EVENT CALCULUS, NOMINALISATION, AND THE PROGRESSIVE

108810FRITZ HAMM/MICHIEL VAN LAMBALGEN

If it can be done, why do it?
Gertrude Stein

1. Introduction

The motto of this paper expresses our surprise upon finding that, while the idea of applying AI techniques such as the event calculus to natural language semantics, in particular the progressive, has been around for some time (see e.g. Steedman's article in the *Handbook of Logic and Linguistics*, Steedman (1997)), no one seems to have done the actual computations. Here, we set out to remedy this situation, because we believe that the event calculus is indeed the proper theory to treat a group of phenomena including nominalisation, *Aktionsart* and the progressive. Clearly, however, ours is not the first treatment of these topics, so we briefly indicate why we believe that previous approaches still leave something to be desired. On the one hand there are the event-based approaches following Davidson's lead, such as that of Parsons (1990). Here, verbs are provided with an argument for events, and the formal language is extended with predicates such as $Cul(e,t)$ and $Hold(e,t)$, meaning respectively that event e culminates at time t, and that e holds at t. The meaning of these predicates is not axiomatised, although sometimes the set of events is equipped with a lattice structure (i.e. in the work of Bach (1986*b*), Link (1987) and Krifka (1989)); but even then the trouble with this approach is that the set of events has much less structure than is necessary for the intended applications. On the other hand, there exist the more intensional approaches based on possible worlds, as in for example Dowty's use of so called *inertia worlds* to explain the semantics of the progressive. The trouble with this approach is that it is formally rather unconstrained, so that its set of predictions is not sharply delineated.

This brief discussion indicates what we think are desirable features for a formal theory in this area: it should have great expressive power; it should allow for intensionality; and it should be presentable in axiomatic form, so that it is entirely clear what it does, and does not, predict.

The event calculus and its precursor, the situation calculus, were developed in AI to model reasoning with time and change. Formally, it is a many sorted predicate logic with sorts for individual objects, timepoints and two kinds of events, or rather event types: the *fluents*, which are time-dependent properties, and the time-independent event types proper. There are a number of distinguished predicates, for instance the predicate *HoldsAt*(f, t), which expresses that fluent f is true at instant t, and axioms which connect the distinguished predicates. To the novice the event calculus initially may seem bewildering, because what is usually conceived of as a predicate, e.g. $Go(x, y)$, is freely treated here as a term $go(x, y)$, a function which maps pairs of objects on event types. This process is known as *reification*, and is usually kept outside the formal system itself.

Interestingly, however, we cannot just take existing formalisms from the literature and apply them. To accommodate the linguistic phenomena we are interested in, a major change is necessary, namely the formalisation of the reification procedure alluded to above, within the theory itself (see section 5). This turns the predicate *HoldsAt* into a fullfledged self-referential truth predicate, applicable to fluents defined in terms of itself. We therefore have to graft a formal theory of truth onto the event calculus, and for this purpose we have chosen Feferman's type-free calculus.

We close with a few remarks on the paper's methodology. We are roughly in agreement with the main tenets of cognitive or conceptual semantics, in that meaning should be explicated with reference to a system of mental representations in terms of which reasoning, planning, or formation of beliefs and intentions takes place. The primitives of such a conceptual system may include CAUSE, EVENT, PATH, GO, etc. Where we differ from most adherents of conceptual semantics, is that we think that this approach is entirely compatible with, and even invites, logical and mathematical methods, even though these are different from possible worlds semantics in any of its guises.

The results of this paper do raise several possibly interesting questions on cognition. The fact that there is a 'pre-established harmony' between different kinds of nominalisations and the event calculus may just be due to the fact that the originators of the event calculus are native speakers of English, but it may also indicate that this way of carving up the world is essential to cognition. Typological linguistics might be of help here. The study of Koptjevskaja-Tamm 1993 shows that, syntactically speaking, the *action nominal constructions* of 70 languages can be ordered in a spectrum one end of which is a sentence, while the other is an NP corresponding to a perfect nominal. Koptjevskaja-Tamm remarks on this classification

> Thus in English, noun phrases which refer to events are further from the corresponding independent sentences than those which refer to facts and propositions. As we shall see, this situation is not at all unique and is also reflected in the internal structure of [action nominal constructions] across languages.

The main problem then would be to determine whether or not all constructions across the spectrum can be mapped semantically onto either events or fluents. There exist some data from Akatek Maya and Japanese which support this conjecture (Hamm et al. (1998)), but many more languages remain to be checked.

The remainder of the paper is organised as follows. The next section discusses nominalisation in English and introduces the data to be explained. Sections 3, 4 and 5 are the technical heart of the paper: they introduce Feferman's theory of truth, the event calculus, and their combination. Section 6 applies the resulting theory to nominalisation, and section 7 does the same for *Aktionsart* and aspect, in particular the progressive.

2. Nominalisation: syntax and compositionality

In chapter five of *Linguistics in Philosophy*, Zeno Vendler discusses two classes of nominalised predicates, the class of perfect and the class of imperfect nominals, and further two classes of verbal contexts which are sensitive to these nominals. In the following two sections, we introduce the most important characteristics of the notions involved. A more detailed discussion of these issues can be found in Vendler (1967) and Vendler (1968). In section 2.3 we briefly describe the syntax of these constructions and the last section of this chapter contains a short discussion of Chierchia's motivation for introducing a kind of type lowering operation into semantic frameworks dealing with nominalisations.

2.1. **Perfect and Imperfect Nominals.** Vendler's differentiation between perfect and imperfect nominals and his observations about their most important properties are illustrated in (1) and (2). Perfect nominals like those in (1) occur with determiners, can be modified by adjectives but not by adverbs, and cannot appear in different tenses or be modalised. Further, it is impossible to negate perfect nominals. To summerize, perfect nominals are nominalised forms which have lost their verbal characteristics and behave like "real" nouns. This is why Vendler dubbed them "perfect".

(1) a. The singing of the song.
b. beautiful singing of the song.
c. *quickly cooking of the dinner.
d. *having cooked of the dinner.
e. *being able to cook of the dinner.
f. *not revealing of the secret.

Imperfect nominals show the opposite behaviour, as the examples in (2) demonstrate. They cannot occur with nominal determiners, they can be modified by adverbs[1] but not by adjectives, they can occur in different tenses or be modalised, and it is possible to negate them.

(2) a. *The singing the song.
b. *beautiful singing the song.
c. Singing the song beautifully.
d. quickly cooking the dinner.
e. having cooked the dinner.
f. being able to cook the dinner.
g. not revealing the secret.

[1] They therefore can occur with adverbial determiners like *always*.

So, imperfect nominals can occur externally in noun phrase positions, but their internal structure strongly resembles the structure of the VP or the S they are derived from. This is, of course, the reason why Vendler called them "imperfect". We shall henceforth use the term *perfect nominal* both for the respective nominal and for the NP which contains a perfect nominal.

Abney (1987) develops a detailed syntactic account of gerunds, which are part of the classes of perfect and imperfect nominals. He distinguishes four types of gerunds:

(3)
a. Acc-ing: John being a spy.
b. PRO-ing: singing loudly.
c. Poss-ing: John's knowing the answer.
d. Ing-of: singing of the song.

Assuming that PRO-ing is a special case of Acc-ing or Poss-ing, there are three classes of gerunds, which differ with respect to their syntactic properties. For example, Abney shows that Acc-ing and Poss-ing constructions show differences with regard to agreement, long distance binding, pied piping, etc.. But what about semantic differences? Of course, Ing-of gerunds and Poss-ing gerunds are among the perfect and imperfect nominals[2] introduced in this section, and Vendler's thesis is (see section 2.2) that there is a category distinction, i.e. something genuinely semantic, involved with these notions. In this paper it will be assumed that Acc-ing and Poss-ing constructions are semantically in the same class, the class of imperfect nominals.

Vendler (1968) demonstrates that the genitive in Poss-ing gerunds is not a "real" genitive like *John's* in *John's house*. This is shown by the following examples:

(4)
a. John's house
b. The house of John
c. John's singing the song
d. *The singing the song of John

Example (4-b) is a paraphrase of (4-a). An analogous paraphrase for (4-c) does not exist.

Compared with the genitive in Poss-ing gerunds the genitive of perfect nominals behaves like a "real" genitive. This is shown by the following observation: It is possible to delete the genitive of embedded imperfect nominals if it is coreferential with the matrix subject. Deletion in the case of perfect nominals however leads to ungrammaticality.

[2]The concepts *perfect* and *imperfect* nominal are used by Vendler primarily to refer to sets of structural properties, which are assumed to be conditioned by two different semantic types. This is especially clear when imperfect nominals are considered. This is a huge and structurally heterogeneous class including Poss–ing, Acc–ing gerunds, absolutive constructions, infinitives and even *that*–clauses, which are traditionally not thought of as *nominal* at all. Perfect nominals however are more coherent. This class contains Ing–of gerunds and some derived nominals.

(5) a. He shocked us by telling a dirty joke.
b. *He entertained us by singing of arias. (Vendler (1968):50)

For more arguments in favour of the claim that the genitive of Poss-ing gerunds is not the same as the genitive in Ing-of nominals see Vendler (1968).

2.2. **Narrow and Loose Containers.** Vendler also considers verbal contexts, which somehow discriminate between the above two classes of nominals. Expressions like *surprised us, is unlikely* are examples of loose containers. Their name derives from the fact that they accept both kinds of nominals as arguments as is shown in (6).

(6) a. The beautiful singing of the aria surprised us.
b. John's not revealing the secret is unlikely.
c. The singing of the song is fun.
d. John's quickly cooking the dinner surprised us.
e. They were surprised by the sudden coming in of a stranger.[3]
f. They were surprised by a stranger coming in suddenly.

Verbal contexts like *was slow, occurred*, etc. which are called narrow by Vendler, show a more restrictive behaviour. They accept only perfect nominals as is shown in (7).

(7) a. *The soprano's singing the aria was slow.
b. The soprano's singing of the aria was slow.
c. John's revealing of the secret occurred at midnight.
d. *John's revealing the secret occurred at midnight.
e. *John's not revealing the secret occurred at midnight.

Narrow containers can be negated and they stay narrow under negation as the following examples demonstrate.

(8) a. The singing of the song didn't occur at noon.
b. John's kicking the cat didn't occur at noon.

Note that the nominals *arrival of the train* and *non-arrival of the train* in the following examples, though similar to the perfect and imperfect nominals, respectively, nevertheless behave differently. It may well be that *arrival of the train* is a perfect nominal, but *non-arrival of the train* is not an imperfect nominal in Vendler's sense because it can occur with nominal determiners and adjectives but not with adverbs.

(9) a. The arrival of the train surprised us.
b. The non-arrival of the train surprised us.

[3]This example is from Jespersen (1933),p 327]

c. The arrival of the train occurred at noon.
d. *The non-arrival of the train occurred at noon.
e. The unexpected non-arrival of the train
f. *The non-arrival of the train unexpectedly

Narrow containers are typical examples for extensional contexts in contrast to loose containers[4].

(10) a. The beheading of the tallest spy occurred at noon.
b. The beheading of the tallest spy surprised us.

If the king and the tallest spy happen to be the same person, then it follows from (10)(a) that *The beheading of the king occurred at noon.* But certainly *The beheading of the king surprised us* does not follow from (10)(b).

Vendler description of the meanings of perfect and imperfect nominals and their respective containers is rather vague but he clearly suggests that a category distinction between events and facts or results forms the philosophical basis for these empirical findings. Events are taken to somehow be related to the meaning of perfect nominals, and facts or results to the meaning of imperfect nominals. We think it is fair to interpret Vendler as claiming that the relationship between the nominals and their respective containers is determined by this category distinction, but it is certainly open whether he wants the other findings to be interpreted in this way or as conditioned by structural (i.e. syntactic) properties of English.

Schachter suggests that some gerunds – his gerundive nominals – behave like names.

> To return to gerundive nominals, I would claim that gerundive nominals without initial possessives or other determiners are also class names, naming a type of activity in which one can participate, a type of condition, etc.
> Schachter (1976), p 215

If we assume that imperfect nominals are like names then this assumption accounts immediately for the lack of determiners in such phrases since names can in general not occur with determiners. This assumption is further supported by the following observation due to Pullum (Pullum (1991)):

(11) *his leaving her that you predicted.

Neither Acc–ing nor Poss–ing gerunds tolerate restrictive relative clauses. One further observation supporting Schachter's proposal is that Ing–of nominals can sometimes be pluralised but Acc–ing and Poss–ing gerunds definitely can't. The following example is from Poutsma.

[4] The examples are from Parsons (1990)

(12) He ignored the sayings and doings of the ladies of his family.
Poutsma (1923), p 113.

Observations from Abney (1987), pp 244 show that perfect and imperfect nominals also differ in their ability to participate in N-bar deletion. For instance, an ellipsis with a Poss-ing construction as in (13)(a) is bad, while it is possible with an Ing-of gerund and a narrow container as is shown in (13)(b).

(13) a. *John's fixing the sink was surprising, and Bill's was more so.
b. John's fixing of the sink was skillful, and Bill's was more so.

Abney claims that the gerund *John's fixing of the sink* is ambiguous and can either refer to the manner in which John fixed the sink - called the Act-reading by Abney - or the fact that John fixed the sink (Fact-reading). N-bar deletion is only possible under the Act-reading.

Of course Abney does not develop a formal semantics for his Fact– and Act–readings. In his work these concepts are just labels which are used to name the intuitive reason for observations as the ones above. In the following chapters we will develop a formal theory which allows us to give a precise reconstruction of Abney's notions. His Act–reading will be described in terms of *event types* and his Fact–reading in terms of *fluents*. These formal concepts are introduced in section 4.

Finally we note the following examples of iterated nominalisations, a phenomenon which was not observed by Vendler.

(14) a. John's supporting his son's not going to church
b. John's improving his singing
c. John's watching the dog's playing
d. My discovering her not leaving
e. his discussion of John's revealing the secret

We are interested in these examples because the negation in say, (14-a) seems to have antonymic force and all examples seem to be factive in the sense that they presuppose that the fact expressed by the embedded nominal holds. For instance (14-a) implies that John's son is not going to church.

In this paper only the Act- and Fact-readings of gerunds are considered[5]. The habitual reading of a gerund like *eating apples* will be neglected[6].

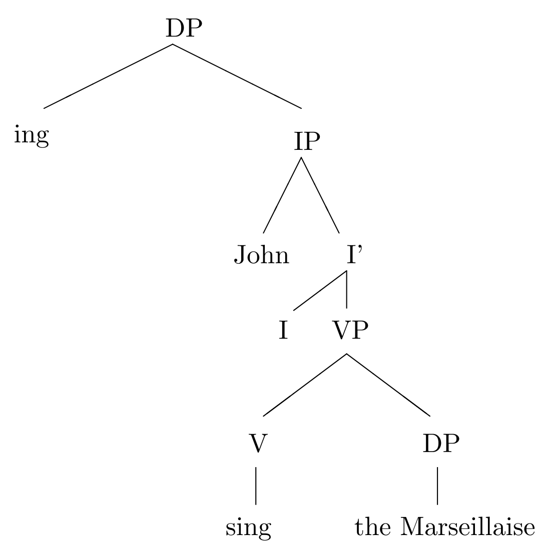
2.3. **Syntax.** Although this is a paper on the semantics of nominalisations we will briefly discuss in this section two theories about the syntactic structure of at least

[5]For a thorough empirical discussion of Vendler's examples the reader is advised to consult Asher (1993) and Zucchi (1993).

[6]See Portner (1991) for a discussion of such examples.

some nominalisations introduced in the previous paragraphs. Our main purpose here is to show that the formal apparatus we will develop in chapters 3 and 4 allows a strictly compositional interpretation of the discussed nominalisations; but we also want to stress that this interpretation process is not tied to a specific syntactic framework. We therefore discuss first Abney's government and binding approach and then the GPSG–based theory of Pullum (1991). We have to admit that this choice is not well balanced since Pullum only analyses what he calls *nominal gerund phrases*, which are Abney's Poss–ing gerunds. Pullum's main interest is in theoretical syntax. He wants to show that the GPSG–analysis of constructions, which exhibit verbal as well as nominal features, doesn't necessarily lead to a trivialisation of the concept *head* of a phrase. Nevertheless these two approaches are the theoretically most explicit accounts of the syntax of at least some of the constructions we are interested in. Therefore we will show in chapter 6 how to interpret the respective syntactic structures compositionally.

Abney's account is based on a conservative extension of classical $\overline{X}$–theory. It is conservative in the sense that it does not eliminate any inferences of $\overline{X}$–theory on the phrasal level. Abney's approach differs from the classical theory only insofar as he assumes that the function of the affix *-ing* is to convert a verbal category into a nominal one. The essence of his analysis is then that the differences in the structures of the various types of English gerunds reduce to the question where in the projection path of the verb this conversion takes place. It is presumed that *-ing* can only be adjoined to the lexical category V and to maximal projections; i.e. VP and IP[7]. If *–ing* is sister of IP the resulting structure is that of Acc–ing.



(15) [Acc–ing]

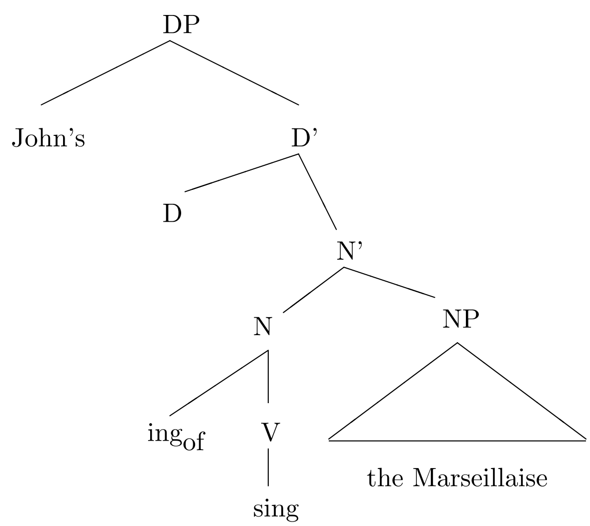
[7] A structure like $[_{CP} C[_{DP} -ing[_{IP} \ldots]]]$ is excluded because it violates the selection properties of C.

In case *–ing* is sister of the VP-node, we get in a similar way the structure of the Poss–ing gerund. The third possibility is that *–ing* is sister of the lexical category V. In this case we have the structure of the Ing–of phrases.

It should be noted that *–ing* does nothing but convert a verbal projection into a nominal one. This abstract morphological element does not have a syntax of its own because it does not project any structure. This is the reason why Abney's system is a conservative extension of classical $\overline{X}$–theory.

In order to give a strictly compositional interpretation we have to deviate slightly from Abney's analysis. We have to assume that the *–ing* which is sister of the lexical category V is different from the one which is sister of maximal projections. This assumption is due to the different semantic effects this affix has when it converts a V to a N in contrast to the conversion of maximal projections. We will write –ing$_{of}$ for the *–ing* which is sister to V and *–ing* for the one which is sister to maximal projections.

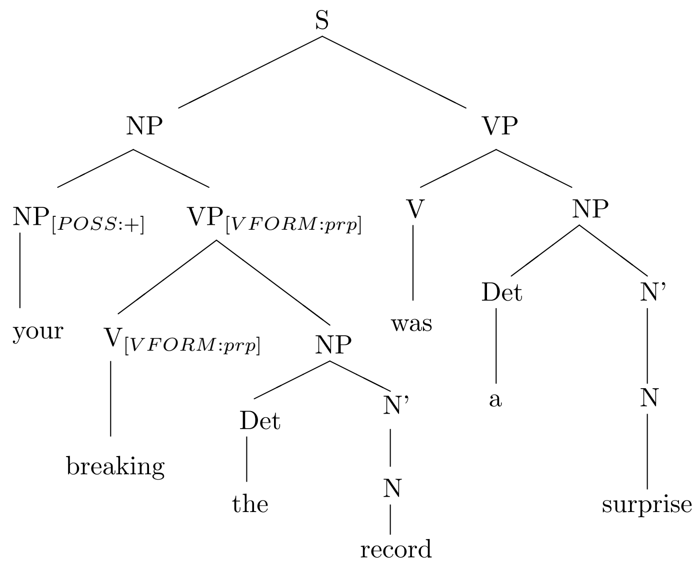
If we moreover assume Chomsky's rule of *of*–insertion we get the following tree which will be compositionally interpreted in section 6.2.



