Lambek–Grishin Calculus Extended to Connectives of Arbitrary Aritiy

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Outline

Why formal linguistics?

Cognitive Artificial Intelligence:

- Understanding humans leads to better software
- Software leads to better understanding humans

Why formal linguistics?

Cognitive Artificial Intelligence:

- Understanding humans leads to better software
- Software leads to better understanding humans

Within formal linguistics:

- Constrains on human language can be applied in software
- Formal language theory can shine light on human language processing

Lambek calculus **NL**

Set of **types** T:

- p with $p \in \mathit{Atoms}$
- a/b with $a, b \in T$
- $b\backslash a$ with $a, b \in T$
- $a \otimes b$ with $a, b \in T$

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Formulas: $a \longrightarrow b$ with a, b types

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Axiom:

$a \longrightarrow a$ (Axiom)

Transitivity:

$$\frac{a \longrightarrow b \qquad b \longrightarrow c}{a \longrightarrow c}$$

Residuation rules:

$$\frac{b \longrightarrow a\backslash c}{a \otimes b \longrightarrow c} \qquad \frac{a \otimes b \longrightarrow c}{a \longrightarrow c/b}$$

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Example:

$$\frac{a\backslash b \longrightarrow a\backslash b}{a \otimes (a\backslash b) \longrightarrow b}$$

Lambek grammar

The **yield** of a formula

- $\text{yield}(a \otimes b) = \text{yield}(a), \text{yield}(b)$
- $\text{yield}(a) = a$ in other cases

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Lambek grammar $\mathcal{L}(\Sigma, D, \varphi)$:

- Σ Terminal symbols
- D Goal symbol
- $\varphi : \Sigma \to \mathcal{P}(T)$ assigns types to terminal symbols

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The *language generated by a Lambek grammar* $\mathcal{L}(\Sigma, D, \varphi)$ is the set of expressions $t_1 \dots t_n$ over the alfabet Σ such that there is a derivable formula with yield $b_1 \dots b_n \to D$ such that $b_i \in \varphi(t_i)$ for all $i \leq n$.

Example: Alice sees the house

$$
\begin{aligned}
\varphi(\mathit{Alice}) &= \{np\} \\
\varphi(\mathit{sees}) &= \{(np\backslash s)/np\} \\
\varphi(\mathit{the}) &= \{np/n, np\} \\
\varphi(\mathit{house}) &= \{n\}
\end{aligned}
$$

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With brackets: $\mathit{Alice} \otimes (\mathit{sees} \otimes (\mathit{the} \otimes \mathit{house}))$

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$$\frac{\Gamma \to b \quad \Delta \to b\backslash a}{\Gamma \otimes \Delta \to a}\ \backslash E$$

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Lambek–Grishin calculus [Moortgat, 2007]

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Grishin interactions:

$(a \oslash b) \otimes c \to a \oslash (b \otimes c)$ $\quad$ $a \otimes (b \oslash c) \to (a \otimes b) \oslash c$

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Motivation generalized **LG**

	Lambek calculus [Lambek, 1958]	

Motivation generalized **LG**

Assymetric (Context-free)	Lambek calculus [Lambek, 1958]	
Symmetric (Mildly context-sensitive)	Lambek–Grishin calculus [Moortgat, 2007]	

Motivation generalized **LG**

	Binary	**Arbitrary arity**
Assymetric (Context-free)	Lambek calculus [Lambek, 1958]	*n*-ary Lambek calculus [Buszkowski, 1986]
Symmetric (Mildly context-sensitive)	Lambek–Grishin calculus [Moortgat, 2007]	

Arbitrary arity: useful for ‘give him the present’ and ‘coffee or tea’

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Assymetric (Context-free)	Lambek calculus [Lambek, 1958]	*n*-ary Lambek calculus [Buszkowski, 1986]
Symmetric (Mildly context-sensitive)	Lambek–Grishin calculus [Moortgat, 2007]	*n*-ary Lambek–Grishin calculus (This work)

Arbitrary arity: useful for ‘give him the present’ and ‘coffee or tea’

The types

Non-atomic types

		Multiplicative	**Implicative**
Binary	**Left**	$a \otimes b$	a/b, $a\backslash b$
	Right	$a \oplus b$	$a \oslash b$, $a \obslash b$

