21. Retro-virus

```
CTGCTACTGACT ->6
CTGCTACTGAT ->7
CTGCTACTAT ->3
CTGCTACT ->6
CTGCTAT ->3
CTGCT ->6
CTGT ->6
TGT
```

Hence: not convertible to others