(16)

We skip the motivation for this rule and refer the reader to Chomsky (1981)(rule 9 on p. 50).

Since we want to make clear that our analysis does not depend on any of the syntactic details of Abney's analysis it will be shown that Pullum's approach allows a compositional analysis too. Pullum argues for an analysis of *nominal gerund phrases* (Abney's Poss–ing gerunds) which results in the following tree.



(17) [Pullum's NGP]

The feature $[VFORM : prp]$ says that the verb is in its participle form. Similarly the feature $[POSS : +]$ indicates that the NP is a possessor NP. These forms are assumed to be derived in the morphological component of grammar and therefore only the results of such processes occur in syntax.

2.4. **Chierchia.** In this section we will briefly sketch Chierchia's most important arguments for introducing "type–lowering" operations into the formal system[8]. This part will also be used for the illustration of our official notation of the formal nominalisation operation in Chierchia's sense. We will introduce this operation here by way of simple examples. The precise formal details of our formalism, which is based on work by S. Feferman, are given in section 3.

Chierchia points out that the usual analysis of gerunds and infinitives in terms of type shifting[9] has some serious disadvantages.

Consider the following examples from Chierchia (1988);

(18) a. To be home is nice.
b. Being home is nice.
c. John is nice.

and contrast them with the unacceptable expressions in (19)

(19) a. *Are home is nice.

[8]We will not discuss his arguments against weak intensional theories here.
[9]For details see Gamut (1991), p 103

b. *Is home is nice.

Like a proper name – *John* in the example above – and unlike finite elements infinitives and gerunds are allowed in argument positions. From examples like these Chierchia concludes that VPs like *is nice* play a double role in the grammatical system. Disregarding intensionality they can be considered as elements of $D_{\langle e,t\rangle}$ when they are finite but in their infinitive or gerundial form they can also be taken as objects. This immediately explains the contrast in (18) and (19).

The type shifting strategy would not only have to assign two different meanings to *is nice* in (18) but two different types of meanings; i.e. $\langle e,t\rangle$ for (18-c) and $\langle\langle e,t\rangle,t\rangle$ for the first two examples. This however proves rather unfortunate once slightly more complicated examples are considered.

(20) a. Having fun is extremely nice.
b. John is extremely nice.

Now we also have to assign two different types to the adverb *extremely*. Therefore the adverbial component of the grammar is infected by the decision to give *is nice* a lower and a higher type. It is not hard to see that proceeding this way forces to assign differing types to practically every major grammatical category. Therefore the conclusion is that the type shifting strategy infects the whole grammatical system which is not a very welcome result.

In order to avoid such consequences Chierchia requires that the formal system contains an operation which transforms predicates and sentences into terms. His notation for this operation is $^{\cap}$.

In this paper we will deviate from Chierchia's notation for the formal nominalisation operation. Instead we will use the notation introduced in Feferman (1984) which has the advantage to clearly indicate which variables are bound and which are free in the nominalised terms resulting from formulas. Let us illustrate this by representing example (18-a) in our system. We assume that *nice* and *home* are translated as $nice(x,s)$ and $home(x,t)$. The variable t in these translations ranges over times.

(21) $nice(home[\hat{x},\hat{t}],s)$.

The term $home[\hat{x},\hat{t}]$ denotes an object as required by Chierchia. The brackets [,] indicate the result of the nominalisation operation applied to the formula $home(x,t)$. The variables x and t are bound by abstraction in this term. By contrast $home[x,\hat{t}]$ with x a free variable denotes a function which when applied to an appropriate argument yields an object.

The formal rendering of the examples in (19-a) runs as follows:

(22) $is\ nice(home(y,t), s)$

Now the expression in (22) is not well formed since $is\ nice(x, t)$ is a function defined on objects not on functions like $home(y, t)$. Therefore the ungrammaticality of (19-a) and likewise that of (19-b) is explained by a type mismatch.

Moreover the type lowering strategy now provides a uniform analysis of the adverb *extremely* in (20). This is so because in both cases the denotation of the adverb can be a function which when applied to the function *is nice* yields a function of the same type as *is nice*[10]. This resulting function can then be applied to the object $home[\hat{x}, \hat{t}]$ as in (21).

It is thus clear that with the help of Feferman's notation we can account for the data considered by Chierchia. The formal properties of this operation are the subject of the next section.

Let us finally remark that Zucchi (1993) uses a kind of nominalisation operation in Chierchia's sense too. Zucchi works within Cresswell's type theory (Cresswell (1973)), which differs primarily from Montague's in the assumption that all functional types contain *partial* functions. The type 1 is the type of names and 0 the type of propositions, which are assumed to be sets of possible worlds. The type of a one–place propositional function in Cresswell's notation is $\langle 0, 1\rangle$[11].

Zucchi assumes that the imperfect nominal *The soprano's singing the song* denotes a proposition. In order to avoid assigning two different types to *is one of my favourite things* in (23-a) and (23-b) he introduces the following semi–formal device, where V is the valuation function of the model.

$$V(i)\ is\ the\ function\ \omega \in D_{\langle 1,0\rangle}\ such\ that\ for\ every\ a \in D_0, \omega(a) = a$$

This means that $V(i)$ maps a proposition to an entity. On the level of logical form i therefore transforms a sentence into a term, an expression of type 1.

(23) a. The soprano's singing the song is one of my favourite things.
b. The soprano's nose is one of my favourite things.

Using function i example (23-a) may then be roughly formalised in the following way:

(24) Is one of my favourite things(i(The soprano's singing the song)).

Analogous to Chierchia's nominalisation operator Zucchi's function i allows a uniform type assignment for the VP *is one of my favourite things*.

[10]In Montague's notation the adverb would be of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$ for both examples in (20).

[11]Note the reversed ordering in this type notation. In Montague's type theory $\langle 0, 1\rangle$ would be written as $\langle e, t\rangle$.

3. Feferman's type–free calculi

As our brief review of the literature has made clear, providing a formal semantics for nominalisation requires a coding procedure whereby formulas are transformed into terms. Extending this semantics to the progressive and iterated nominalisations furthermore requires the use of a truth predicate. Here, we introduce these formal prerequisites in the form of a synopsis of Feferman's type–free calculi (Feferman (1984))[12].

The Russell paradox shows that one cannot have unrestricted comprehension, whereas Tarski showed that one cannot consistently add a truth predicate to first order logic. That is, it is impossible to extend a first order theory S_0 to a first order theory S having truth predicate T and relation of elementhood $\in$ satisfying

(25) a. (i) Every formula φ in the language L of S has a name in L; formally, in L there exists a closed term $\ulcorner\varphi\urcorner$ for φ.
(ii) For every formula $\psi(x)$ one can construct a formula φ such that in S, $\psi(\ulcorner\varphi\urcorner)$ is equivalent to φ.
b. S satisfies classical logic.
c. In S, the truth predicate satisfies the following axiom: for all formulas φ in L,
$$T(\ulcorner\varphi\urcorner) \leftrightarrow \varphi$$
d. For every formula $\varphi(x)$ there exists in L a term $\{x \mid \varphi(x)\}$, in which x is a bound variable.
e. S satisfies the following comprehension axiom for every formula $\varphi(x)$ in L:
$$\forall y[y \in \{x \mid \varphi(x)\} \leftrightarrow \varphi(y)].$$

Feferman investigates several ways out, among which dropping (25-b) in favour of a three–valued logic, and modifying (25-c) and (25-e) in such a way that the role of negation is made more explicit.

3.1. **Coding and truth predicates.** A necessary ingredient of languages containing a truth predicate is a coding scheme which maps formulas φ to terms $\ulcorner\varphi\urcorner$. Let L_0 be some first order language, S_0 a theory formulated in L_0. We briefly review the requirements on L_0 and S_0 which allow such coding.

First, one requires that L_0 contains an individual constant $\bar{0}$, a binary function symbol π and two unary function symbols π_1 and π_2. We shall often write (τ_1, τ_2) for $\pi(\tau_1, \tau_2)$. Secondly, we assume that S_0 proves the following statements concerning these functions

(26) a. $(x, y) \neq \bar{0}$
b. $\pi_1(x, y) = x \wedge \pi_2(x, y) = y$

[12]For a comprehensive study of theories of truth and abstraction the reader is referred to Cantini (1996).

If $M_0 \models S_0$, $(\cdot, \cdot)$ is a pairing function in M_0, and π_1 and π_2 are the corresponding projection functions.

One may now define tuples inductively by putting: $(\tau) = \tau$ and $(\tau_1, \ldots, \tau_{k+1}) = ((\tau_1, \ldots, \tau_k), \tau_{k+1})$. Similarly one may define the corresponding projection operations π_i^k $(1 \leq i \leq k)$ such that: $\pi_i^k(x_1, \ldots, x_k) = x_i$.

These constructs suffice to define an abstract form of Gödel numbering (see Mendelson (1987)).

Definition 1. *Let L be some extension of L_0 (e.g. by means of a truth predicate). Then we may code formulas of L as terms in L_0. We write $\ulcorner \varphi \urcorner$ for the Gödel number in L_0 of φ in L. This notation will be used interchangeably both for the term in L_0 and for the object denoted by that term in a model M_0.*

We will now put this machinery to work. Let φ be a formula with free variables among $x_1, \ldots, x_k, y_1, \ldots, y_n$. The L_0–term $(\ulcorner \varphi \urcorner, y_1, \ldots, y_n)$ contains $x_1, \ldots, x_k$ as bound variables and $y_1, \ldots, y_n$ as free variables. Since the $x_1, \ldots, x_k$ are bound by abstraction, the following notation makes sense

Definition 2. Δ_n $\quad \varphi[\hat{x}_1, \ldots, \hat{x}_n, y_1, \ldots, y_m] = (\ulcorner \varphi \urcorner, y_1, \ldots, y_m)$. *For $n = 1$ we will use standard set theoretical notation Δ_1 $\{x \mid \varphi(x, y_1, \ldots, y_n)\} = \varphi[\hat{x}, y_1, \ldots, y_n]$. If both m and n are equal to 0, we write $\ulcorner \varphi \urcorner$.*

To formalise the truth definition and the comprehension axiom, we add predicates T_n to L_0. The intuitive meaning of $T_n(x_1, \ldots, x_n, z)$ is: the tuple $(x_1, \ldots, x_n)$ satisfies (the formula coded by) z. This leads to the following axiom scheme, which generalises both truth definition and comprehension axiom:

Axiom 1. $(T_n A)$

$$T_n(x_1, \ldots, x_n, \varphi[\hat{u}_1, \ldots, \hat{u}_n, y_1, \ldots, y_m]) \leftrightarrow \varphi(x_1, \ldots, x_n, y_1, \ldots, y_m)$$

Important special cases are the axioms for T_0

(27) a. $(T_0 A)$ $\quad T_0(\varphi[y_1, \ldots, y_m]) \leftrightarrow \varphi(y_1, \ldots, y_m)$
b. For $m = 0$: $T_0(\ulcorner \varphi \urcorner) \leftrightarrow \varphi$

and for $(T_1 A)$,

(28) $(T_1 A)$ $\quad T_1(x, \{u \mid \varphi(u, y_1, \ldots, y_m)\} \leftrightarrow \varphi(x, y_1, \ldots, y_m)$.

The latter statement shows that one may write $\in$ for T_1. T_1 will be of special importance for us, since it is identical to the *HoldsAt* predicate of the event calculus that will be used in our treatment of the progressive. The purpose of the next section is to show that the axiom scheme 1 can be consistently added to S_0, provided one makes some alterations to the basic set up.

3.2. **Three–valued logic and partial models.** Tarski's paradox of truth can be derived in any logic containing minimal logic, so a way out has to be sought in a different direction. Feferman observed that Kleene's (strong) three–valued logic provides the solution. Kleene's three–valued logic differs from Łukasiewicz' in the status of the third value u, which in this case means *not yet known* instead of 'intermediate'. In other words, u is not a degree of truth, but rather means that the truth value is undecided. The set of truth values $\{u, 0, 1\}$ thus has the partial order $u \leq 0$ and $u \leq 1$.

Let M_0 be a classical model with universe M. Feferman shows how to expand M_0 with a truth predicate which is a partial relation.

Definition 3. *A partial model $\mathcal{M}$ is a tuple of the form $(\mathcal{M}_0, \mathsf{R}_1, \ldots, \mathsf{R}_n, \ldots)$, where $\mathcal{M}_0$ is a classical first order structure and the R_i are partial relations*[13]*on M.*

The ordering $\leq$ on partial relations naturally extends to an ordering $\leq$ on partial models with fixed classical component. The logic of partial models will be given by the Strong Kleene operations (Kleene (1952)).

p	$\neg p$
1	0
0	1
u	u

p	q	$p \wedge q$	p	q	$p \vee q$	p	q	$p \to q$	p	q	$p \leftrightarrow q$
1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	1	0	0	1
u	u	u	u	u	u	u	u	u	u	u	u
1	0	0	1	0	1	1	0	0	1	0	0
1	u	u	1	u	1	1	u	u	1	u	u
0	1	0	0	1	1	0	1	1	0	1	0
0	u	0	0	u	u	0	u	1	0	u	u
u	1	u	u	1	1	u	1	1	u	1	u
u	0	0	u	0	u	u	0	u	u	0	u

The universal quantifier is defined by means of a generalised conjunction $\bigwedge$ and the existential quantifier is then defined as the dual of $\forall$.

The distinguishing feature of the Kleene operations (including $\bigwedge$) is that they are monotone:

Definition 4. *A mapping O from $\{u, 0, 1\}^n$ to $\{u, 0, 1\}$ is* m*onotone if $p_i \leq q_i$ for all $1 \leq i \leq n$, implies $O(p_1, \ldots, p_n) \leq O(q_1, \ldots, q_n)$.*

Now let L be the language which results from L_0 by the addition of the truth predicates T_n, for all n. Feferman shows the consistency of the truth axioms with an arbitrary theory S_0 in L_0 by means of a fixed point construction.

Definition 5. *a. Let M be a set. A n–ary inductive definition (also called a monotone operation) on M is a function $\mathcal{F}$ which maps n–ary relations on*

[13]A n–ary *partial* relation R is a a pair $(R, \overline{R})$, where $R, \overline{R} \subseteq M^n$ and $R \cap \overline{R} = \emptyset$. The ordering on the truth values leads to an ordering on relations in the following manner: $\mathsf{R} \leq \mathsf{R}'$ if for all $m_1, \ldots, m_n \in M$, $\mathsf{R}(m_1, \ldots, m_n) \leq \mathsf{R}'(m_1, \ldots, m_n)$.

M to n–ary relations on M, and which is monotone in the following sense: for all $R, S \subseteq M^n$ one has

$$R \subseteq S \text{ implies } \mathcal{F}(R) \subseteq \mathcal{F}(S)$$

b. *If $\mathcal{F}(R) = R$, then R is called a fixedpoint of $\mathcal{F}$.*

Theorem 1. *For every model $\mathcal{M}_0$ of S_0 there exists a smallest expansion to a partial model $\mathcal{M} = (\mathcal{M}_0, T_0, \ldots, T_n, \ldots)$ which satisfies the property: for every T_n in L and every formula $\varphi(x_1, \ldots, x_n, y_1, \ldots, y_m)$ from L one has*

$$[\![\, T_n(c_1, \ldots, c_n, \varphi[\hat{u}_1, \ldots, \hat{u}_n, c_{n+1}, \ldots, c_{n+m}]) \,]\!]_{\mathcal{M}} = [\![\, \varphi(c_1, \ldots, c_n, c_{n+1}, \ldots, c_{n+m}) \,]\!]_{\mathcal{M}},$$

where $[\![\, \varphi \,]\!]_{\mathcal{M}}$ denotes the truth value of φ in $\mathcal{M}$.

Proofsketch. We do the proof for T_1, the general case is similar. Let M_0 be a model for S_0 and suppose a partial interpretation for T_1 has been constructed, i.e. we have predicates E and $\overline{E}$ representing positive and negative parts of T_1. We now define a monotone operation $\mathcal{F}$ on the partial model $(M_0, E, \overline{E})$, by putting $\mathcal{F}(E, \overline{E}) = (E', \overline{E'})$, where

$$(\ulcorner \varphi \urcorner, \mathbf{c}) \in E' \text{ iff } \varphi(c) \text{ is true in } (M_0, E, \overline{E}),$$

and

$$(\ulcorner \varphi \urcorner, \mathbf{c}) \in \overline{E'} \text{ iff } \varphi(c) \text{ is false in } (M_0, E, \overline{E}).$$

Since the semantics is monotone, monotonicity of $\mathcal{F}$ is immediate. Starting from any pair $(E,\overline{E})$, let $\mathcal{M}$ be the smallest fixed point of $\mathcal{F}$, then by definition one has $\mathbf{c} \in \{x \mid \varphi(x)\}$ is true (false) in $\mathcal{M}$ iff $\varphi(c)$ is true (false) in $\mathcal{M}$. $\square$

For our later purposes it is important that one can start from any pair $(E,\overline{E})$, and not just from, say, the pair $(\emptyset, \emptyset)$, since in this way one may put additional constraints on the truth predicates.

The preceding theorem is not yet quite sufficient to show the validity of the truth axioms since if, in the above theorem, both sides equal u, the use of the Kleene biconditional in the axioms gives u as truth value. In this case, the Łukasiewicz biconditional would give truth value 1, but its semantics is not monotone, so it cannot be used in this context. We shall skip the intermediate steps that Feferman uses, and jump directly to the final result.

3.3. **A fully classical type-free system.** Feferman's trick is to code Kleene's three-valued logic into classical logic by splitting the truth predicate T_n into a definitely positive part, also denoted T_n, and a definitely negative part, denoted $\bar{T}_n$, such that $T_n \cap \bar{T}_n = \emptyset$, although $T_n \cup \bar{T}_n$ is not necessarily the whole space M^n. This last possibility corresponds to the value u of the Kleene truth tables.

Again, let L_0 be some first order language, and let L be L_0 expanded with $\{T_n, \bar{T}_n \mid n \in I\!N\}$. A formula is called *positive* over L_0, if it is equivalent to a formula constructed from atomic formulas of L, and negations of atomic formulas of L_0, without using negation. Inductively, one defines φ^+ and φ^-as follows:

Definition 6. *a.* $\varphi^+ = \varphi$, *for atomic φ in L.*

b. *If* φ *in* L_0 *is atomic,* $\varphi^- = \neg\varphi$*; if* $\varphi = T_n(\ldots)$*,* $\varphi^- = \bar{T}_n(\ldots)$*; if* $\varphi = \bar{T}_n(\ldots)$*,* $\varphi^- = T_n(\ldots)$*.*
c. $(\neg\varphi)^+ = \varphi^-$ *and* $(\neg\varphi)^- = \varphi^+$*.*
d. $(\varphi \wedge \psi)^+ = \varphi^+ \wedge \psi^+$ *and* $(\varphi \wedge \psi)^- = \varphi^- \vee \psi^-$*.*
e. $(\varphi \vee \psi)^+ = \varphi^+ \vee \psi^+$ *and* $(\varphi \vee \psi)^- = \varphi^- \wedge \psi^-$*.*
f. $(\forall x\varphi)^+ = \forall x\varphi^+$ *and* $(\forall x\varphi)^- = \exists x\varphi^-$*.*
g. $(\exists x\varphi)^+ = \exists x\varphi^+$ *and* $(\exists x\varphi)^- = \forall x\varphi^-$*.*

Let S_0 be a theory in L_0 comprising at least the axioms for the pairing operation. S is the extension of S_0 by means of the following axioms

Axiom 2. *DIS*$(T_n, \bar{T}_n)$ $\neg(T_n(x_1, \ldots, x_n, z) \wedge \bar{T}_n(x_1, \ldots, x_n, z))$.