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Generalized, $n = 2$

	Multiplicative	Implicative
Left	$f_\bullet(a, b)$	$f^1_\rightarrow(a, b)$, $f^2_\rightarrow(a, b)$
Right	$g_\bullet(a, b)$	$g^1_\rightarrow(a, b)$, $g^2_\rightarrow(a, b)$

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Generalized

	Multiplicative	Implicative
Left	$f_\bullet(a_1, \ldots, a_n)$	$f^i_\rightarrow(a_1, \ldots, a_n)$
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The axioms and rules

Identity:

$$\frac{}{a \longrightarrow a}$$

Transitivity:

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$$\frac{f_\bullet(a_1, \ldots, a_n) \to b}{a_i \to f^i_{\to}(a_1, \ldots, a_{i-1}, b, a_{i+1}, \ldots, a_n)}$$

$$\frac{b \to g_\bullet(a_1, \ldots, a_n)}{g^i_{\to}(a_1, \ldots, a_{i-1}, b, a_{i+1}, \ldots, a_n) \to a_i}$$

The Grishin interactions

Grishin interactions **Gr(i, j)** ($1 \leq i \leq n$, $1 \leq j \leq n$):

$$\frac{g^i_{\rightarrow}(b_1, \ldots, b_{i-1}, f_{\bullet}(a_1, \ldots, a_{j-1}, b_i, a_{j+1}, \ldots, a_n), b_{i+1}, \ldots, b_n) \rightarrow d}{f_{\bullet}(a_1, \ldots, a_{j-1}, g^i_{\rightarrow}(b_1, \ldots, b_n), a_{j+1}, \ldots, a_n) \rightarrow d}$$

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Example: Grishin interaction for $n = 3$, $i = 1$, $j = 2$.

$$\frac{g^1_{\rightarrow}(f_\bullet(a_1, b_1, a_3), b_2, b_3) \rightarrow d}{f_\bullet(a_1, g^1_{\rightarrow}(b_1, b_2, b_3), a_3) \rightarrow d}$$

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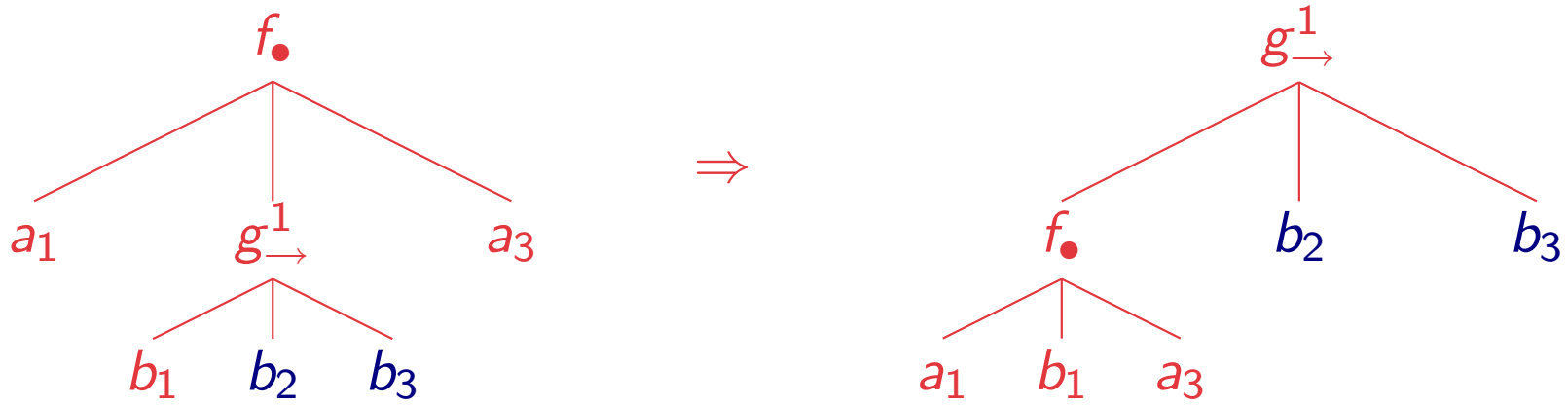

Properties of the calculus

Advantages:

- Branching of arbitrary order
- At least mildly context-sensitive
- Decidability
- Complete with respect to Kripke semantics
- Derivations can be interpreted using continuation semantics

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Disadvantage:

- Hard to find the right types

Generative capacity of **LG**

Chomsky hierarchy:

1. Regular
2. Context-free

4. Context-sensitive
5. Recursive
6. Recursively enumerable

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Complexity **LG**:

- At least mildly context-sensitive [Moot, 2008]
- At most recursive
- New result: stronger than context-sensitive

Generative capacity of **LG** (2)

Theorem

Any language that is the intersection of a context-free language and the permutation closure of a context-free language can be recognized by **LG**.

Examples:

- $\pi(a^n b^n c^n)$ (permutation closure of context-free language)
- $a^n b^n c^n d^n e^n$ (intersection of $a^i b^j c^k d^l e^m$ and permutation of $(abcde)^n$

Spinal Ajdukiewicz–Bar-Hillel grammar $\mathbf{AB_s}$

$\mathbf{AB_s}$ (spinal Ajdukiewicz–Bar-Hillel-grammar)

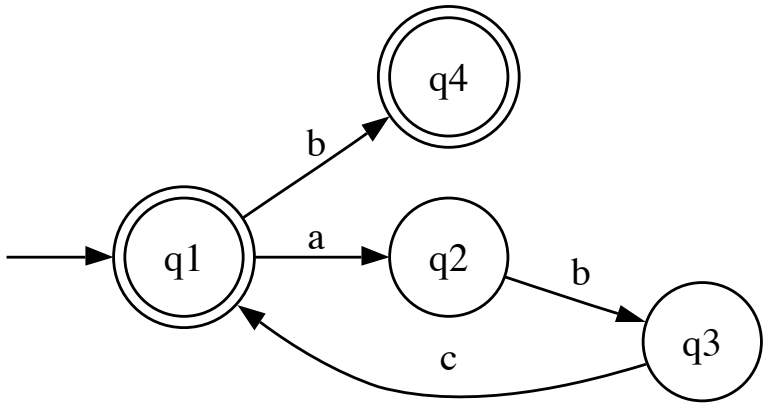
- Types: a and $a \backslash b$ where a and b atoms
- Derivation rule: only $a, (a \backslash b) \rightarrow b$
- Set of goal types instead of one goal type

All derivable sequents have the following form:
$a_0, (a_0 \backslash a_1), (a_1 \backslash a_2), \ldots, (a_{n-1} \backslash a_n) \rightarrow a_n$

Modelling of finite automata in $\mathbf{AB_s}$

Finite automata can be modelled in $\mathbf{AB_s}$

Example:



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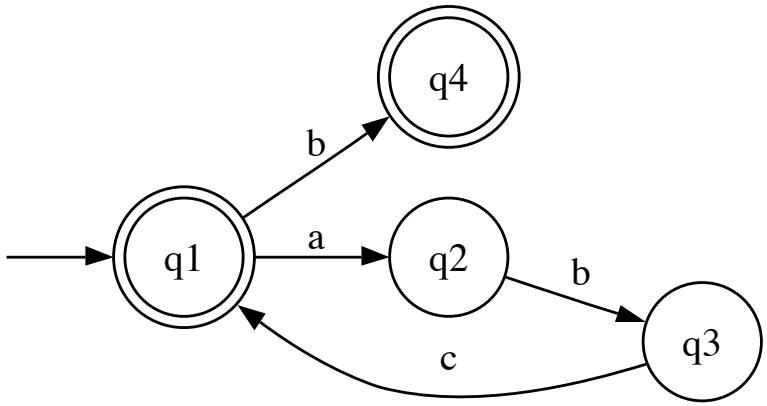

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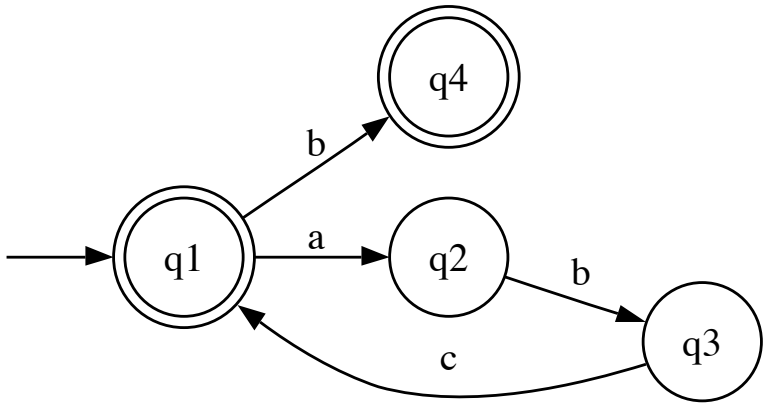