Axiom 3.

$$T_n(x_1, \ldots, x_n, \varphi[\hat{u}_1, \ldots, \hat{u}_n, y_1, \ldots, y_m]) \leftrightarrow \varphi^+(x_1, \ldots, x_n, y_1, \ldots, y_m)$$
$$\bar{T}_n(x_1, \ldots, x_n, \varphi[\hat{u}_1, \ldots, \hat{u}_n, y_1, \ldots, y_m]) \leftrightarrow \varphi^-(x_1, \ldots, x_n, y_1, \ldots, y_m)$$

Important special cases of these axioms are

(29) For every sentence φ:

$$T(\ulcorner\varphi\urcorner) \leftrightarrow \varphi^+$$
$$\bar{T}(\ulcorner\varphi\urcorner) \leftrightarrow \varphi^-$$

(30) For every formula $\varphi(x, y_1, \ldots, y_n)$:
a. $x \in \{u \mid \varphi(u, y_1, \ldots, y_n)\} \leftrightarrow \varphi^+(x, y_1, \ldots, y_n)$
b. $x \bar{\in} \{u \mid \varphi(u, y_1, \ldots, y_n)\} \leftrightarrow \varphi^-(x, y_1, \ldots, y_n)$.

But when treating nominalisation we shall also have occasion to use the T_n for $n > 1$.

An easy induction on the construction of φ^+ and φ^- shows

Lemma 1. $(\varphi^+ \to \varphi)$ *and* $(\varphi^- \to \neg\varphi)$ *for every* φ*.*

For Feferman, the following is then the main result

Theorem 2. *S is a conservative extension of S_0, hence if S_0 is consistent, so is S.*

For our purposes, this is not yet sufficient, since in our case the theory S_0 will be the event calculus, which already contains axioms involving the truth predicate T_1, in the guise of the predicate *HoldsAt*. Thus the analogue of Theorem 2 requires a separate proof, but it will be seen that Feferman's main steps can be copied in our case.

3.4. **Extensions of L.** We will sometimes have occasion to consider an extension of L with generalised quantifiers. It is required that these quantifiers $\mathbf{Q}$ have a monotone semantics in the following sense: $\mathbf{Q}$ is a set of subsets of a set M satisfying

(31) $$(X \subseteq Y \subseteq M) \wedge X \in Q \Rightarrow Y \in Q$$

The reason for this requirement is that Theorem 2 can be extended to these generalised quantifiers. Thus we have

Theorem 3. *Let $L_0(Q)$ be an expansion of the first order language L_0 with a set of generalised quantifiers having monotone semantics, and let S_0 be a theory in $L_0(Q)$. Then S_0 can be conservatively extended to a theory S incorporating the axioms 3 for the expanded language.*

3.5. **Intensionality.** If $\varphi(x)$ is a formula, Feferman's coding trick allows one to introduce the set-like object $\varphi[\hat{x}]$, alternatively written as $\{x \mid \varphi(x)\}$. It is important to realise, however, that these sets are unlike classical sets in that they do not necessarily satisfy the axiom of extensionality

$$\forall x(x \in a \;\leftrightarrow\; x \in b) \;\rightarrow\; a \;=\; b.$$

The analogue would be

$$\forall y(y \in \{x \mid \varphi(x)\} \;\leftrightarrow\; y \in \{x \mid \psi(x)\}) \;\rightarrow \varphi[\hat{x}] \;=\; \psi[\hat{x}],$$

but this is in general false.

4. Event calculus and circumscription

The event calculus is a first-order formalism that was originally developed (by Kowalski and Sergot Kowalski & Sergot (1986)) in the context of solving robot planning problems, and as such is an alternative to the earlier situation calculus of McCarthy and Hayes (McCarthy & Hayes (1969)). Surprisingly, the rich ontology (which distinguishes among different kinds of events) necessary for the robotics domain turns out to be extremely helpful for semantics. The event calculus comes with a nonmonotonic inference mechanism (here we use McCarthy's *circumscription* for that purpose) which turns out to be well suited to solve the imperfective paradox. Before we delve into technicalities, some motivation is provided by section 4.1. [14]

4.1. **The frame problem.** Although the frame problem has its roots in artificial intelligence, it is relevant for the semantics of natural language as well (cf. Steedman (1997)), insofar as the use of events has gained prominence there.

In a nutshell, the frame problem is this[15]. When describing the effects of actions or events on the state of the world, it is also necessary to describe the properties

[14] Many variants of the event calculus have been proposed in the meantime; the interested reader may consult the special issue of the *Journal of Logic Programming* devoted to logics of action (1995) for further references.

[15] The frame problem was first identified in McCarthy & Hayes (1969).

that do not change as the result of the action or the event. Not doing this actually renders acting in the world impossible. Suppose I am deliberating whether to clean my desk. The intended effect of cleaning my desk is that I may actually use it again. However, the following facts about the world should also be considered:

- Cleaning my desk does *not* change its shape.
- Cleaning my desk does *not* cause my neighbour's desk to be cleaned also.
- Cleaning my desk does *not* cause an earthquake.
- *etc.* ...

If I would remain agnostic about these properties of the world, I would be unable to act; cf. the third fact. But clearly the explicit description of the things that don't change is incomparably more involved than a description of the things that do change. Of course, in ordinary life one never appeals to such an explicit description; instead one appeals to the *common sense law of inertia*, which can be roughly formulated as follows:

(32) Nothing changes, except when there is explicit evidence to the contrary.

4.2. **The event calculus.** Shanahan's version of the Event Calculus $\mathcal{EC}$[16] is formulated in many-sorted first order logic, which has (at least) sorts for *individuals*, *instants*, *event(types)* and *fluents*. Sometimes we will need an additional sort for numerical parameters such as lengths or angles. A fluent is a time-dependent property[17]. Fluents may be initiated or terminated by an event(token), or they may undergo continuous change. The various interactions of events and fluents are codified in nine primitive predicates.

4.2.1. *Syntax of the event calculus.* Let S be the set of sorts, comprising at least the sort of individuals, the sort of instants,the sort of fluents, and the sort of event(types). The language of the event calculus contains variables $x_{1s}, x_{2s}, \ldots$ and constants $c_{1s}, c_{2s}, \ldots$ for each sort $s \in S$. For the sake of legibility we shall often write for a variable over e.g. the sort of fluents not x_f but f, and similarly for the other sorts. Furthermore, for each n, each vector $\overline{s}$ of sorts, and sort r there exist function symbols $f^0, f^1, \ldots$ which map a tuple of type $\overline{s}$ to an element of type r. Similarly, for each vector $\overline{s}$ of sorts, there are relation symbols $R^0, G^1, \ldots$ which take arguments of this type.

The event calculus contains nine distinguished predicates, whose meaning is determined by axioms or explicit definitions.

[16] The version used in this paper was developed in a series of papers Shanahan (1996*a*), Shanahan (1995), Shanahan (1996*b*) and Shanahan (1990). Shanahan's discussion of the frame problem and his proposed solution can also be found in the book Shanahan (1997).

[17] The name derives from Newton's approach to differential calculus, where each variable was assumed to be implicitly dependent on time.

The first four predicates concern 'normal' change, where a time-dependent property is switched on or switched off by an event, as when the fluent *(water)flowing* is initiated (or terminated) by turning on (off) a tap.

Initially(f): *Initially* is a property of fluents, which singles out those fluents f which are true at the beginning of the history considered.

Happens(e,t): *Happens* is a binary relation between an event(type) e and a time point t, with the intended interpretation that e occurs at t. Accordingly, the set $\{(e,t) \mid \mathit{Happens}(e,t)\}$ can be interpreted as the set of event tokens corresponding to e.

Initiates(e,f,t): *Initiates* is a ternary relation between event types e, fluents f and instants t, whose intended interpretation is that the realisation of event type e at t causes f to hold after t. For definiteness it is assumed that f does not yet hold at t.

Terminates(e,f,t): *Terminates* is again a ternary relation between event types e, fluents f and instants t, which says that f does no longer hold after e occurred at t. Here, it is assumed that f holds at t.

The next two predicates are concerned with continuous change, as when water *flowing* into a bucket makes the *height* of the water in the bucket increase.

Trajectory(f_1,t,f_2,d): the *Trajectory*–predicate takes four arguments, fluents f_1, f_2, an instant t_1 and a length of time d. In a typical application of this predicate to continuous change, the second fluent-argument represents a property that may vary with time, such as the height of water in a bucket. Formally, such a property is represented by a fluent-valued function, such as *height*(x), where x, the height, may be a function of time. For example, if the height of the water in a bucket rises linearly when the tap is on, we may represent this by

$$\forall x_1 \forall t \forall d (\mathit{HoldsAt}(\mathit{height}(x_1),t) \;\rightarrow\; \mathit{Trajectory}(\mathit{filling},t,\mathit{height}(x_1 + d),d)),$$

which (in conjunction with axiom 4 below) says that if the height at time t is x_1, then at time $t+d$ it will be x_1+d.

Releases(e,f,t): *Releases* is a ternary relation between event types e, fluents f and instants t, which is necessary to model continuous change consistently. Consider the fluent *height*(0), which expresses that the height of the water in the bucket is 0 units. This is true *initially*, but when water starts *flowing*, i.e. when a *tap–on* event occurs, *height*(0) should no longer be true; we then obtain true fluents of the form *height*(x), for every x in some interval $(0,b]$, where each *height*(x) is true for a single instant only. Clearly however, there will be no separate events which initiate and terminate the fluents *height*(x) for various values of x, so the vocabulary previously introduced is of no help here. The predicate *Releases* indicates that a special mechanism for continuous change takes over.

For a smooth formulation of the axioms, it is useful to have two ternary predicates explicitly defined in terms of the preceding.

Definition 7. $\mathit{Clipped}(t, f, t') := \exists e, s(\mathit{Happens}(e, s) \wedge t < s < t' \wedge (\mathit{Terminates}(e, f, s) \vee \mathit{Releases}(e, f, s)))$

Definition 8. $\mathit{Declipped}(t, f, t') := \exists e, s(\mathit{Happens}(e, s) \wedge t < s < t' \wedge (\mathit{Initiates}(e, f, s) \vee \mathit{Releases}(e, f, s)))$

Lastly we need a truth predicate, analogous to Feferman's T_1:

$\mathit{HoldsAt}(f, t)$: *HoldsAt* is a binary relation between a fluent f and a time point t, whose intended interpretation is that f is true at t.

The event calculus in its standard form does not contain the characteristic axiom for a truth predicate. This would require a formal mechanism to map a formula $\varphi(t)$ on a fluent-term f, so that we could write

$$\mathit{HoldsAt}(f, t) \;\leftrightarrow\; \varphi(t).$$

The event calculus itself does not possess such a mechanism; although it is built on the idea of *reifying* properties, the reification of properties is not itself part of the calculus. We will use Feferman's type-free system for this purpose.

4.2.2. *Axioms for $\mathcal{EC}$.* In the following, all variables are assumed to be universally quantified.

Axiom 4. $\mathit{Initially}(f) \wedge \neg \mathit{Clipped}(0, f, t) \rightarrow \mathit{HoldsAt}(f, t)$

Axiom 5. $\mathit{Happens}(e, t) \wedge \mathit{Initiates}(e, f, t) \wedge \; t < t' \; \wedge \neg \mathit{Clipped}(t, f, t') \rightarrow \mathit{HoldsAt}(f, t')$

Axiom 6. $\mathit{Happens}(e, t) \wedge \mathit{Terminates}(e, f, t) \wedge \; t < t' \; \wedge \neg \mathit{Declipped}(t, f, t') \rightarrow \neg \mathit{HoldsAt}(f, t')$

Axiom 7. $\mathit{Happens}(e, t) \wedge \mathit{Initiates}(e, f, t) \wedge \; t < t' \; \wedge \; t' = t + d \; \wedge \mathit{Trajectory}(f_1, t, f_2, d) \wedge \neg \mathit{Clipped}(t, f, t') \rightarrow \mathit{HoldsAt}(f_2, t')$

We add some comments on the axioms.

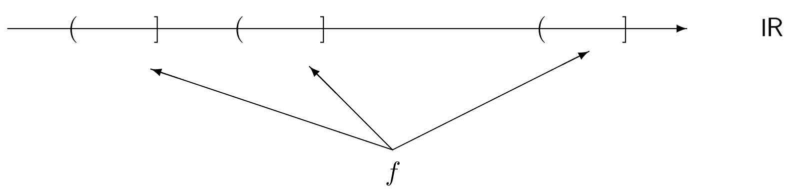
Suppose for a moment that the *Release*–predicate does not occur in the definition of *Clipped*. Then the first axiom just says that if a fluent holds at time 0 and no event has terminated it at time $t > 0$, then it still holds at t. The presence of *Release* allows us to say in addition that this is only so when the fluent is all the time subject to the law of inertia.

The second axiom treats the analogous case where the fluent is initiated at some time $t_1 > 0$. Again, assume first that the *Release*–predicate does not occur in the definition of *Declipped*. The third axiom then says that a fluent f does not hold at t_2, when it has been terminated before t_2 and no initiating event has occurred after termination. However, when f is not subject to the law of inertia, we have no reason to expect this; the *Release*–clause takes care of this case.

The fourth axiom is best explained by means of the example of filling a bucket with water. So let x_f be instantiated by *filling*, and y_f by *height*(x). If *filling* has been going on from t_1 until t_2, then for a certain x, *height(x)* will be true at t_2, the particular x being determined by the law of the process as exemplified by the *Trajectory*–predicate.

4.3. **A model for $\mathcal{EC}$.** To facilitate the reader's comprehension of the axioms, we will provide an intuitively appealing model of $\mathcal{EC}$, incidentally showing its consistency. The result to be presented is very weak. In practice, the $\mathcal{EC}$ axioms are always used in conjunction with some first order theory (e.g. detailing the properties of the sensors and effectors of a robot), but this first result says nothing about extending an arbitrary first order theory with the $\mathcal{EC}$ axioms.

The most important concept to be defined is that of a fluent. We interpret fluents as sets of halfopen intervals $(a, b]$, where a is the instant at which an initiating event occurs, and b is the instant where 'the next' terminating event occurs[18]. Talk about 'the next' seems justified due to the inertia inherent in fluents. A typical fluent therefore looks as follows:



For the purpose of constructing the model, we think of event(types) as derivative of fluents, in the sense that each event either initiates or terminates a fluent, and that fluents are initiated or terminated by events only. The *instants* are taken to be nonnegative reals. Each fluent f is a finite set of disjoint halfopen intervals $(a, b]$, with the possible addition of an interval $[0, c]$. Event types e are of the form $e = e_f^+$ or $e = e_f^-$ where $e_f^+ := \{(f, r) \mid \exists s((r, s] \in f)\}$ and
$e_f^- := \{(f, s) \mid \exists r((r, s] \in f)\}$.

This then yields the following interpretations for the distinguished predicates.
$HoldsAt := \{(f, t) \mid \exists I \in f(t \in I)\}$
$Initially := \{f \mid \exists s > 0[0, s] \in f)\}$
$Happens := \{(e, t) \mid \exists f((e = e_f^+ \vee e = e_f^-) \wedge (f, t) \in e)\}$
$Initiates := \{(e, f, t) \mid e = e_f^+ \wedge (f, t) \in e\}$
$Terminates := \{(e, f, t) \mid e = e_f^- \wedge (f, t) \in e\}$
$Releases := \emptyset$
$Trajectory := \{(f_1, t_1, f_2, d) \mid [\exists e(e = e_{f_1}^+ \wedge$
$(f_1, t_1) \in e) \wedge \forall t(t_1 < t \leq t_1 + d \rightarrow HoldsAt(f_1, t))] \rightarrow HoldsAt(f_2, t_1 + d)\}$.

[18]We allow b to be ∞.

Obviously these stipulations enforce the following interpretations for *Clipped* and *Declipped*:
Clipped $:= \{(t_1, f, t_2) \mid \exists t(t_1 < t < t_2 \wedge (f,t) \in e_f^-)\}$
Declipped $:= \{(t_1, f, t_2) \mid \exists t(t_1 < t < t_2 \wedge (f,t) \in e_f^+)\}$

Proposition 1. *$\mathcal{EC}$ is true under the above interpretation.*

Proof Given the above interpretation of the distinguished predicates, the meaning of axiom 4 can be rendered formally as:

$\exists s > 0([0,s] \in f) \wedge \forall t^{'}(0 < t^{'} < t \rightarrow (f,t^{'}) \notin e_f^-) \rightarrow \exists r, s(t \in (r,s] \in f)$

Define $s_0 := \sup\{s \mid \exists I([0,s] \subseteq I \in f)\}$; s_0 exists and is greater than 0. It suffices to show that $t \in [0, s_0] \in f$. $[0, s_0]$ is clearly in f. Suppose that $t \notin [0, s_0]$, i.e. $s_0 < t$, then we would have $(f, s_0) \notin e_f^-$. By definition of s_0 and of terminating event we have, however, that $(f, s_0) \in e_f^-$.

The second axiom (5) receives as interpretation:
$\exists f^{'}((e = e_{f'}^+ \vee e = e_{f'}^-) \wedge (f^{'}, t_1) \in e) \wedge t_1 < t_2 \wedge e = e_f^+ \wedge (f,t) \in e \wedge \forall t(t_1 < t < t_2 \rightarrow (f,t) \notin e_f^-) \rightarrow \exists I(t_2 \in I \in f)$.

It suffices to show that
$(e = e_f^+ \wedge (f, t_1) \in e) \wedge \forall(t(t_1 < t < t_2) \rightarrow (f,t) \notin e_f^-) \rightarrow \exists I(t_2 \in I \in f)$, since we have
$e = e_f^+ \wedge (f,t) \in e \rightarrow \exists f^{'}((e = e_{f'}^+ \vee e = e_{f'}^-) \wedge (f^{'}, t) \in e)$.

Argue as in the previous case, but now define $s_0 = \sup\{s \mid \exists r < t_2 \exists I(r,s] \subseteq I \in f\}$. We must show that $t_2 \leq s_0$. By definition of e_f^- we have:
$(f, s_0) \in e_f^-$ and $t_1 < s_0$. If $s_0 < t_2$, the hypothesis of the axiom would give: $(f, s_0) \notin e_f^-$, a contradiction.

The remaining two axioms are easy: 6 follows by contraposition and 7 is true by definition. □

Clearly this easy construction works only due to the lack of additional axioms. If a theory were to contain, in addition to $\mathcal{EC}$, infinitely many axioms of the form *HoldsAt*(f, t), and axioms involving the *Trajectory*–predicate, such simple models are not likely to be forthcoming. However, the model captures an important intuition, and we shall come back to the question when models of this type exist.

4.4. **Circumscription.** In terms of the language introduced above, we may now state the common sense law of inertia as follows

> Normally, given any action [or event] and any fluent, the action doesn't affect the fluent.
> McCarthy & Hayes (1969)

It is clear that the axioms given have as yet little bearing on this desideratum. The axioms do embody a notion of persistence: a fluent not affected by an action will continue to hold (or not to hold, as the case may be). But in itself this carries no information about which actions affect which fluents. This will be accomplished by a nonmonotonic reasoning scheme called circumscription. The idea is to minimise the occurrence of actions and of the influence of actions as much as is compatible with the data. For example, if the data say that there are two events e_1 and e_2 such that $Happens(e_1, t_1)$ and $Happens(e_2, t_2)$ then minimising the occurrence of actions would be equivalent to adding the statement

$$Happens(e, t) \leftrightarrow (e = e_1 \vee e = e_2) \wedge (t = t_1 \vee t = t_2).$$

Circumscription provides a general formulation of this idea[19].

4.4.1. *Definitions.*

Definition 9. *Let P, Q be predicate symbols of the same arity. Put*
$P = Q := \forall x(P(x) \leftrightarrow Q(x))$
$P \leq Q := \forall x(P(x) \rightarrow Q(x))$
$P < Q := P \leq Q \wedge \neg(P = Q)$

Definition 10. *Let $\varphi(P)$ be a sentence containing an occurrence of the predicate symbol P. The circumscription of P in $\varphi(P)$ is defined as the following formula of second order logic:*

$$\varphi(P) \wedge \neg\exists p[\varphi(p) \wedge p < P],$$

where p is a predicate variable of the same arity as P. The resulting formula will be denoted by $CIRC[\varphi; P]$.