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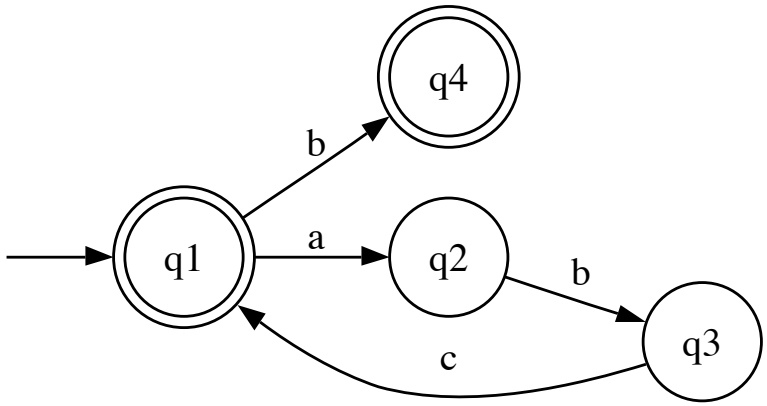

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- φ:

t	$\varphi(t)$
a	$q1 \backslash q2$
b	$q2 \backslash q3$
c	$q3 \backslash q1$

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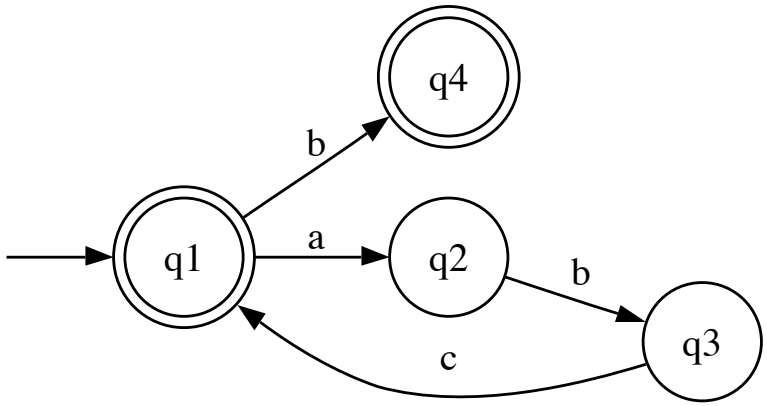

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b	$q4$	$q2 \backslash q3$
c		$q3 \backslash q1$

Conversion of permutation of $\mathbf{AB_s}$ into **LG**

$\mathbf{AB_s}$-grammar

- symbols Σ
- goal types D
- type dictionary φ_1

NL-grammar

- symbols Σ'
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LG-grammar

- symbols $\Sigma \cap \Sigma'$
- fresh goal type d
- type dictionary φ: see below

$\varphi_1(p)$	$\varphi_2(p)$	$\varphi(p)$
a (atom)	c	$\{(a \oslash d) \oslash c\}$
$a \backslash b$	c	$\{(b \oslash a) \oslash c\}$

Example: permuation of $a^n b^n c^n$

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- symbols $\{a, b, c\}$
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- type dictionary

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NL-grammar

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$$\frac{}{\frac{}{\frac{}{(\underbrace{((b \oslash a) \obslash s)}_{\text{b}} \otimes \underbrace{((s \oslash b) \obslash (s\backslash s))}_{\text{c}}) \otimes \underbrace{((a \obslash s) \obslash (s\backslash s))}_{\text{a}} \to d}}}$$

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$$\frac{\dfrac{(a \oslash s) \oslash ((b \oslash a) \oslash ((s \oslash b) \oslash ((s \otimes (s\backslash s)) \otimes (s\backslash s))))) \to d}{(\underbrace{((b \oslash a) \oslash s)}_{b} \otimes \underbrace{((s \oslash b) \oslash (s\backslash s))}_{c}) \otimes \underbrace{((a \oslash s) \oslash (s\backslash s))}_{a} \to d}}{}$$

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$$\frac{\dfrac{\dfrac{}{(s \otimes (s\backslash s)) \otimes (s\backslash s) \to (s \oslash b) \oplus ((b \oslash a) \oplus ((a \oslash d) \oplus d))}}{(a \otimes s) \otimes ((b \oslash a) \otimes ((s \oslash b) \otimes ((s \otimes (s\backslash s)) \otimes (s\backslash s)))) \to d}}{\underbrace{(((b \oslash a) \otimes s)}_{b} \otimes \underbrace{((s \oslash b) \otimes (s\backslash s))}_{c} \otimes \underbrace{((a \otimes s) \otimes (s\backslash s))}_{a} \to d}$$

Example: permuation of $a^n b^n c^n$

AB_s-grammar

- symbols $\{a, b, c\}$
- goal types $\{s\}$
- type dictionary

  $\varphi_1(a) = \{a, s\backslash a\}$
  $\varphi_1(b) = \{a\backslash b\}$
  $\varphi_1(c) = \{b\backslash s\}$

NL-grammar

- symbols $\{a, b, c\}$
- goal types $\{s\}$
- type dictionary

  $\varphi_2(a) = \{s, s\backslash s\}$
  $\varphi_2(b) = \{s, s\backslash s\}$
  $\varphi_2(c) = \{s, s\backslash s\}$

LG-grammar

- symbols $\{a, b, c\}$
- goal type d

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(s \otimes (s\backslash s)) \otimes (s\backslash s) \to (s \oslash b) \oplus ((b \oslash a) \oplus a)}}{(s \otimes (s\backslash s)) \otimes (s\backslash s) \to (s \oslash b) \oplus ((b \oslash a) \oplus ((a \oslash d) \oplus d))}}{(a \oslash s) \otimes ((b \oslash a) \otimes ((s \oslash b) \otimes ((s \otimes (s\backslash s)) \otimes (s\backslash s)))) \to d}}{\underbrace{((\underbrace{(b \oslash a)}_{b} \otimes s)}_{} \otimes \underbrace{((s \oslash b) \otimes (s\backslash s))}_{c}) \otimes \underbrace{((a \otimes s) \otimes (s\backslash s))}_{a} \to d}$$

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NL-grammar

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- goal types $\{s\}$
- type dictionary

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$\varphi_2(b) = \{s, s\backslash s\}$
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LG-grammar

- symbols $\{a, b, c\}$
- goal type d

$$\dfrac{\dfrac{\dfrac{\dfrac{s \to s}{(s \otimes (s\backslash s)) \otimes (s\backslash s) \to (s \oslash b) \oplus ((b \oslash a) \oplus a)}}{(s \otimes (s\backslash s)) \otimes (s\backslash s) \to (s \oslash b) \oplus ((b \oslash a) \oplus ((a \oslash d) \oplus d))}}{(a \otimes s) \otimes ((b \oslash a) \otimes ((s \oslash b) \otimes ((s \otimes (s\backslash s)) \otimes (s\backslash s)))) \to d}}{\underbrace{(((b \oslash a) \otimes s)}_{b} \otimes \underbrace{((s \oslash b) \otimes (s\backslash s))}_{c}) \otimes \underbrace{((a \otimes s) \otimes (s\backslash s))}_{a} \to d}$$

Proof

- Any language generated by a finite automata can be recognized by some $\mathbf{AB_s}$-grammar
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- Regular languages can be recognized by finite automata
- **CFG** languages are equal to **NL**-languages
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- The permutation of context-free grammars is equal to the permutation of regular grammars
- Therefore, any intersection of a permutation of a context-free grammar and a context-free language can be recognized by some **LG**-grammar

Conclusions and future work

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- We extended binary Lambek–Grishin calculus to a calculus allowing for connectives of arbitrary arity
- The calculus has some other good properties, such as decidability, completeness and a connected continuation semantics
- The binary calculus recognizes all languages that are the intersection of a context-free language and the permutation closure of a context-free language
- The generative capacity is slightly more than mildly context-sensitivity and for this reason a good candidate for modelling natural language

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Future work

- Apply formalism to 'real' (natural) language
- Find upper bound generative complexity

References


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