Example 1. *Let a be an individual constant and $\varphi(P) = P(a)$. By definition, $CIRC[P(a); P]$ is equal to $P(a) \wedge \neg\exists p[p(a) \wedge p < P]$. It follows that $CIRC[P(a); P]$ is equivalent to the first order formula $\forall x[P(x) \leftrightarrow x = a]$. We shall later provide a number of conditions on φ which ensure that $CIRC[\varphi; P]$ is first order.*

Example 2. *Let $\varphi(P) = \forall x(Q(x) \rightarrow P(x))$. Then we have $CIRC[\phi; P]$: $\forall x(Q(x) \rightarrow P(x)) \wedge \neg\exists p((\forall x(Q(x) \rightarrow p(x)) \wedge p < P)$; that is, $CIRC[\phi; P] \leftrightarrow \forall x(Q(x) \leftrightarrow P(x))$.*

Definition 10 is too restrictive for most applications. Note that the effect of 10 is that P is minimised under the assumption that the interpretations of the other predicates are kept constant. In many contexts it is important however to study the *effect* of minimising P on the interpretations of (some) other predicates. This idea is better captured by the next definition.[20]

Definition 11. *Let $\varphi(P, Z_1, \ldots, Z_m)$ be a sentence which contains an occurrence of the predicate symbol P. Let $Z_1, \ldots, Z_m$ be individual constants, predicate symbols or function symbols different from P. The circumscription of P in φ with varying $Z_1, \ldots, Z_m$ is the following formula of second order logic:*

$$\varphi(P, Z_1, \ldots, Z_m) \wedge \neg\exists p z_1 \ldots z_m(\varphi(p, z_1, \ldots, z_m) \wedge p < P),$$

[19] It is also possible to use the theory of completions of (constraint) logic programs for this purpose, as has been done in van Lambalgen & Hamm (2001).

[20] It should be noted though that Shanahan's proposed solution of the frame problem involves a return to the earlier definition.

abbreviated by $CIRC[\varphi; P; \overline{Z}]$ *where* $\overline{Z}$ *is shorthand for* $Z_1, \ldots, Z_m$. *(Similarly,* $z_1, \ldots, z_m$ *will often be written as* $\overline{z}$.*)*

Example 3. *It is easy to see that* $CIRC[P(a) \wedge P(b); P]$ *equals* $\forall x[P(x) \leftrightarrow (x = a \vee x = b)]$. *Now consider* $CIRC[P(a) \wedge P(b); P; a, b]$. *The difference with the situation above is that we may now give* P *a smaller extension by identifying* a *and* b. *Indeed,* $CIRC[P(a) \wedge P(b); P; a, b]$ *is by definition equivalent to*

$$P(a) \wedge P(b) \wedge \neg\exists p z_1 z_2 [P(z_1) \wedge P(z_2) \wedge p < P],$$

which in turn is equivalent to the first order formula $\forall x[P(x) \leftrightarrow x = a] \wedge a = b$.

We will now make the semantic intuition behind circumscription more precise. As already indicated, the models of $CIRC[\varphi; P; Z]$ should be those models of φ in which the extension of P cannot be made smaller even when the interpretation of Z is allowed to vary. This intuition leads to the following partial order on the class of models

Definition 12. *Let models* $\mathcal{M}_1$ *and* $\mathcal{M}_2$ *be given.* $\mathcal{M}_1 \leq^{P;Z} \mathcal{M}_2$ *iff*
- $M_1 = M_2$
- $[\![c]\!]_{\mathcal{M}_1} = [\![c]\!]_{\mathcal{M}_2}$, *for all* c *not in* $\{P\} \cup Z$.
- $[\![P]\!]_{\mathcal{M}_1} \subseteq [\![P]\!]_{\mathcal{M}_2}$

Hence, $\mathcal{M}_1 \leq^{P;Z} \mathcal{M}_2$ means that the structures differ only with respect to the interpretations of P and Z and furthermore, that the extension of P in $\mathcal{M}_1$ is a subset of the extension of P in $\mathcal{M}_2$. When $\mathcal{M}_1 \leq^{P;Z} \mathcal{M}_2$ but not $\mathcal{M}_2 \leq^{P;Z} \mathcal{M}_1$, we write $\mathcal{M}_1 <^{P;Z} \mathcal{M}_2$.

Since $\leq^{P;Z}$ is reflexive and transitive, it makes sense to talk of minimal structures with respect to this order (which is not to say that such structures must exist!).

Definition 13. *A structure* $\mathcal{M}$ *is minimal with respect to* $\leq^{P;Z}$ *if and only if there does not exist a structure* $\mathcal{M}'$ *with* $\mathcal{M}' <^{P;Z} \mathcal{M}$.

One then easily proves

Lemma 2. *A structure* $\mathcal{M}$ *is a model of* $CIRC[\varphi; P; Z]$ *if and only if* $\mathcal{M}$ *is minimal with respect to* $\leq^{P;Z}$.

4.4.2. *Rules for computing circumscriptions.* In this section we collect a few recipes for computing circumscription that will be useful later. Proofs will be omitted; they can be found in Lifschitz (1994).

The first result is a generalisation of example 2, which will be referred to as *predicate completion.*

Lemma 3. *If* $\psi(x)$ *does not contain* P, *then the circumscription of* $\forall x(\psi(x) \rightarrow P(x))$ *with respect to* P

$$CIRC[\forall x(\psi(x) \rightarrow P(x)); P]$$

is equivalent to

$$\forall x(\psi(x) \leftrightarrow P(x))$$

The second result is an immediate, but useful, consequence of Definition 11.

Lemma 4. *Let ψ be a sentence which does not contain P and Z. Then we have* $CIRC[\varphi(P,Z) \wedge \psi; P; Z] \leftrightarrow CIRC[\varphi(P,Z); P; Z] \wedge \psi$.

The next lemma is mostly of theoretical interest (although a special case will be applied in Section 6.5) ; it shows that circumscription with varying constants can in a sense be reduced to the basic case, Definition 10.

Lemma 5. *The formula*

$$CIRC[\varphi(P,Z); P; Z]$$

is equivalent to

$$\varphi(P,Z) \wedge CIRC[\exists z\varphi(P,z); P]$$

The case most relevant for our purposes is where Z is a set of individual constants. For example, according to Lemma 5 $CIRC[P(a) \wedge P(b); P; a, b]$ is equivalent to

$$P(a) \wedge P(b) \wedge CIRC[\exists z_1 z_2(P(z_1) \wedge P(z_2)); P],$$

which in turn is equivalent to

$$P(a) \wedge P(b) \wedge \exists x\forall y(P(y) \leftrightarrow x = y),$$

since $\exists z_1 z_2(P(z_1) \wedge P(z_2))$ is equivalent to $\exists z P(z)$ and circumscribing the latter formula forces P to contain one element only.

Stronger results can be obtained when one considers specific classes of formulas. The occurrence of a predicate symbol in a formula is *positive*, if it occurs in the scope of an even number of negations, *negative* otherwise[21]. A formula $\varphi(P)$ is *positive* with respect to P if all occurrences of P in φ are positive; $\varphi(P)$ is *negative* with respect to P if all occurrences of P in φ are negative.

Lemma 6. *If $\varphi(P,Z)$ is positive with respect to P then*

$$CIRC[\varphi(P,Z); P; Z]$$

is equivalent to

$$\varphi(P,Z) \wedge \neg\exists xz[P(x) \wedge \varphi(\lambda y(P(y) \wedge x \neq y), z)].$$

The analogue for negative $\psi(P)$ is:

Lemma 7. *If $\psi(P)$is negative, then*

$$CIRC[\varphi(P) \wedge \psi(P); P]$$

is equivalent to

$$CIRC[\varphi(P); P] \wedge \psi(P).$$

Unfortunately, for our applications below positive and negative formulas do not suffice. Some statements of interest are only *definite in* P, that is, of the form $F(P) \rightarrow P$, where P occurs only positively in F.

[21]We assume that $\rightarrow$ and $\leftrightarrow$ are defined in terms of $\neg$, $\wedge$ and $\vee$.

Lemma 8. *Let $A(P)$ be the universal closure of a formula definite in P, then $CIRC[A(P); P]$ is equivalent to $A(P) \wedge \forall x[P(x) \leftrightarrow \forall p(A(p) \rightarrow p(x))]$.*

We have now seen several examples where the circumscription of a sentence φ, which in principle is a sentence of second order logic, can be reduced to a first order sentence, and one example where there does not appear to be such a reduction. The obvious question is, whether these results can be subsumed under a single theorem which gives necessary and sufficient conditions for the existence of a first order equivalent. That cannot be done in general, but the next result goes a little way toward that unattainable goal:

Lemma 9. *Let $\varphi(P)$ be positive and $\psi(P)$ be negative, both first order. Then the circumscription $CIRC[\varphi(P) \wedge \psi(P); P]$ is equivalent to a first order formula.*

4.4.3. *Parallel circumscription.* In the applications of circumscription to the event calculus, we will usually need to circumscribe several predicates simultaneously. This is not yet covered by Definition 11, so we need a generalisation:

Definition 14. *Let $\vec{P} = (P_1, \ldots, P_n)$ and $\vec{Q} = (Q_1, \ldots, Q_n)$ be sequences of predicate constants.*
$\vec{P} = \vec{Q}$ iff $P_1 = Q_1 \wedge \ldots \wedge P_n = Q_n$.
$\vec{P} \leq \vec{Q}$ iff $P_1 \leq Q_1 \wedge \ldots \wedge P_n \leq Q_n$.
$\vec{P} < \vec{Q}$ iff $\vec{P} \leq \vec{Q} \wedge \neg\vec{P} = \vec{Q}$.

$$CIRC[\varphi; \vec{P}; Z] \text{ iff} \varphi(\vec{P}, Z) \wedge \neg\exists\vec{p}z[\varphi(\vec{p}, z) \wedge \vec{p} < \vec{P}].$$

It is easy to see how to generalise the definition of the ordering relation $\mathcal{M}_1 \leq^{P,Z} \mathcal{M}_2$ and the proposition relating minimal models in the ordering and circumscription. For our purposes, the following theorem, reducing parallel circumscription to a conjunction of circumscriptions (albeit only in special cases) is of great importance:

Theorem 4. *Let $\vec{P}$ be a sequence of predicate constants. If $\varphi(\vec{P}, Z)$ is positive with respect to each element P_i of the sequence $\vec{P}$, then the parallel circumscription*

$$CIRC[\varphi(\vec{P}, Z); \vec{P}; Z]$$

is equivalent to

$$\bigwedge_i CIRC[\varphi(\vec{P}, Z); P_i; Z]$$

4.4.4. *Existence.* We have already noted that talk of minimal models of a (consistent) theory is not to imply that these models exist. Theories which do have this desirable property are isolated by means of the following definition:

Definition 15. *A consistent first order theory Δ will be called* wellfounded *with respect to (P;Z) if and only if for each model $\mathcal{M}$ of Δ there exists a model $\mathcal{M}' \models \Delta$ which is minimal with respect to $\leq^{P;Z}$ and which satisfies $\mathcal{M}' \leq^{P;Z} \mathcal{M}$.*

Wellfoundedness of a theory is related to its syntactical form. A first order formula is *universal*, if it is of the form $\forall\vec{x}\varphi$ is, where φ is quantifier free. A first order formula is called *almost universal with respect to* P, if the formula φ satisfies the condition that P does not occur positively in the scope of quantifiers. The relevance of almost universality to wellfoundedness is given by

Theorem 5. *Suppose* Δ *can be axiomatised in such a way that each axiom is almost universal with respect to* P*. Then* Δ *is wellfounded with respect to* P*.*

This existence theorem thus pertains only to the basic case of circumscription. For the general case we have

Theorem 6. *Suppose the theory* Δ *has a universal axiomatisation and suppose furthermore that* Z *only contains predicate symbols (i.e. no individual constants). Then* Δ *is wellfounded with respect to* $(P; Z)$*.*

4.5. **Scenarios.** In the applications of the event calculus to natural language we must formulate information about the specific situation at hand in the language of fluents and events. Such a situation-description will be called a *scenario* (in the literature one also finds the term *narrative*). In order for circumscription to apply, scenarios are subject to some syntactic restrictions[22].

Definition 16. *A* state *at time* t *is a conjunction of*

(1) *literals of the form* $(\neg)\mathit{HoldsAt}(f,t)$*, for* t *fixed and possibly different* f*,*
(2) *equalities between fluent terms, and between event terms*
(3) *statements formulated in the language* $\{0, 1, +, \times, <\}$ *of the real numbers.*[23]

In principle we may distinguish between two kinds of lawlike statements $F \rightarrow G$ (assumed to be universally quantified): the type where the time variables occurring in G are different from those in F (called *dynamic* laws), and the type where this is not the case (the *static* laws). It seems sensible to keep the dynamics outside the circumscription, and to minimise the distinguished predicates only in accordance with the static laws. In each time slice, we assume only those initiating and terminating events that we are forced to, and we then let the dynamics do its thing. Generally speaking, the effect of the dynamics is a change of state. On the other hand, the static laws seem to be of one of the following types, where $S(t)$ is a state at t:

(1) $\mathit{Initially}(f)$
(2) $S(t) \rightarrow \mathit{Initiates}(e,f,t)$
(3) $S(t) \rightarrow \mathit{Terminates}(e,f,t)$
(4) $S(t) \rightarrow \mathit{Releases}(e,f,t)$
(5) $S(t) \wedge \mathit{Happens}(e',t) \rightarrow \mathit{Happens}(e,t)$.

That is, an action (or event) may initiate or terminate a fluent, if a precondition on the current state is fulfilled; and certain events may trigger others under suitable conditions. The latter type appears to be dynamic, but since we want to reserve the

[22] These restrictions can to a large extent be lifted when the theory is reformulated in a logic programing framework; see van Lambalgen & Hamm (2001).

[23] Every such statement is equivalent to one which is quantifier free.

name 'dynamic' for continuous change, to be formalised by means of the *Trajectory* predicate, we classify instantaneous change as static here. (An example of 5 is provided by a bump sensor of a robot registering a collision of the robot with a wall.)

There is one more feature which distinguishes the dynamic and the static parts of a description: the static part is concerned only with *concrete* fluents and events. In the case of fluents, this includes the possibility that the fluent is a function like $\mathit{height}(x)$, which for each real x yields the time-varying property of being of that height.

The appropriate circumscription policy now seems to be to minimise the scenario, while leaving the dynamics untouched. This will be the policy adopted in the sequel, when treating aspect and the progressive. Note that, formally, circumscription of 1,2, 3 and 4 requires simple predicate completion (Lemma 3), whereas circumscription of 5 requires the more involved lemma on definite formulas (Lemma 8), with attendant loss of first order characterisability. This has important consequences for the structure of fluents (as functions of time), and we shall leave conditions of type 5 aside for the moment. Instead, we use a simpler form which leaves out the *Happens* clause in the antecedent. A precondition of the form $S(t)$ can still force events to happen at single instants only; for instance, if the fluent $\mathit{height}(g(s))$ is monotone increasing in time s, for a fixed height h $\mathit{HoldsAt}(\mathit{height}(d), t)$ will be true at one t only, and may thus trigger an event to happen (exactly at) t. However, we do not force events to be instantaneous; it may very well be the case that $\{t \mid \mathit{Happens}(e,t)\}$ is a set of intervals. In fact, it would be rather awkward to force events to be instantaneous; compare 'the burning of the house'. Thus, events and fluents are distinguished not so much by their time profile as by the respective roles they play in the theory.

These considerations motivate the following

Definition 17. *A* scenario *is a conjunction of statements of the form*

(1) $\mathit{Initially}(f)$, *or*
(2) $\forall t(S(t) \rightarrow \mathit{Initiates}(e,f,t))$, *or*
(3) $\forall t(S(t) \rightarrow \mathit{Terminates}(e,f,t))$, *or*
(4) $\forall t(S(t) \rightarrow \mathit{Releases}(e,f,t))$, *or*
(5) $\forall t(S(t) \rightarrow \mathit{Happens}(e,t))$,

where $S(t)$ is a state in the sense of Definition 16. These formulas may contain additional constants for objects, reals or time points and can be prefixed by universal quantifiers over time points, reals and objects. We do not allow Skolem functions of time points etc. so that the above formulae do not have implicit existential quantifiers. Quantifiers over fluents or events are not allowed.

Applying circumscription (i.e. predicate completion) to a scenario in this sense entails that in a minimal model all events and fluents of interest are definable. For in the axioms of the event calculus, the quantifiers over events and fluents are always relativised to one or more of the distinguished predicates, so that in a minimal model of a scenario the quantifiers must range over definable elements only.

5. Reification formalised

In this section we show that the event calculus can be extended conservatively by the truth axioms 3. As already observed, this does not follow automatically from Theorem 2, since the natural interpretation of the predicate *HoldsAt* makes it a special case of T_1. To ensure that every model of the combined calculi satisfies this constraint, we have to devise a separate proof. Before we embark on this, we give a brief sketch of the history of the problem.

The event calculus and its predecessor, the situation calculus, make heavy use of the procedure of reification, whereby properties are transformed into terms, i.e. objects or functions. Although fundamental, the procedure was not itself formalised. It was thus impossible to formulate the axioms required to turn *HoldsAt* into a fullfledged truth predicate. It is somewhat surprising that this apparently has not been done before, although a related problem has cropped up in the context of meta(logic)programming. Interpreters, compilers and debuggers take programs as input, and thus require coding. The solution sometimes adopted in this area is *ambivalent syntax*, in which there is literally no distinction between formulas and terms (originally proposed by Richards (Richards (1974)); see Kalsbeek and Jiang (M.B.Kalsbeek & Jiang (1995)) for an overview). This suits our purpose less well, because there is no analogue of the comprehension axioms. A more sophisticated solution is due to Sato (Sato (1992)), who observes that every metaprogram can be defined from a truth predicate, and who actually provides an executable version of Feferman's T_0 in a three-valued setting. Although this comes close, we need all T_n with their associated axioms, and we want to use classical logic, so we cannot use Sato's work.

The formal language we need in order to provide a semantics for nominalisation and the progressive generally consists of the following parts

(1) an arbitrary first order language L_0, representing the nouns, verbs etc. of natural language; this language may be extended with symbols for monotone generalised quantifiers.
(2) a language for talking about elementary properties of the real numbers, e.g. the language $\{0, 1, +, \times, <\}$.
(3) the distinguished predicates of the event calculus (which are assumed not to occur in L_0)[24].
(4) the predicates T_n.

Let R be an axiomatisation of the reals in the language $\{0, 1, +, \times, <\}$, S_0 a consistent theory in L_0, and EC the axiomatisation of the event calculus. Then

[24] Actually we only have to require that *HoldsAt* does not occur in L_0, but since the axioms of the event calculus involve *HoldsAt* along with the other predicates, this apparent weakening has no real import.

$R + S_0 + EC$ is consistent; call the combined language L_1. R furnishes the necessary machinery for coding formulas. Interestingly, coding may be used to obtain important subclasses of the fluents and the event types. Let $\varphi(t, \overline{x})$ be a formula of L_1, then a fluent function is obtained by considering $\varphi[\hat{t}, \overline{x}]$[25], and an event type by $\exists t.\varphi[t, \overline{x}]$. It is then clear that we must have

$$\mathit{HoldsAt}(\varphi[\hat{t}, \overline{x}], t) \text{ iff } T_1(\varphi[\hat{t}, \overline{x}], t) \text{ iff } \varphi(t, \overline{x}).$$

Sometimes the T_n for $n > 1$ are also useful, for example when one wants to evaluate formulas at pairs of instants, or when one wants to consider possible worlds (although we believe that intensionality is best treated differently). Event tokens may be obtained from event types by means of the *Happens* predicate:

$$\mathit{Happens}(\exists t.\varphi[t, \overline{x}], s).$$

One may similarly construct fluents and event types by coding formulas involving one or more of the T_n, although now something interesting starts to happen.

Reformulate EC in terms of T_1. In axiom 6, replace $\neg T_1$ by its positive counterpart $\overline{T_1}$. Let S be the theory resulting from the addition of the axioms 2 and 3 to $R + S_0 + EC^{'}$, where $EC^{'}$ is EC thus modified.

Theorem 7. *S is a conservative extension of $R + S_0 + EC^{'}$.*

Proof. The main obstacle to the direct use of Theorem 3 is the occurrence of T_1, in the form of *HoldsAt*, in EC. Since $\neg$*HoldsAt* occurs in the consequence of an axiom, EC is not monotone in *HoldsAt*. Since in $EC^{'}$, $\neg$*HoldsAt* has been replaced by its positive counterpart, the resulting axiom system is monotone in (*HoldsAt*, $\overline{\mathit{HoldsAt}}$), and Feferman's construction is applicable. □

Lemma 10. *For fluents $f = \varphi[\hat{t}, \overline{x}]$ definable in L_0, $\neg\mathit{HoldsAt}(f, t)$ iff $\overline{\mathit{HoldsAt}}(f, t)$.*

Proof. This follows from the Formulas (30-a) and (30-b). □

Now let EC_0 be the event calculus which arises from the original EC by erasing quantification over fluents, and replacing each axiom by an axiom scheme, one axiom for each L_0–definable fluent. Then we have

Theorem 8. *The truth axioms 2 and 3 can be added conservatively to $R+S_0+EC_0$.*

This is no yet sufficient for providing a semantics for the progressive, because there the event calculus has to be extended by a scenario and a dynamics. Since the scenario does not contain nested *HoldsAt*–predicates it presents no problems. It will be seen that statements characterising the dynamics are typically of the form

$$\mathit{HoldsAt}(\ldots) \rightarrow \mathit{Trajectory}(\ldots).$$

Replacing $\rightarrow$ by its definition in terms of $\neg$ and $\vee$, we see that at least for L_0–definable fluents, addition of the dynamics presents no problems. We thus have

Theorem 9. *Let $SCEN$ be a scenario and DYN a dynamics. Then the truth axioms 2 and 3 can be added conservatively to $R + S_0 + SCEN + DYN + EC_0$.*

[25] Compare Definition 2.

To conclude this section, we add some remarks on the lattice structure of fluents. We have seen that in the standard model for EC, fluents form a Boolean algebra, since they are defined as sets of halfopen intervals. In the general case we have less control over the fluents, but the above material shows that always

Lemma 11. *The L_0–definable fluents form a Boolean algebra.*

Interestingly, in case the scenarios are finite, fluents representing activities can again be represented as finite sets of halfopen intervals, see Theorem 10 below. Both observations will be of some importance when we discuss the distributional properties of imperfect nominals.

6. Nominalisation formalised

We first make some general remarks on the roles of time and event variables. It has become customary to provide the formal denotation of verbs with an argument for events, and to assume that events are partially ordered by inclusion. However, we wish to deviate from this convention because in our setup 'event' is the notion to be analysed; and we also want to construct various kinds of events from linguistic expressions. We therefore have to say a few words about the formal representation of verbs. In natural language, verbs do not appear to have truth conditions which depend on explicitly given time points, but only on time points given indexically (e.g. by the time of utterance). Stassen for example observes in his representative study of intransitive predication (Stassen (1997)) that in the majority of the languages in his sample, which contains more than 400 languages, tense is not marked in the grammatical system at all. This suggests that verbs are represented by predicates which do not have a parameter for time, and that temporal reference is accomplished by a separate Priorean operator $At(t, \varphi)$, where t does not occur free in φ. Thus, from $B(x, y)$ we may derive a time-dependent predicate $A(t, x, y)$ by putting $A(t, x, y) := B(x, y) \wedge At(t, B(x, y))$. However, formally there is not much of a difference between this and a representation where time is included in the verb. If $A(t, x, y)$ is a predicate, then we may define a formula $B(x, y)$ and At such that $A(t, x, y) \leftrightarrow B(x, y) \wedge At(t, B(x, y))$, by putting $B(x, y) := \exists t A(t, x, y)$ and $At(t, B(x, y)) \leftrightarrow A(t, x, y)$. We shall therefore opt for the simpler representation and incorporate the time parameter into the predicate.

We now choose an interpretation for perfect nominals. As the above discussion shows, it is reasonable to equate the event type generated by $A(t, x, y)$ with $\exists t A(t, x, y)[x, y]$, which represents the subset $\{(a, b) \mid \exists t A(t, a, b)\}$ of the domain of the model. There is a subtlety hidden here: $\{(a, b) \mid \exists t A(t, a, b)\}$ is a real, i.e. extensional, subset, not a term in Feferman's calculus. We have chosen this option in order to stay close to the traditional interpretation of determiners as relation between (extensionally conceived) sets. If A is constructed from $B(x, y)$ and At, this operation has the effect of recapturing $B[x, y]$. More generally:

Definition 18. *If $\varphi(t_1 \ldots t_n, \overline{x})$ is a formula, the event type generated by φ will be $\exists t_1 \ldots t_n.\varphi(t_1 \ldots t_n, \overline{x})[\overline{x}]$. (This notation, due to Feferman, was introduced in Definition 2).*

The form involving several variables for time is useful to derive a lattice structure on event types, as will be seen below. When no confusion can arise, we shall usually write φ instead of $\varphi(t_1 \dots t_n, \overline{x})$. We allow event types constructed in this way to occur as event-arguments in applications of the event calculus. It should be noted at this point that there is a syntactic difference between the *At* operator and the *Happens* predicate, since the latter, but not the former, is a relation between terms.

We have seen that imperfect nominals, as witnessed by their distributional properties, still contain some vestiges of time. The following definition therefore seems appropriate.

Definition 19. *The denotation of the imperfect nominal deriving from an expression $\varphi(t, \overline{x})$ is the term $\varphi[\hat{t}, \overline{x}]$*[26].

Clearly $\varphi[\hat{t}, \overline{x}]$ may be substituted for a fluent-argument in the event calculus. Both from a logical and a linguistic point of view it is then interesting to determine what the structure of sets of the form $\varphi[\hat{t}, \overline{x}]$ can be. If the imperfect nominal is an accomplishment such as 'building a house', one would expect that the set of instants satisfying this description is something like a finite set of intervals. By Tarski's theorem on quantifier elimination[27] we know that every set of reals definable with a formula involving only $+$, $\times$, 0,1 and $\leq$ is a finite union of open intervals and points. The question then becomes whether a result like this is still true when the theory of the reals is expanded with the predicates and axioms of the event calculus, together with a scenario. Theorem 10 provides some answers to this question.

With the above definitions in place, we can now explain in greater detail the similarities and differences between the distinguished predicates *HoldsAt* and *Happens*. In a sense, both are truth predicates; but whereas *HoldsAt* codes truth in models $(\mathcal{M}, t)$, *Happens* has a different interpretation. Recall that according to Austin (1961), an indicative sentence φ has two components of meaning: the descriptive conventions of language yield an event type, whereas the demonstrative conventions yield an event token. Thus, translated into our language, if e is an event type, and (e', t) is an event token, then the truth condition $(e', t) \models e$ is simply $e' \leq e \ \wedge \ \mathit{Happens}(e', t)$, where $\leq$ derives from the implication between the corresponding formulas. As will be seen in Section 6.3, there are intimate connections between the two predicates, and in some cases *Happens* can be defined in terms of *HoldsAt*. However, this does not mean that we can do without *Happens*, since we want to minimise the number of occurrences of events without simultaneously minimising *HoldsAt*.

[26]This definition does not say that imperfect nominals denote *propositions*. But the righthand-side in the following biconditional may denote a proposition:

$$\mathit{HoldsAt}(\phi(x, \hat{t}) \leftrightarrow \phi(t)$$

It is compatible with the approach taken here to think of propositions as sets of worlds but the system does not force this view. Many different notions of proposition are compatible with the proposed framework. For instance one could also think of propositions as structured meanings. To develop a novel concept of proposition is not one of the aims of this paper.

[27]For a proof, one may consult Hodges (Hodges (1993)), Section 8.4.

An important issue that now has to be addressed is that of extensionality versus intensionality of fluents and event types. It seems advantageous to take fluents as intensional entities. If we say

(33) Mary predicted the king's beheading.

then, even in the case that the king is actually identical to the red-haired spy, we still do not want to infer from this that

(34) Mary predicted the red-haired spy's beheading.

This can easily be modeled in Feferman's calculus. Even when the formulas $\varphi(t,x)$ and $\psi(t,x)$ are logically equivalent, the terms $\varphi[\hat{t},x]$ and $\psi[\hat{t},x]$ are different, and there is no axiom of extensionality which can force equality of the sets these terms represent. Here is another example of the same phenomenon: if one doesn't know that Bill is John's friend, the following two sentences involving imperfect nominals can be true simultaneously

(35) a. John's greeting Bill surprises me.
b. John's greeting his friend does not surprise me.

A last example is one discussed by Zucchi (Zucchi (1999), p. 185). Suppose Gianni was going by train from Milan to Florence, but due to a strike of the railroad workers, he only got as far as Piacenza. On a Parsons-type approach to events, there is the following problem. Let e be the trip that Gianni took on this occasion and t the time at which he reached Piacenza. Event e does not culminate at t, since e is an unfinished trip to Florence, and Gianni is at Piacenza. But e is also a trip to Piacenza, which does culminate at t. On the present analysis there is no problem at all, since the trips to Florence and Piacenza would be represented by different fluents, which simply happen to share their space-time behaviour from Milan to Piacenza. The predicate *Terminates* can very well be true of one, but not of the other fluent. That is, if f is the fluent corresponding to a trip to Florence, and g the fluent corresponding to a trip to Piacenza, a the event of reaching Piacenza, b the strike, then the scenario would feature the conditions $\mathit{Terminates}(a,g,t)$ and $\mathit{Terminates}(b,f,t)$; applying circumscription would then have the effect of enforcing $\neg\mathit{Terminates}(a,f,t)$, as required.

6.1. **Systematic Translation: Imperfect Nominals.** We start this section with a systematic translation of Poss–ing gerunds or *nominal gerund phrases* (NGP) in Pullum's terminology. Consider example (36):

(36) John's singing the Marseillaise

Let us first assume Abney's analysis. According to our assumption the verb *sing* is represented by the predicate *sing(x,y,t)*. Applying this propositional function to the object *the Marseillaise (m)* in the standard way yields: *sing(x,m,t)*. Following Definition 19 the semantic effect of *ing* adjoined to VP is: $sing[x, m, \hat{t}]$. Now an application of this fluent valued function with regard to *John's* results in the fluent $sing[j, m, \hat{t}]$, a fluent object.

We therefore arrive at a strictly compositional interpretation of Abney's analysis of Poss–ing gerunds. The derivation of the interpretation for Acc-ing gerunds is analogous. The semantic interpretation of Pullum's analysis however is slightly different.

Pullum's structure is much more in accord with traditional grammar and certainly less abstract than Abney's. The formation of the participle form of the verb *break* is here part of the morphological component of grammar and therefore does not show up in the syntactic tree. Hence we will assume here that the transformation of $break(x, y, t)$ to $break[x, y, \hat{t}]$ is taken over by the lexicon. The interpretation process then works more or less as in the previous case. The fluent–valued function $break[x, y, \hat{t}]$ is first applied to the *record (r)* yielding $break[x, r, \hat{t}]$. Applying this function to the pronoun *your* results in the fluent $break[your, r, \hat{t}]$, which is again a fluent object. As pointed out in the first chapter we do not consider *your* or *John's* as real (possessive) genitive phrases when these phrases occur in Poss–ing gerunds in contrast to the case of Ing–of gerunds.

The following assumption concerning the denotations of narrow and loose containers will explain some of Vendler's observations.

(37) a. Loose containers denote **sets of fluents** or propositional functions defined on the set of fluents.
b. Narrow containers denote propositional functions defined on the **set of event tokens** or a set of event tokens, i.e. they are subsets of $Happens$.

With these assumptions in place the explanation for the contrast in (38) is immediate.

(38) a. Your breaking the record was a surprise.
b. *Your breaking the record took place at ten.

In (38-a) *was a surprise* denotes a set of fluents and the fluent $break[your, r, \hat{t}]$ may well be an element of this set. By contrast the expression *took place at ten* denotes a set of event tokens which does not tolerate fluent elements. Therefore the unacceptability of (38-b) is due to the type conflict postulated in (37). Note especially that an expression like $Happens(break[your, r, \hat{t}], t)$ is not well formed.

Let us further illustrate the present approach by analyzing a classical example involving infinitives from Chierchia (1988). Chierchia considers the following data:

(39) a. John runs.
b. *John to run.
c. John tries to run.
d. *John tries runs.

Our formal representations for these examples are as shown in (40).

(40) a. $run(j, t)$.
b. $run[\hat{x}, \hat{t}](j)$.
c. $try(j, run[\hat{x}, \hat{t}])$
d. $try(j, run)$.

Here $run[\hat{x}, \hat{t}]$ is the fluent object derived from the propositional function $run(x, t)$. It is now easy to see why the pattern in (39) results. The function run is defined for the individual $John$ but since $run[\hat{x}, \hat{t}]$ is an object the denotation of this expression cannot take $John$ as an argument. The contrast in (39-c) and (39-d) can be derived in a similar way once one assumes that *try* denotes a relation between individuals.

This representation of infinitives is in complete accordance with the one given in classical GB–theory (Chomsky (1981)). An infinitive like the one in *He promised to come* is roughly analysed as:

(41) He$_i$ promised [PRO$_i$ to come]

Here PRO is an ungoverned empty category subject to the theory of control which has to account for the fact that *He* and the empty subject of the infinitive have to be coreferential. Our formalism represents PRO as the variable x bound by abstraction.

6.2. **Systematic Translation: Perfect Nominals and Determiners.** We now turn to the translation of Ing–of gerunds. Consider (42):

(42) John's singing of the Marseillaise

Since ing_{of} is adjoined to a lexical category in the above example the semantic effect is slightly different than that of *ing*. The affix ing_{of} turns $sing(x, y, t)$ into the function $\exists t.sing[x, y, t]$, which maps two objects to an event type. Since the temporal variable t is bound by the existential quantifier temporal modification is no longer possible. Applying this function to the NP *the Marseillaise* yields $\exists t.sing[x, m, t]$.

In contrast to Poss–ing gerunds the possessive *John's* will be analysed as a determiner, i.e. as the universal quantifier restricted to the set of actions that have John as an agent.

Let us first illustrate the semantic role of determiners with a concrete example.

(43) Every singing of the aria took place at noon[28].

The intuitive idea is that determiners relate event types to event tokens via the *Happens*–predicate. Since fluents cannot occur as arguments of the *Happens*–predicate this strategy immediately explains why (44) is unacceptable.

(44) *Every singing the aria

The precise formalisation of example (43) is now as follows:

(45) $\forall x, s(Happens(\exists t.sing[x,a,t],s) \ \rightarrow \ took\ place\ at\ noon(\exists t.sing[x,a,t],s).$

If we want to interpret the determiner *every* in a strictly compositional way, we have to use λ–notation. But note that we use this notation here only as a kind of book–keeping device.

With the help of this device *every* will be represented by the term $\lambda P \lambda Q \forall x, s(Happens(P(x), s) \rightarrow Q(P(x), s))$. The compositional interpretation of the NP *Every singing of the aria* is then given by the application of *every* to $\lambda y \exists t.sing[y,a,t]$, which results in:

$$\lambda Q \forall x, s(Happens(\lambda y \exists t.sing[y,a,t](x), s) \rightarrow Q(\lambda y \exists t.sing[y,a,t](x), s))$$

This formula reduces further to:

$$\lambda Q \forall x, s(Happens(\exists t.sing[x,a,t], s) \rightarrow Q(\exists t.sing[x,a,t], s)).$$

The general scheme for determiners in Ing–of gerunds is thus:

$$\lambda P \lambda Q\ Det\ x, t(Happens(P(x), t), Q(P(x), t))$$

We will from now on skip such details and even deviate from this official notation when we think that a simpler and more transparent formalisation helps with the presentation of the topic under discussion.

In Section 6.5 we will show that sometimes determiner relate event tokens also to certain types of fluents. This will allow us to account for Vendler's observation that in the context of loose container perfect nominals tend to be interpreted as imperfect.

[28]We introduce *every* explicitly since the more naturally occurring determiners like *the* or *John's* are special cases of *every*; for instance *the* is *every* with the additional requirement that the restrictor set has cardinality one. For more detailed information about the semantics of determiners see Westerståhl (1989).

6.3. **Derived nominals.** Derived nominals such as *arrival of the train*, *destruction of the city* show a much less systematic behaviour than ing–of gerunds. This was one of the reasons why Chomsky excluded them from a syntactic analysis in his *Remarks on nominalisations* (Chomsky (1970)). Often it is quite idiosyncratic how the meaning of the nominalisation is related to the meaning of the verbs it is derived from. For example there seems to be no significant general pattern that forms the basis of nominalisations like *construction* in *the Anglo–Saxon genitive construction* and *revolution* in *the French revolution.* The relation between *construct* and *construction* and *revolve* and *revolution* in these cases clearly differs considerably[29]. But although many derived nominals are highly ambiguous some of them have the eventive reading described for Ing–of gerunds among their meanings. For example *destruction of the city* has a resultative meaning but the eventive reading as well. This aspect of the meaning of *destruction of the city* will therefore be analysed in the following way:

(46) $\exists t.destroy[x, c, t]$

Other types of nominalisations however don't have any of the readings discussed in this paper, for instance *referee* or *amusement* [30].

It is more interesting to compare our proposal with Reichenbach's observations on nominalisation presented in Paragraph 48 of his *Elements of Symbolic Logic* (Reichenbach (1947)).
Reichenbach correctly observes that the following sentences have the same truth conditions

(47) Amundsen flew to the North Pole in May 1926.

(48) A flight by Amundsen to the North Pole took place in May 1926.

Here, *flight* is the nominal derived from *fly*. Sentence (47) is an example of *thing splitting*, whereas sentence (48) is an example of *event splitting*[31]. The equivalence of these two sentences imparts a certain structure to nominalisation which he explains as follows. He uses an operation * to create a (perfect) nominal from a sentence; the nominal is an event type viewed as a set of event tokens.[32] Thus, if $\varphi(a, t)$ is a sentence as in (47), the corresponding perfect nominal would be denoted as $[\varphi(a, t)]^*$, and (48) is written as $\exists v[\varphi(a, t)]^*(v)$. Reichenbach also gives a second form of (48), which is actually very much analogous to our proposal: first construct

---

[29]See Scalise (1984) for a more thorough discussion of this topic.

[30]See Spencer (1991) for an overview of theories dealing with these kinds of nominalisation.

[31]Reichenbach uses the term *splitting* because he thinks that the predicate–subject form of a sentence splits the situation it describes into a part corresponding to the predicate and a thing–part corresponding to the subject.

[32]Reichenbach conflates *events* and *facts*; we find him writing:

> Synonymously with the word *event* we shall use the word *fact*.
> Reichenbach (1947), p. 269

$\exists t\varphi(a,t)$ from $\varphi(a,t)$, and then write (48) as $\exists v[\exists t\varphi(a,t)]^*(v,t)$. Since Reichenbach's proposal is similar to ours, we shall conduct the discussion in the remainder of this section in our own notation.

The equivalence of (47) and (48) for a formula φ can then be rendered formally as

(1) $$\varphi(x,s) \ \leftrightarrow\ \mathit{Happens}(\exists t.\varphi[x,t],s)$$

or, equivalently but more conveniently,

(2) $$\mathit{HoldsAt}(\varphi[x,\hat{t}],s) \ \leftrightarrow\ \mathit{Happens}(\exists t.\varphi[x,t],s).$$

This constraint on the *Happens* predicate has not been included explicitly so far, but can be easily incorporated. One may observe that the direction from left to right in equation 2 is of a syntactic form allowed in scenarios (cf. condition 5 in Definition 17), so that minimising the *Happens* predicate yields the desired equivalence. This can be done only for finitely many formulas φ at a time, but that suffices. Thus, we have extensional equivalence of (47) and (48).

We close our discussion of this topic with a formal remark. The reader may have wondered why we did not add an axiom of the form 2 to the axioms of the event calculus. One reason is that the proposed axiom's syntactical form is unpleasant when combined with the Feferman calculus. A more important reason is that the axiom would force us to include in a minimal model every event which can be derived from an (already present) fluent via 2. This may lead to spurious events which do not occur in the *Initiates* or *Terminates* predicates. Putting 2, when necessary at all, in the scenario, obviates this problem.

6.4. **Boolean operations.** We have seen in Section 2 that imperfect nominals can be combined by means of conjunction, disjunction and classical negation. Formally, this requires defining these operations on the terms interpreting imperfect nominals. This does not present a problem, since we have already seen in Lemma 11 that these terms, at least when L_0-definable, form a Boolean algebra. Thus, if f_1, f_2 are L_0-definable fluents, we have

Lemma 12. (1) $\mathit{HoldsAt}(f_1 \wedge f_2, t) \leftrightarrow \mathit{HoldsAt}(f_1,t) \wedge \mathit{HoldsAt}(f_2,t)$ *and similarly for* $\vee$;
(2) $\neg\mathit{HoldsAt}(f_1,t) \leftrightarrow \mathit{HoldsAt}(\neg f_1,t)$.

Some residual problems remain, however. The first of these is that conditionals apparently cannot be nominalised. Although the proposition *If the professor gives a talk, then he submits a paper* is fully correct, nominalising the proposition as in *if the professor's having given a talk, then his submitting a paper* appears to yield word-salad. Now clearly the above construction allows the definition of a conditional on fluents at least in the sense of material implication, and although the material implication is only the crudest approximation to a conditional, we appear to have a problem. And in fact, the imperfect nominal *the professor's not having given a talk or his submitting a paper* does not seem to be ungrammatical. Also, the problem does not appear to lie in the subordinating construction, because

(49) A student's not getting a degree when having submitted a thesis, is a cause for surprise.

is not ungrammatical. However, replacing *when* by *if* destroys grammaticality; the best we can do is something like

(50) If a student has submitted a thesis, his not getting a degree is a cause for surprise.

The trouble thus seems to arise when a purely hypothetical conditional is used: this is not perceived as defining a single fluent, but a relation between two fluents.

The second problem has to do with imperfect nominals which cannot be interpreted as L_0-definable fluents, because they somehow already involve a truth predicate. This can occur for essentially three reasons. The first has to do with the *coercion* of perfect nominals in the context of a wide container, as will be explained more fully in Section 6.5: *the collapse of the Germans is unlikely* means something like *that the Germans will collapse is unlikely*, so that the perfect nominal *the collapse of the Germans* is interpreted as the imperfect nominal *that the Germans will collapse*. Now any perfect nominal e gives rise to an imperfect nominal f by putting $f = Happens[e, \hat{t}]$. However, a glance at the definition of scenario (17) shows that in models of the scenario which are minimal with respect to *Initially*, *Happens*, *Initiates*, *Terminates* and *Releases*, each of these predicates is definable in using only *HoldsAt* and equations between terms. It follows that in such models, which, as we shall see when discussing the progressive, are actually the only models of interest, the fluent $Happens[e, \hat{t}]$ is extensionally equal to a fluent involving one or more occurrences of *HoldsAt*, i.e. a truth predicate. Now consider what happens when such a fluent is negated. By Feferman's axiom 3, occurrences of $\neg HoldsAt$ then have to be replaced by their positive counterparts $\overline{HoldsAt}$. In linguistic terms, this means that negation in such contexts has antonymic force. And indeed, although opinions differ as to whether Cooper's example

(51) Andrew's not stopping before the traffic light took place at noon.

is quite grammatical, no such problems seem to arise when the container is wide:

(52) Andrew's not stopping before the traffic light caused a commotion.

except that now 'not stopping' appears to have the meaning of an antonym to 'stopping', in line with the above analysis. Thus we see that we really need the full strength of Feferman's theory, and cannot content ourselves with a truth predicate that operates on L_0-formulas only, as is customary in the treatments of the event calculus current in artificial intelligence (if done formally at all).

This point is corroborated when we look at iterated nominalisations. Consider

(53) a. John supports his son's not going to church.
b. John's supporting his son's not going to church caused me much chagrin.

The implication of the use of *support* is that John's son is actually not going to church, so the semantics of *support* is roughly of the following form

$$S(x,f,t) \ \leftrightarrow \ \mathit{HoldsAt}(f,t) \ \wedge \ S^{'}(x,f,t).$$

If f is the fluent *John's son going to church*, then (53-a) is formalised by $S(j,\neg f,t)$, so that the fluent occurring in (53-b) is $S[j,\neg f,\hat{t}]$. Sticking this fluent inside *HoldsAt* then means that a negative occurrence of *HoldsAt* is in the scope of *HoldsAt*, so that a proper truth condition can only be formulated in terms of $\overline{\mathit{HoldsAt}}$.

6.5. **Coercion.** Vendler observed that in the context of wide containers, perfect nominals tend to be interpreted as being imperfect: a sentence like

(54) The collapse of the Germans is unlikely.

seems to have the interpretation

(55) That the Germans will collapse is unlikely.

This suggests that the denotation of perfect nominals given in Definition 18 is reinterpreted when they occur in the context of a wide container.

Definition 20. *Let e be an event type, then there exists a canonical fluent f associated to e defined by $f \ = \ \mathit{Happens}[e,\hat{t}]$.*[33]

Hence it appears to depend on the containers whether interpretation 18 or 20 is chosen. An example like (56) will therefore be represented as in (57).

(56) The beheading of the king surprised us.

(57) $\mathit{The}\ x,s(\mathit{Happens}(\exists t.\mathit{behead}[x,\mathit{the\ king},t],s),$
$\mathit{surprised\ us}(\mathit{Happens}[\exists t.\mathit{behead}[x,\mathit{the\ king},t],\hat{t}],s)$

We assume here an interpretation of *the* like the one in Section 6.2 but with the modification due to coercion that *the* relates event tokens to fluents derived from the *Happens*-predicate.

The obvious question that now arises is: given that we have two possibilities for forming a fluent, i.e. an imperfect nominal, from a formula φ, how are these possibilities related? This question has actually already been answered implicitly in

[33]Corollary 3 of Theorem 10 shows that these fluents have the same general characteristics as those definable in L_0.

Section 6.3, when discussing Reichenbach's analysis of derived nominals. Equations 1 and 2 both ensure that the fluents $\varphi[x,\hat{s}]$ and $Happens[\exists t.\varphi[x,t],\hat{s}]$ are equal, at least extensionally. To enforce strict identity, the circumscription policy would have to be changed. This can be done by dropping the relevant uniqueness–of–names assumptions, and putting fluent–constants among the varied constants, as in example 3 of Section 4.4. Lemma 5 then shows that also in this case, circumscription reduces to predicate completion.

This type of coercion is also important for the intensionality that some containers enforce. Compare sentences (58) and (59)

(58) The beheading of the tallest spy occurred at noon.

(59) Mary predicted the beheading of the tallest spy.

Even when *the king* = *the tallest spy*, (59) does not imply

(60) Mary predicted the beheading of the king.

whereas we of course do have

(61) The beheading of the king occurred at noon.

This can now be explained, since the formal translations run as follows:

(62) $predict(Mary, The\ x(Happens[\exists t.behead[x, the\ king, t], \hat{t}], s) \wedge\ s <\ now$[34]

Replacing *king* by *the tallest spy* would result in the fluent
$Happens[\exists t.behead[x, the\ tallest\ spy, t], \hat{t}]$ which is different from
$Happens[\exists t.behead[x, the\ king, t], \hat{t}]$, hence (59) and (60) are not equivalent.

The formalization of example (58) is (63).

(63) $The\ x, t(Happens(\exists t.behead[x, the\ tallest\ spy, t], t),$
$occurred\ at\ noon(\exists t.behead[x, the\ tallest\ spy, t], t)$

[34]The reader may wonder why we choose to interpret *the* as a unary quantifier in (62). The reason is that this is enough to explain our point concerning intensionality versus extensionality. To insist on a completely uniform representation of *the* as a binary quantifier would have enforced the following rather messy formula:
$predict(Mary, \lambda B\ The\ x, r(Happens(\exists t.behead[x, the\ king, t], r),$
$B(Happens[\exists t.behead[x, the\ king, t], \hat{t}], r), s) \wedge\ s <\ now$, where B is a variable for a relation between fluents and times.

The crucial difference is that *occurred at noon* like $Happens$ is a relation between event types and times.

The extensionality inherent in example (58) can then be enforced by the following axiom:

Axiom 8. *Let B be a relation between event types and times and let ϕ and ψ be logically equivalent formulas, then*

$$B(\exists t_1, \dots, t_n.\phi[\overline{x}], t) \leftrightarrow B(\exists t_1, \dots, t_n.\psi[\overline{x}], t)$$

Empirical evidence suggests that narrow containers can always be expressed as a Boolean combination of the *Happens*-predicate. On the basis of such a *semantic universal* it is possible to state Axiom 8 in a more restricted form.

Axiom 9. *Let ϕ and ψ be logically equivalent formulas, then*

$$Happens(\exists t_1, \dots, t_n.\phi[\overline{x}], t) \leftrightarrow Happens(\exists t_1, \dots, t_n.\psi[\overline{x}], t)$$

6.6. **Lattice structure of event types, and the role of negation.** By Definition 18 of event types as terms of the form $\exists t_1 \dots t_n.\varphi(t_1 \dots t_n, \overline{x})[\overline{x}]$, closure under $\vee$ and $\wedge$ is immediate. Since *Happens* is not a truth predicate, we have to augment the event calculus with the

Axiom 10. *$Happens(e \wedge e', t) \ \leftrightarrow \ Happens(e, t) \wedge Happens(e', t)$, and similarly for $\vee$.*

We then have to be a bit careful in minimising a scenario with respect to *Happens*; the result will not just be predicate completion, but *Happens* still has to be closed off under $\wedge$ and $\vee$. In any case it is clear that the lattice structure of perfect nominals is mirrored in that of the event types.

This lattice structure is of interest in view of the observation of Bach (1986*a*) and others (e.g. Link (1987), Krifka (1989), Lasersohn (1995), Eckardt (1998)), that there exists a close parallel between the pair mass/count nouns on the one hand, and the pair processes/events on the other. Bach puts this in the form of the following equation:

events:processes :: things:stuff

Now just as there exists a mapping which associates to things the stuff they are made of, there should exist a mapping which associates to an event type a process, so that, e.g., a running event is mapped onto the 'stuff' it consists of, namely the activity running. This mapping should commute with conjunction and should respect temporal relationships such as 'overlaps'. A remark is in order here: we require commutation with conjunction whereas Bach and Link require commutation with *disjunction*. This is because we have a different view of plural events: whereas Link (*op. cit.*, p. 247) considers *John and Bill hit each other* to consist of the *sum* of the events *John hit Bill* and *Bill hit John*, we believe it might as well be described as a conjunction. Now clearly our set up yields such a mapping for free, namely the mapping $e \ \mapsto \ Happens[e, \hat{t}]$. This homomorphism satisfies Bach's main desiderata: it is many-to-one (thus explaining why processes do not correspond to unique event types), and event types may be mapped onto fluents which are equivalent as functions of time, even though the event types themselves are different. Bach's examples are

(64) Jones poison the populace.

(65) Jones pour poison into the water main.

in the situation where Jones intentionally pours poison in the water main (to get rid of bedfish) without having the intention to poison the populace.

Krifka (1989) uses the equation

events:processes :: things:stuff

as the starting point of an investigation into the relation between the mass:count distinction in the nominal domain and the atelic:telic distinction in the verbal domain. A striking phenomenon here is that the object argument of a verb seems to determine its temporal constitution; compare

(66) a. Mary drank beer (for ten minutes)/(*in ten minutes).
b. Mary drank a glass of beer (*for ten minutes)/(in ten minutes).

In this example, use of the mass noun *beer* forces an atelic interpretation on *drank*, whereas *glass of beer* enforces a telic interpretation.

These phenomena call for a different kind of mapping, from objects to fluents, where the sort of objects is now equipped with a lattice structure as in Link (1987). We shall discuss this mapping in detail in Hamm & van Lambalgen (2002).

It appears that the negation[35] of an event type can only marginally be an event type itself, as (perhaps) in Cooper's example (51). This observation has been made several times in connection with perception verb complements. Higginbotham's example (Higginbotham (1983)) is

(67) John saw Mary not smoke.

Insofar as this sentence is grammatical, the *not* seems to turn *smoke* into an antonym. This situation can also occur in our context. In general, there seems to exist some evidence that negation turns an event type into a stative predicate (cf. Verkuyl (1993)); again, this is analogous to a construction introduced below, which equates the negation of an event type with a certain fluent.

[35] An extensive discussion of negation related to nominalisation is contained in Asher (1993).

We shall first show that there are formal obstacles to introducing a negation-like operation on event types, and then proceed to give an interpretation of negation which transforms event types into fluents (but not conversely!).

Suppose then that $\sim$ is a negation on event types. Perhaps $\sim$ can be identified with a function whose domain and range are the set of formulas of the form $\exists t.\varphi$; but we do not assume this, and $\sim$ may create genuinely new event types. However that may be, $\sim$ should at least satisfy

$$\forall e(\mathit{Happens}(\sim e,t) \ \rightarrow \ \neg \mathit{Happens}(e,t)). \tag{3}$$

This makes $\sim$ into an operation which yields an antonym of an event type, but not yet necessarily a classical negation. For this we need

$$\forall e(\neg \mathit{Happens}(e,s) \ \rightarrow \ \mathit{Happens}(\sim e,s)). \tag{4}$$

The latter statement is highly problematic, since it will be seen to lead to an unacceptable notion of minimal model[36]. The problem is not in the syntactic form of (4); since *HoldsAt* is a truth predicate, (4) can be reformulated as

$$\mathit{HoldsAt}(f,t) \ \rightarrow \ \mathit{Happens}(\sim e,t), \tag{5}$$

where f is the nominalisation of $\neg \mathit{Happens}(e,t)$. Hence, (4) is of the form allowed in scenarios.

It will be seen below that a proper semantics for the progressive vitally needs finite scenarios to which circumscription (with respect to *Happens* etc.) is applied. Let SCEN be such a scenario, which thus mentions only finitely many events and fluents. First consider a model $\mathcal{N}$ of EC+CIRC[SCEN; *Happens*]; it is easy to see that for an event e which is not mentioned in SCEN, $\mathcal{N}$ will satisfy $\forall t \neg \mathit{Happens}(e,t)$. This is in accord with the intuitive way around the frame problem: we may assume that irrelevant events don't occur. Next, consider a model $\mathcal{M}$ for EC+(4)+SCEN, and let e be an event not mentioned in SCEN; suppose furthermore that for some nontrivial set S, we have in $\mathcal{M}$ $\mathit{Happens}(e,t) \leftrightarrow t \in S$. It follows by (4) that, in $\mathcal{M}$, $\mathit{Happens}(\sim e,t) \leftrightarrow t \in \mathbb{R} \setminus S$. If we now move to a model $\mathcal{N} \leq^{\mathit{Happens}} \mathcal{M}$ of EC+(4)+CIRC[SCEN;*Happens*], then it is no longer possible to take away t such that $\mathit{Happens}(e,t)$, because that would increase the set of t for which $\neg \mathit{Happens}(\sim e,t)$. Thus, in the presence of (4) the concept of minimal model changes (too) drastically. A corollary of the above argument is that there is no longer a unique minimal model for EC+SCEN; there are now uncountably many, all differing in their interpretations of irrelevant events. This raises the spectre of the frame problem again.

The conclusion of this argument is, that (4) is not acceptable in our context; but without it $\sim e$ will in general be an antonym of e rather than a true classical negation of e.

[36]This is very clear in the case of minimisation as given by the completion of a logic program, for which see van Lambalgen & Hamm (2001). Readers familiar with both circumscription and logic programming will see how the intuitive considerations given here can be made precise in the latter set up.

The preceding discussion suggests that there *does* exist a uniform negation on event types, which however transforms *event types* into *fluents*

Definition 21. *The* fluent negation $\approx e$ *of an event type* e *is defined by* $\approx e := \neg Happens[e, \hat{t}]$.

As we have seen above, $\approx$ may play a role when negations of event types occur in a wide container, such as in example (52).

The upshot of all this is that there will be no uniform treatment of negation for event types. With this in mind, let us analyse Cresswell's example and some of its relatives:

(68)
a. The non–arrival of the train caused consternation.
b. *The non–arrival of the train unexpectedly ...
c. The unexpected non–arrival of the train caused consternation.
d. The fact that the train did not arrive caused consternation.
e. The train's not arriving caused consternation.
f. The train's not arriving quickly/?unexpectedly caused consternation.
g. *The non–arrival of the train occurred at noon.
h. Every non-arrival of a train causes consternation.

Recall that the first argument of $cause(x, y, t)$ is a wide container, whereas the second is narrow; *consternation* is therefore an event type, but *the/every non-arrival of the train* could be both an event type and a fluent. If Vendler's observation is correct that wide containers coerce their argument to be a fluent (an observation endorsed by Cresswell, who believes (68-d) and (68-e) are good paraphrases of (68-a)), then we should favour an imperfect reading of *the/every non-arrival of the train*; and indeed, as (68-g) shows, a perfect reading seems to be out. Even so, the offending phrase appears to have the internal structure of a perfect nominal.

Formally, then, the obvious interpretation of *non-arrival* as the event type $\exists t.\neg arrive[x, t]$ appears to be not allowed. This is a pity because it would allow us to treat the determiners in (68-a) and (68-h) along the following lines:

(68-h) $\forall x, s(Happens(\exists t.\neg arrive[x, t], s) \rightarrow cause(\exists t.\neg arrive[x, t], c, s)$.

The next option is to take the fluent-negation, which would yield $\neg Happens[\exists t.arrive[x, t], \hat{s}]$. Then the sentence (68-h) can be formalised as

(6) $\forall x, s(\neg Happens(\exists t.arrive[x, t], s) \rightarrow cause(\neg Happens[\exists t.arrive[x, t], \hat{s}], c, s))$.

In general such quantification over fluents is not allowed, although there are such marginal examples as 'the completing a tract', but the presence of the *Happens* predicate in the restrictor of the determiner seems to make the difference between this particular fluent and fluents generally. The reason for this could be that by making the restrictor slightly smaller, and using the property 3, i.e.

$$Happens(\sim e, t) \rightarrow \neg Happens(e, t),$$

Formula 6 can be brought into the canonical form for the application of determiners, where the *Happens* predicate occurs positively in the antecedent. We thus believe that Cresswell's examples show the confusing pattern they do because various notions of negation play a role simultaneously.

7. *Aktionsart* AND THE PROGRESSIVE

The purpose of this section is to show that the event calculus augmented with circumscription can be elegantly put to use in providing a formal representation of the progressive, avoiding the imperfective paradox. As a preparation, we briefly discuss the definition of the various *Aktionsarten* in this framework. One interesting feature of the event calculus is that it does not make a choice between an instant-based and an interval-based representation of time, in the following sense. One primitive concept is that of a fluent, which is implicitly a function of time points. As will be proved in Theorem 10, fluents can generally be represented as the union of a finite set of halfopen intervals and a finite set of points. If moreover the fluent corresponds to an activity, the set of points is empty. Thus, we do not need an a priori discussion on the merits of interval semantics vis à vis point based semantics; the interpretation is dictated by the mathematical properties of the set up.

The section ends with a discussion of some recent work on the progressive, including Bonomi's 'multiple choice paradox' (Bonomi (1997)). Our conclusion will be that this is not a paradox; but it will be seen that the expressive power of the present proposal is rather helpful in analysing what is at stake.

7.1. *Aktionsart.*

Before we embark on a formal analysis of the progressive and the accompanying 'imperfective paradox', we show how Vendler's famous classification of verbs into states, achievements, activities and accomplishments, can be given a theoretical underpinning in the present framework. Needless to say, we can treat *Aktionsart* only insofar as it pertains to inherent properties of a verb; we do not consider the contribution of the discourse context. The strategy we shall follow is largely that of distinguishing *Aktionsarten* by means of the functional role the corresponding fluents play in the event calculus. In general[37], a verb is represented by a structure of the form (f_1, f_2, e, f_3), where f_1 is a fluent which corresponds to an activity, f_2 is a fluent which corresponds to a partial object which changes under the influence of f_1 (thus f_2 will in general contain variables), e is the culminating event of the activity, and f_3 is the resulting consequent state[38]. The fluents in this structure play different roles in a scenario. f_1 is not allowed to occur in *Releases* and in the third argument of *Trajectory*, but may occur in the latter's first argument. For f_2 it is the other way around; and since f_3 will usually be an instantiation of f_2 it inherits the latter's syntactic restrictions. The structure furthermore satisfies the property that *Initiates*(e, f_3, t) and *Terminates*(e, f_1, t). The fluents f_1 and f_2 have a complicated relationship which will be discussed in the section on accomplishments.

[37] A much more elaborate treatment of this topic can be found in Hamm & van Lambalgen (2002).

[38] This is of course strongly related to the 'event nucleus' of Steedman (1997).

Different *Aktionsarten* emphasise different parts of this structure, as will be seen in the following sections.

7.1.1. *States.* Examples are *know*, *be beautiful* and *love*. The distinguishing feature of states seems to be that there does not necessarily exist a natural culmination point. A consequence is that states exhibit relatively unconstrained behaviour with respect to time; in fact one may think of states as an arbitrary combination of a timeless predicate with the *At* operator. Bach remarks that "[p]erhaps it is only states that can be profitably thought of as properties of moments –that is, instants of time" (Bach (1986*b*)). This is true in the sense that in principle it can be decided for each instant independently whether it belongs to the property representing the state; no such independence can be expected for activities. It is often claimed that states involve the unchanged continuation of some condition, or better, involve *viewing* some condition as continuing unchanged. This may be true, but some constructions involving states seem to emphasise change. One can say

(69) She is more beautiful now than she was in her youth.

or

(70) I've come to hate my job.

Therefore, as representations of states we will also allow fluents which contain a variable for a real, indicating a degree. In the event calculus, fluent(functions) representing states may occur as the third argument of the *Trajectory* predicate, and as argument of the *Releases* predicate. Thus, the event of *falling-in-love* at t *Releases* the fluent-function *be-beautiful*(x) at t, which then starts its *Trajectory* as long as the fluent *being-loved* exerts its beneficent influence.

It follows from these stipulations that the time profile of a fluent corresponding to a state, can be fairly arbitrary (but not too much, if the scenario is finite; see Theorem 10). For example, for a particular x, *be-beautiful*(x) may hold at a single instant only. Thus one cannot expect the construction of fluents in Proposition 1 to be applicable generally.

7.1.2. *Activities.* Here, examples are *run*, *push a cart* and *seek*. One difference between states and activities is that even when the latter is conceived of as a set of instants, determining whether a particular instant belongs to this set cannot proceed independently of what we know about other instants. A further difference between states and activities is that, while the latter again do not have explicit natural culmination points, a culmination point is at least implied. As Kamp and Reyle (Kamp & Reyle (1993)) put it, an activity verb is incomplete, and demands that a culmination point is given by the context or by a constituent of the sentence in which it occurs. For us, this means that the scenario characterising the fluent representing an activity, should contain statements characterising (initiation and) termination of that activity. More importantly, we must also forbid that such an activity-fluent occurs as an argument of the *Releases* predicate, since such an occurrence may override the effect of the termination condition (see axiom 6). It will turn out that the previous observations are all related. Thus, supposing we have a predicate *Act* which isolates the fluents corresponding to activities, we may add the following axiom to the event calculus

Axiom 11. $Act(f) \leftrightarrow \forall e \forall t \neg Releases(e, f, t)$.

In a consistent scenario, activity-fluents will then not occur as third argument of the *Trajectory* predicate. An interesting consequence of axiom 11 is that the time profile of activity-fluents conforms to that exhibited in the standard model: a finite set of half-open intervals. That seems to be as it should: roughly speaking for *John runs* to be true at t there should be some interval around t where the sentence is true.[39]

The proof of the next theorem is somewhat involved; for a proof see van Lambalgen & Hamm (2001). The proof of Theorem 11 below will illustrate some of the essential steps in a concrete case. We believe that both Theorem 10 and Theorem 11 hold some potential as prerequisites for the use of the event calculus in natural language processing.

Theorem 10. *Suppose we are given a finite scenario* SCEN *and a finite dynamics* DYN*. In minimal models of* SCEN+DYN+*EC, fluents satisfying Act are composed of finitely many halfopen intervals. All other fluents are composed of finitely many intervals together with finitely many points.*

Corollary 1. *Under the same assumptions as above: for activity fluents f and for all t (except the right endpoints), $HoldsAt(f,t)$ if and only if $\exists s, r\ (s < t < r \wedge \forall t' \in (r,s) HoldsAt(f, t'))$.*

Corollary 2. *Under the same assumptions as above: activity fluents are initiated and terminated solely by events mentioned in the scenario.*

Corollary 3. *Under the same assumptions as above: $\{t \mid Happens(e,t)\}$ is a union of finitely many intervals and finitely many points. Thus, event tokens may take place at both points and intervals.*

A linguistic test which distinguishes activities from the achievements and accomplishments to be discussed next is their behaviour with respect to the adverbial modifiers *for an hour* and *in an hour* (more generally, *in y time* etc.). One says 'he ran for an hour' but 'he reached the summit in an hour'. Moreover, 'he ran for an hour' allows brief interruptions such as waiting for a traffic light. In this context, *for an hour* is a predicate of (activity-)fluents; since such fluents are composed of halfopen intervals, the possibility of interruptions is automatically guaranteed, and

[39] In our opinion, this does not mean that an activity cannot be evaluated at a time point, as has sometimes been maintained. Thus, the following dialogue seems to make perfect sense:

> "I came in to see John at 2:00 pm, but he wasn't there." "Hmm, let me think
> ... at 2:00 pm, John was running in the park."

Hence there is no objection to using the *HoldsAt* predicate for activity fluents. The problem is rather that an activity cannot be an arbitrary set of instants.

the applicability of the *for an hour*-predicate depends both on the lengths of the intervals and those of the interruptions.

Before we move on to the achievements, we will try to answer a query that is probably on the reader's mind: 'Isn't the traditional view that states do not *require* change, whereas activities do? How is that reflected here?' First of all, as Corollary 2 shows, activities are always initiated and terminated by events, i.e. explicit changes; states may 'switch' on or off without any such explicit change. Secondly, to fluents representing an activity one may associate another fluent representing the state that changes as the result of that activity. For example, associated to *running* will be *distance-traversed*(x), where x is a numerical parameter. Activities thus in general use the first two components of the structure outlined above. This is in sharp contrast to the achievements to be discussed.

7.1.3. *Achievements.* Examples are *begin*, *reach* and *recognise.* Achievements seem strongly connected to a change of state: if we begin playing soccer at t, we didn't play immediately before t, and if we recognise someone's face at s, the face was not yet recognised before s (cf. Dowty (1979), p. 76*ff*). This suggests an analysis analogous to what we did for activities, except that here the achievement is represented by the changing state; furthermore, this change is typically instantaneous. Thus, unlike activities (and accomplishments), achievements do *not* refer to the first two components of the general verb structure; they refer rather to the last two components.

Let us take *reach the top* as an example. Reaching the top may be a result of the activity *climbing*, just as *height-gained* is. Formally, such a result can be represented as the third argument in the *Trajectory*–predicate; if *climbing* $= f$, and *reach the top* $= h$, then the relation between these fluents can be formalised by a statement such as

$$\mathit{HoldsAt}(f,t) \wedge \mathit{HoldsAt}(h,t) \rightarrow \mathit{Trajectory}(f,t,h(t+d),d).$$

Intuitively, if I am *climbing* at t, and I haven't *reached the top* yet (i.e. $h(t)=0$), then if all goes well, for some $d>0$, I will have *reached the top* at time $t{+}d$ (i.e. $h(t{+}d) = 1$). Formally, the value of h at any time $t+d$ can then be determined from axiom 7, provided that the effect of the law of inertia embodied in axiom 5 is cancelled by means of the *Releases* or *Terminates* predicates. Thus, the scenario must incorporate an event e ('setting foot on the top'), which releases (or terminates) $h(t)=0$, and initiates $h(t)=1$.

When the temporal adverbial *in an hour* makes sense, as in *he reached the summit in an hour*, it can be formalised thus. Let *Happens*(e,t) and let e' be an event type initiating $\neg f$ such that if *Happens*(e',s), where $s<t$ is maximal (such an s exists in minimal models of scenarios, because there $\{t \mid \mathit{Happens}(e,t)\}$ is a union of a finite set of intervals $(a,b]$ together with finitely many points). Then $t-s$ should be less than one hour.

7.1.4. *Accomplishments.* Examples are *draw a circle*, *write a letter* or *cross the street.* Recall that Dowty (1979) analysed accomplishments such as *Mary draws a* *circle* by decomposing them into two parts, the first part concerned with an activity (*draw*), the second with the result of that activity. The two parts are connected by a causal relationship, so that we obtain

(71) $CAUSE[\textit{Mary draws something}, \textit{a circle comes into existence}]$.

It is clear that the second clause describes a process, so that by the result of the activity *draw* we do not mean the finished circle, but the various stages of its construction. It is of course no simple matter to come up with a *syntactic* analysis which somehow produces *Mary draws a circle* from (71). Nevertheless, we believe this type of semantic analysis makes good sense, but needs to be supplemented by a theory of *CAUSE* to be really informative. The event calculus furnishes all ingredients necessary to formalise (71): *draw something* is a fluent which depends only on time (and the tacitly understood subject, Mary), *a circle comes into existence* is a fluent which depends on time and, say, a parameter for the length of the circumference already drawn (supposing the radius to be known), and *CAUSE* is represented by the *Trajectory*–predicate.

The first type of fluent will be referred to as the cause–fluent, the second as the result–fluent. In principle, the verb *draw* is a ternary predicate, so its corresponding cause–fluent would be a function of time depending on two parameters, but in this context the object variable is quantified existentially. Formally, then, there is a cause–fluent f_1, a result–fluent $f_2(x)$, and a function g such that

(72) $$HoldsAt(f_1, now) \ \wedge\ \forall t(HoldsAt(f_2(g(t)), t) \ \rightarrow \\ \forall d > 0\, Trajectory(f_1, t, f_2(g(t + d)), d)).$$

Here, f_1 denotes an activity, so that its time profile can be visualised as, in general, a set of halfopen intervals. However, if the cause f_1 exerts its influence uninterruptedly from t until $t + d$, the state of the result–fluent will be $f_2(g(t + d))$.

In general, g will be continuous, as in the case of drawing a circle, but sometimes one wants to allow jumps, for instance to treat Landman's example (Landman (1992))

(73) God was creating a unicorn, when he changed his mind.

The scenario is that God, after much preparatory work, was just about ready to create a unicorn in one stroke (no partial unicorns here), when he changed his mind; in this scenario sentence (73) is true. The problem posed by the sentence is, how to interpret the quantifier 'a unicorn', since it clearly cannot quantify over unicorns in the real world. This seems to make (73) analogous to 'Mary tried to find a unicorn', so that the progressive brings with it an intensional context. However, the intensionality uncovered here is not essentially different from the one in example (71), the only difference being the nature of the function g.

Another way of explaining the Formula (72) is by way of referring to Dowty's notion of an 'incremental theme' (cf. Dowty (1991), p. 567). Dowty defines a telic predicate as a homomorphism (with respect to 'part-of' relations) from its structured theme argument into a structured domain of events (modulo its other arguments). But Formula (72) generates such a homomorphism: since f_1 is an activity, it consists of a finite number of intervals, and by the meaning of the *Trajectory*-predicate each such interval corresponds to an interval on which the result-fluent changes.

The analysis just presented shows that accomplishments need the full four–part structure associated to verbs. It is now relatively straightforward to compose scenarios corresponding to each of the *Aktionsarten*; we only have to take care that the relevant components of the four–part structure are correlated with each other. We shall further comment on the analyses of accomplishments found in the literature after stating and proving the main result of this section.

7.2. **The progressive and the imperfective paradox.** We begin this section with an analysis of the progressive as applied to activities.

Consider

(74) John was pushing a cart.

(74) strictly entails that *John pushed a cart*, so our semantics must reproduce this inference. We conceive of the progressive as an operator which transforms a sentence of natural language into a formula of the event calculus involving *HoldsAt*. In more detail, if *now* is a constant for the time of utterance and f the fluent which results from nominalising *John pushes a cart*, then the formalisation of *John is pushing a cart* becomes

(75) $\mathit{HoldsAt}(f, \text{now})$

and that of (74) becomes

(76) $\exists t(HoldsAt(f,t) \wedge t < now)$.

Since *push a cart* is an activity, the fluent f satisfies axiom 11. Intuitively, (74) says that there is some interval earlier than *now* during which John pushed a cart continuously. This intuition can be reproduced formally by using the fact that *HoldsAt* is a truth predicate. Thus *John pushed a cart* follows from (76), as desired.

The treatment of accomplishments such as

(77) John is crossing the street.

is considerably more complicated, and involves the machinery developed by Shanahan to model continuous change (cf. Shanahan (1990), Shanahan (1997)), which was briefly alluded to in Section 4.2.1. For one thing, accomplishments allow the interesting subtlety that the direct object need not exist. That is, although 'the street' in example (77) must exist, this need not be true of 'a circle' in the sentence *Mary was drawing a circle*. We therefore give an analysis of example (77) which also extends to the general case.

Suppose we want to show that a bucket into which water flows continuously will ultimately overflow. This can be formalised by assuming a fluent *filling*, a fluent function *height*(x), a fluent *spilling* and events *overflow*, *tap–on*, *tap–off* which are connected by axioms such as the following (this list is not exhaustive).

(1) $\mathit{Initially}(\mathit{height}(0))$
(2) $\mathit{Initiates}(\mathit{tap\text{–}on}, \mathit{filling}, t)$
(3) $\mathit{Initiates}(\mathit{overflow}, \mathit{spilling}, t)$
(4) $\mathit{Releases}(\mathit{tap\text{–}on}, \mathit{height}(x), t)$
(5) $\mathit{Terminates}(\mathit{overflow}, \mathit{filling}, t)$
(6) $\mathit{HoldsAt}(\mathit{height}(x), t) \rightarrow \mathit{Trajectory}(\mathit{filling}, t, \mathit{height}(x+d), d)$
(7) $\mathit{HoldsAt}(\mathit{height}(10), t) \wedge \mathit{HoldsAt}(\mathit{filling}, t) \rightarrow \mathit{Happens}(\mathit{overflow}, t)$.

The desired result, overflow at $t = 10$, is then derived by applying circumscription to this scenario.

In the description of this scenario, an important role is played by the *Trajectory*–predicate, which allows one to state that *filling* causes increase of *height*. It is precisely this possibility to formulate causal relationships that makes the event calculus relevant for a semantics of accomplishments.

In the case of example (77) we shall proceed by analogy with the above, that is, we identify a cause–fluent (*crossing*(x)) and a result–fluent (the *distance* traversed), as already indicated in our discussion of accomplishments above. However, there is a subtlety here which bears emphasising. In the case of 'Mary is drawing a circle', 'a circle' is not treated by means of existential quantification over circles, to avoid the inference from 'Mary is drawing a circle' to 'a circle exists'. Thus, the fluent generated from the verb 'draw' does not have a free variable representing the object; we may think of this variable as being existentially quantified. In the case of 'cross the street' such scruples are unnecessary; in fact in this case one would like to draw the inference that the street exists. The fluent constructed from 'cross' should therefore have a free variable for which 'street' can be substituted, thus allowing existential generalisation. This being said, in order not to encumber the notation, we continue to write *crossing* for this fluent, it being understood that this is a fluent–function depending on two parameters in which the objects 'John' and 'the street' have been substituted for the two free variables.

Thus, *John is crossing the street* is formalised as

$$\begin{aligned} &HoldsAt(crossing, now) \;\wedge\; (HoldsAt(distance(x), t) \;\rightarrow \\ &\qquad \forall d > 0\, Trajectory(crossing, t, distance(x+d), d) \end{aligned}$$

and the entire street-crossing scenario is described by the following conditions:

(1) $HoldsAt(crossing, now)$
(2) $t_0 < now$
(3) $\neg HoldsAt(other\text{–}side,\ now)$
(4) $Initially(one\text{–}side)$
(5) $Initially(distance(0))$
(6) $Happens(start,\ t_0)$
(7) $HoldsAt(distance(m),\ t) \wedge HoldsAt(crossing,\ t) \rightarrow Happens(reach,\ t)$
(8) $Initiates(start,\ crossing,\ t)$
(9) $Releases(start,\ distance(x),\ t)$
(10) $Initiates(reach,\ other\text{–}side,\ t)$
(11) $Terminates(reach,\ crossing,\ t)$
(12) $HoldsAt(distance(x), t) \rightarrow Initiates(reach,\ distance(x),\ t)$
(13) $HoldsAt(distance(x), t) \rightarrow Trajectory(crossing\ ,t,\ distance(x + d), d)$
(14) $HoldsAt(distance(x_1), t) \wedge HoldsAt(distance(x_2), t) \rightarrow x_1 = x_2$
(15) UNA: uniqueness-of-names assumptions.

A few comments are in order. The constant m denotes the width of the street, and t_0 is the time point at which John commences his crossing of the street, supposed to be before *now*. Also for simplicity we have assumed uniform velocity in 13; nothing hinges on this, of course[40].

Put Δ = 4–7, Σ = 8–12; let Γ be the conjunction of the remaining statements with the axioms of the event calculus. We then have that, barring unforeseen circumstances, John will safely reach the other side of the street:

Theorem 11. *For $s \geq t_0$:*

$CIRC[\Delta; Happens, Initially] \ \wedge\ CIRC[\Sigma; Initiates, Terminates, Releases] \ \wedge\ \Gamma\ \models$ $HoldsAt(other\text{-}side,\ s + m)$.

Proof We show that the premises determine the model as being of the following form:

(1) *crossing* holds in the interval $(t_0, t_0 + m]$, and is false outside this interval.
(2) $distance(0)$ holds on $[0, t_0]$, $distance(x)$ holds at $t_0 + x$, for $x \leq m$, and $distance(m)$ holds after $t_0 + m$.
(3) *start* happens at t_0, *reach* at $t_0 + m$.
(4) *one–side* holds before (and including) t_0, and is false thereafter.
(5) *other–side* holds at and after $t_0 + m$, and is false at other times.

The last line is the desired conclusion. We may observe that this model satisfies $\Delta \cup \Sigma \cup \Gamma$, as can be easily verified. Thus, by Theorem 5, the premiss

$$CIRC[\Delta; Happens, Initially] \ \wedge\ CIRC[\Sigma; Initiates, Terminates, Releases] \ \wedge\ \Gamma$$

[40]But see Section 7.3. The uniform motion assumed in 13 is reasonable for the situation described, where only a short distance has to be traversed. To describe the proper dynamics of the sentence ‘Rebecca is swimming across the Atlantic’ one needs a more complicated function relating time and position.

is consistent.

Due to the simple form of Δ and Σ, the relevant circumscriptions can be easily computed. $CIRC[\Delta; \mathit{Happens}, \mathit{Initially}]$ yields

$$\mathit{Initially}(f) \leftrightarrow f = \mathit{one{-}side} \vee f = \mathit{distance}(0) \tag{7}$$

$$\begin{gathered} \mathit{Happens}(a,t) \leftrightarrow (a = \mathit{start} \wedge t = t_0) \vee \\ (a = \mathit{reach} \wedge \mathit{HoldsAt}(\mathit{crossing}, t) \wedge \mathit{HoldsAt}(\mathit{distance}(m), t)) \end{gathered} \tag{8}$$

Using parallel circumscription we may decompose
$CIRC[\Sigma; \mathit{Initiates}, \mathit{Terminates}, \mathit{Releases}]$ into $CIRC[\Sigma; \mathit{Releases}] \wedge$
$CIRC[\Sigma; \mathit{Terminates}] \wedge$ $CIRC[\Sigma; \mathit{Initiates}]$ so we get

$$\mathit{Releases}(a,f,t) \leftrightarrow a = \mathit{start} \wedge f = \mathit{distance}(x) \tag{9}$$

$$\mathit{Terminates}(a,f,t) \leftrightarrow a = \mathit{reach} \wedge f = \mathit{crossing} \tag{10}$$

$$\begin{gathered} \mathit{Initiates}(a,f,t) \leftrightarrow (a = \mathit{start} \wedge f = \mathit{crossing}) \vee \\ (a = \mathit{reach} \wedge f = \mathit{other{-}side} \vee \\ (a = \mathit{reach} \wedge f = \mathit{distance}(x) \wedge \mathit{HoldsAt}(\mathit{distance}(x), t)) \end{gathered} \tag{11}$$

Accordingly, a model $\mathcal{M}$ of the premiss has the above interpretations of the distinguished predicates; we have to compute the interpretation of *HoldsAt* on $\mathcal{M}$, both for *crossing* and for *distance*(x).

Thus, in order to determine for which t one has *HoldsAt*(*crossing*,t), it suffices (by axiom 5), to find e, y, t such that

$$\mathit{Happens}(e,y) \wedge \mathit{Initiates}(e, \mathit{crossing}, y) \wedge y < t \wedge \neg \mathit{Clipped}(y, \mathit{crossing}, t).$$

¿From the circumscriptions 8 and 11 it is immediate that $e = \mathit{start}$ and $y = t_0$. It thus remains to compute an upper bound on t from the clause $\neg \mathit{Clipped}(t_0, \mathit{crossing}, t)$.

Suppose then, $\mathit{Clipped}(t_0, \mathit{crossing}, t)$. This means that for some e, s: $t_0 < s < t \wedge \mathit{Happens}(e,s) \wedge \mathit{Terminates}(e, \mathit{crossing}, s)$. We claim that the pair e, s is unique. From 10 it follows that $e = \mathit{reach}$. Furthermore, we have that $\neg \mathit{Declipped}(t_0, \mathit{crossing}, t)$, since the only action initiating *crossing* can be *start*, which however only happens at time t_0. By axiom 6, it follows that $\neg \mathit{HoldsAt}(\mathit{crossing}, t^{'})$, for $s < t^{'} \leq t$. However, if for some $r > s$, *Happens*(*reach*,r), then by circumscription property 8 *HoldsAt*(*crossing*,r), a contradiction. Thus s is unique, and we have that $\neg Clipped(t_0, crossing, s)$. By axiom 7, it follows that $HoldsAt(distance(s - t_0), s)$. By circumscription property 8, we must have that $HoldsAt(distance(m), s)$, so that by the definition of the dynamics, 14, $m = s - t_0$. It follows that *crossing* is not clipped before $t = t_0 + m$, hence $Clipped(t_0, crossing, t)$ implies $t \in (t_0 + m, \infty)$.

We are now in a position to compute the time profile of *crossing*. By axiom 7, $HoldsAt(distance(m), t_0 + m)$. By condition 7 of the scenario, $Happens(reach, t_0 + m)$, and thus $Terminates(reach, crossing, t_0 + m)$. Since *crossing* is not resumed thereafter, $HoldsAt(crossing, t)$ is true only for $t \in (t_0, t_0 + m]$.

The time profile of $distance(x)$ is then easily computed from the dynamics, 13, which yields for $t \leq t_0 + m$, $HoldsAt(distance(x), t)$ iff $x = t - t_0$, together with condition 12 of the scenario, which says that for $t \geq t_0 + m$, $HoldsAt(distance(m), t)$.

All in all we have that at time $t_0 + m$, the fluent *other–side* is initiated, and since there is no event which can terminate it, for $s \geq t_0 + m$ we have that $HoldsAt(other\text{–}side, s)$, as desired. □

Before moving on to the 'imperfective paradox', we add a few remarks on the proof just given. First, note the role of the *Releases* predicate in this proof: if we wouldn't have $Releases(start, distance(0), t_0)$, then axioms 4 and 7 together with the scenario would yield a contradiction. Furthermore, it is interesting to see the role that the various conditions of the scenario play in providing a definite interpretation for the fluents. For example, the condition 12 was added to make the fluent $distance(x)$ definite for all values of x and t, in the sense that all minimal models give the same interpretation to $distance(x)$. It would be equally natural to say nothing about $distance(x)$ after *other–side* has been reached, in which case minimal models would differ in their interpretation of $distance(x)$. Lastly, the condition $Terminates(reach, crossing, t)$ has the effect of making the event *reach* occur at one instant only. Without this condition, the other postulates would force *reach* to happen at every instant in $(t_0 + m, \infty)$.

The 'imperfective paradox' is the observation that accomplishments in the progressive tense are not veridical. Under suitable circumstances, (77) entails that John reaches the other side of the street; we have seen how to provide 'suitable circumstances' with an exact semantics. But the entailment can be canceled, as in the following example

(78) John was crossing the street, when the truck hit him.

It is easy to analyse (78) in the present framework. This sentence leads to the following additions to scenario (77):

15. for some r, $t_0 < r < t_0 + m$, $Happens(hit, r)$
16. $Terminates(hit, crossing, t)$.

The additional data change the interpretations of the distinguished predicates in such a way that *Clipped*(t_0, *crossing*, $t_0 + m$) becomes derivable, and Theorem 11 is no longer true.

A similar analysis applies to statements such as 'John was building a house' or 'Mary was drawing a circle'. In both cases there are two fluents involved, one describing a cause, the other describing the result. In the first example, the cause–fluent is *building* and the dynamic fluent is some such function as *construction-stage-of-house*(x), where x is a summary of the relevant numerical parameters (e.g. height). We may add an axiom to the scenario for building a house, stating that for x large enough, the house exists:

(79) $\mathit{HoldsAt}(\mathit{construction\text{-}stage\text{-}of\text{-}house}(x),t) \wedge x > x_0 \rightarrow$
$\mathit{house}(\mathit{construction\text{-}stage\text{-}of\text{-}house}(x))$.

Thus, we interpret the predicate *house* not only on the sort of objects, but also on the sort of fluents, saying in effect that some fluents can be treated as objects. This is our analogue of the meaning postulates introduced in Zucchi (Zucchi (1999), p. 189).

7.3. **Comments on the literature.** A. Bonomi (see Bonomi (1997)) has tried to isolate another aspect of the intensionality of the progressive in what he calls the *Multiple–Choice Paradox*, which he takes to be as central as the imperfective paradox. To solve the problem, he develops an intricate semantics for the progressive, which we shall not go into, since we believe that the paradox can be treated with the machinery developed here. This being said, we happily acknowledge that the paradox is quite fruitful, and raises many issues about the progressive. We hope the analysis below covers the main aspects.

The Multiple–Choice Paradox can be illustrated by the following story:

> Leo, who has just left Dijon in his car, has decided to spend the night in one of the following three cities: Besançon, Metz or Paris. He drives on the autoroute which runs in the direction of these cities (before branching into three different roads). In each of the cities he has reserved a room. However, before he has made up his mind, his car breaks down. Now suppose that Besançon, Metz and Paris are the only cities in France where there will be a concert of Baroque music that night. Then the sentence (80), uttered shortly before the car breaks down, appears to be true

(80) Leo is going to a French city where today there is a concert of Baroque music.

Bonomi claims that (80) is ambiguous. On one reading the sentence is true, since the city that Leo is going to has the relevant characteristics; but the sentences *Leo* *is going to Besançon*, *Leo is going to Metz* and *Leo is going to Paris* appear to be false, hence so is the disjunction.[41]This means that the second reading of (80), where it is claimed that Leo is going to a *specific* city, is false.

We believe that the breakdown of the car is a red herring, and that the readings differ even without this flourish to the story. The problem seems to be that *is going* creates an intensional context, which blocks exportation of the disjunction (or existential quantifier). Informally, one may analyse the apparent paradox by means of the *Trajectory* predicate of the event calculus. Each of the sentences *Leo is going to Besançon*, *Leo is going to Metz* and *Leo is going to Paris* specifies a distinct trajectory. On the other hand, on the reading of (80) which makes that sentence true, we are concerned with a trajectory which branches out into three trajectories. The branching point is determined by an event distinct from the one initiating the journey. The two readings thus appear to be distinguished by the underlying dynamics.

The situation is actually a little more complicated, since, on the semantics we have provided, it is not quite clear what a reading is. When proving that 'John is crossing the street' implies nonmonotonically that John will eventually reach the other side, we used a nontrivial scenario and dynamics to describe the situation. Do all statements involved belong to the meaning of 'John is crossing the street'? The issue is of some relevance to the present example, since there are various ways to tease the two readings apart, depending on how one answers this question.

A formal analysis might run as follows. Let $go(t,x,y)$ be the predicate expressing that at time t, x is travelling in the direction of y, $distance(t,u,v,y)$ the predicate which says that at time t, u has travelled v kilometers in direction y, and finally, let $C(y,z)$ mean that z is a concert of Baroque music in (city) y. We use the constant l for Leo, and b,m,p for the cities. We shall freely combine these expressions into fluents. Since we will refer both to fluents and to the predicates from which they orginate, we use Feferman's notation for abstraction. The most important axiom describing the situation sketched is then

$$\exists t_0 \forall t \leq t_0 \forall y \forall y^{'} (go(t,l,y) \leftrightarrow go(t,l,y^{'})).$$

We shall not formalise the full scenario, but we want to draw attention to one point, involving intensionality. There has to be an axiom saying that at time 0, a *start* action initiates the fluent 'Leo is going somewhere', formally

$$Initiates(start, \exists y.go[\hat{t},l,y],0).$$

By the preceding axiom, $\exists y.go(0,l,y)$ is equivalent to *each* of $go(0,l,b)$, $go(0,l,m)$ and $go(0,l,p)$. However, by the intensionality of the setup, all these formulas determine different fluents, so that the axiom does not imply $Initiates(start, go[\hat{t},l,b],0)$ etc. This gives a quick way to dissolve the paradox, since the statement $Initiates(start, go[\hat{t},l,b],0)$ would have to belong to the scenario for 'Leo is going to Besançon'; if it is false, so is 'Leo is going to Besançon'. However, this appears to be a little too quick, so we shall assume a moderate form of extensionality for

[41]One might think that this is due to intentionality; on that reading, *Leo is going to Metz* entails that Leo has *decided* to go there. Bonomi constructs a more complicated example where intentionality plays no role.

the moment, and hence locate the root of the paradox elsewhere. The extensionality resides in the assumption that $go[\hat{t}, l, b]$, $go[\hat{t}, l, m]$ and $go[\hat{t}, l, p]$ are activated simultaneously; reaching a branching point then terminates (at least) one of them. Under this assumption, the root of the paradox seems to lie in the different dynamics involved.

The second reading of (80) can be formalised by

$$\exists y(\exists z C(y,z) \wedge \mathit{HoldsAt}(go[\hat{t},l,y], now) \ \wedge \ \mathit{HoldsAt}(\mathit{distance}[\hat{t},l,g(now,y),y], now)) \ \wedge \\ \forall y \forall t \forall d(\mathit{HoldsAt}(\mathit{distance}[\hat{t},l,g(t,y),y],t) \rightarrow \\ \mathit{Trajectory}(go[\hat{t},l,y],t,\mathit{distance}[\hat{t},l,g(t+d,y),y],d))).$$

The first conjunct could have been formulated equivalently by pulling the logical operations inside the *HoldsAt*, in virtue of the axioms governing T_1. We also see that one may eliminate the quantifiers $\exists y$ and $\forall y$ in favour of a disjunction featuring the three cities; the resulting formula then describes three different trajectories.

The second conjunct defines the dynamics, and it is this part which is false in the situation described. Indeed, its negation is

$$\exists y \exists t \exists d(\mathit{HoldsAt}(\mathit{distance}[\hat{t},l,g(t,y),y)],t) \ \wedge \\ \neg\, \mathit{Trajectory}(go[\hat{t},l,y],t,\mathit{distance}[\hat{t},l,g(t+d,y),y],d)).$$

But this is true if we take for $t = 0$ the time point at which Leo leaves home (i.e. before the branching point). By the assumption of extensionality for fluents, *start* initiates $go(0,l,b)$, $go(0,l,m)$ and $go(0,l,p)$. In order to establish the clause $\neg\, \mathit{Trajectory}$, we have to show for some y, *start* initiates a fluent $go(0,l,y)$ such that at some time d, Leo *cannot* be expected to be at $\mathit{distance}(d,l,g(t+d,y),y)$. But since the three fluents are activated simultaneously, if d is large enough, Leo will have passed the branching point, so that he would have to be at three different points at the same time, a contradiction. Thus on the second reading, (80) is false.

It remains to provide a formalisation of the first reading. The first conjunct remains the same, but this reading differs from the other one in its underlying dynamics, which is now described by

$$\forall t(\mathit{HoldsAt}(\exists y(\exists z C(y,z) \wedge go)[\hat{t},l,y],t) \ \rightarrow \ \exists y(\exists z C(y,z) \ \wedge \\ \forall d > 0\, \mathit{Trajectory}(go[\hat{t},l,y],t,\mathit{distance}[\hat{t},l,g(t+d,y),y],d))).$$

Again, the discourse context allows us to eliminate the quantifier $\exists y$ in favour of the three cities, but one cannot pull the resulting disjunction outside the quantifier $\forall t$. Indeed, the dynamics just says that as long as Leo keeps going in the direction of a city where there is a concert of Baroque music, there will be some destination such that if Leo initiates the fluent of going in that direction (by taking the right exit of the autoroute), then he will get there eventually, if nothing comes in between. It follows that the disjunction cannot be pulled out, because the disjunct chosen depends on t.

The paper Naumann & Piñón (1997) contains a number of interesting observations on the progressive. We will therefore discuss some of their examples here. Whereas our analysis of the imperfective paradox has been mostly concerned with the question, what can be inferred from a sentence in the progressive form, their examples focus attention on cases where sentences in the progressive form can or cannot be inferred.

Consider for example the sentence

(81) The coin is coming up heads,

uttered after flipping a coin and before the coin has landed. In this context, the utterance of (81) seems to be infelicitous, and likewise when 'is' is replaced by 'isn't'. The characteristic feature of the situation is of course that for all practical purposes, coin tossing is indeterministic. If our analysis is right, part of the meaning of the progressive is a dynamic law, which in this case would deterministically relate an initiating action (flipping the coin) and an outcome (heads). However, the true dynamics of the situation is given by

$$\begin{aligned}&\mathit{HoldsAt}(\mathit{vertical\text{–}speed}[\hat{x},\hat{s}],t)\ \wedge\ \mathit{HoldsAt}(\mathit{angular\text{–}velocity}[\hat{y},\hat{s}],t)\\ &\quad\rightarrow\ \mathit{Trajectory}(\mathit{angular\text{–}velocity}[\hat{y},\hat{s}],t,\mathit{outcome}(g(x,y,t+d)),d),\end{aligned}$$

which says that the outcome is completely determined by the initial vertical speed and angular velocity. Since the values of these initial conditions cannot be determined with any precision, the first conjunct of the formalisation of (81) is essentially something like

(12) $\mathit{HoldsAt}(\exists x(\mathit{vertical\text{–}speed}[x,\hat{s}]),t) \wedge \mathit{HoldsAt}(\exists y(\mathit{angular\text{–}velocity}[y,\hat{s}]),t)$,

from which one can only derive that there will be some outcome, barring unforeseen circumstances such as catching the coin in mid-air.

Now consider the sentence

(82) Rebecca is drawing a square,

uttered when Rebecca has just drawn a single straight edge (which could also from part of, say, a triangle). Naumann and Piñon argue that, unless we make some assumptions concerning Rebecca's intentions, the truth value of (82) cannot be established. Accordingly, in their proposed semantics, intention (modeled as a primitive accessibility relation) plays an important part. In our setup, the intention is part of the dynamics: the activity of drawing is supposed to result in a square. But clearly in the situation indicated, where we only see a single straight edge, we have just enough information to conclude that Rebecca is drawing (i.e. the first conjunct of the formalisation of (82)), but we do not have enough information to infer the dynamics, the second conjunct. Thus we would likewise say that the truth value of (82) cannot be established.

Lastly, consider

(83) Rebecca is swimming across the North Sea,

uttered when she is 100 meters off shore at Zandvoort Beach, heading west. This is an interesting example, because it may be considered true or false, depending on whether Rebecca's intention is taken into account. To formalise the difference, we have to avail ourselves of the intensionality of Feferman's calculus. The fluent *crossing* is derived from the verb 'x crosses y at time t' and hence of the form $cross[\hat{t}, x, y]$. There are two ways in which the fact that it is *Rebecca* who is attempting to cross, can be formalised. Firstly, there is a 'universal' way which makes r (= Rebecca) independent of the act of crossing: $cross[\hat{t}, r, y]$. Secondly, there is a 'particular' way, which ties up Rebecca and the act of crossing, by nominalising instead 'Rebecca crosses y at t', so that we obtain *Rebecca* $cross[\hat{t}, y]$. Although the truth axioms force these fluents to have the same time profile, they still are different terms.

Suppose first that the sentence is uttered by an observer who has no access to Rebecca's intention, only to the objective dynamics. In this case one is likely to say that (83) is false. Again, the trouble seems to reside in the dynamics, but this time in a rather subtle way. Recall that, when discussing the example of John crossing the street, for the sake of simplicity we assumed uniform motion, as embodied in 13. This seemed reasonable, because one can maintain constant speed over a short distance. This is clearly no longer possible over large distances, so that the true dynamics should now be formulated as

(13)

$$HoldsAt(distance(g(t)), t) \ \rightarrow \ Trajectory(crossing[\hat{t}, r, sea], t, distance(g(t+d)), d),$$

where g is a function such that for some s_0, for all $s, s' > s_0$, $g(s) = g(s')$. For suitable values of s_0, it is then no longer possible to copy the argument of Theorem 11 in order to obtain the conclusion that Rebecca gets to the other side, eventually. It seems that, for a statement 'A is φ–ing' to be true, the statement 'A has φ–ed' should at least be possible. This observation sets (83) apart from the sentence 'Rebecca was running across a minefield' to be discussed below.

On the other hand, if it really is Rebecca's intention to swim across the North Sea, then sentence (83) is generally considered to be true, never mind the objective dynamics which make it unlikely that she will get to the other side. This suggests that the dynamics involved should be her personal view, formulated in terms of the fluent *Rebecca* $cross[\hat{t}, y]$.

We close with a few remarks comparing our proposal with Asher's work (Asher (1992)), which was actually the first to provide a formalised treatment of the progressive within a nonmonotonic logic. The characteristic feature of Asher's theory is that he assimilates the progressive to generic expressions. This means that the following entailment holds by virtue of the meaning of the progressive

(84) When you are crossing the street, you typically get to the other side eventually.

The conditional in sentence (84) is interpreted formally as a nonmonotonic implication $>$[42], which satisfies $p, p > q \vdash q$, but not $p, r, p > q \vdash q$. This allows for canceling the implied consequent when the antecedent is expanded with, say, 'and are hit by a truck'.

While we agree completely with Asher in his insistence on formalisation, we have some doubts whether the progressive is really analogous to generics. The following example is from Naumann & Piñón (1997):

(85) Rebecca was running across the minefield.

It now seems that the use of the progressive is not governed by the analogue of (84)

(86) When you are running across a minefield, you typically get to the other side eventually;

the default assumption is rather

(87) When you are running across a minefield, you typically don't make it to the other side.

Asher solves the problem by assigning priorities to defaults, in such a way that specific defaults such as (87) get priority over general defaults of type (86) (general, because based only on the running across, without taking into account the object of the preposition). Asher's procedure requires, first, that one is able to assign a typicality-judgment to each use of the progressive and, second, that these judgments can be ordered according to priority. Not the least problem raised by these requirements is to say what 'typically' means: does it mean 'usually', or is it a conventional expectation? There is a subtle difference between 'typically' and 'in the absence of information to the contrary', both of which are used by Asher as intuitive motivations. Formally, 'typically' is an expression that belongs to the object language, and hence can be modeled by a generalised quantifier or conditional, whereas the second expression denotes a concept of the meta-language, for which circumscription is a more appropriate formalisation. By dispensing with a genericity interpretation of the progressive, we can do both without the two requirements above and without the need to provide an interpretation for 'typically'. In our approach, we must provide scenarios and dynamic laws which, upon applying circumscription, yield predictions which can then be tested against whatever typicality-judgments are available. However, the machinery can be put to work even when these judgments are absent; it is then just a matter of establishing, what, if anything, is true in certain minimal models. Asher's approach is thus compatible with ours, but the claim is that here defaults such as (84) or (87) are derivable, not introduced as meaning postulates. Furthermore, it seems that the distinction

[42] Actually, in the context of predicate logic it is a binary generalised quantifier.

between 'John is crossing the street' and 'Mary is drawing a circle' is not easily explained in Asher's setup.

References

Abney, S. (1987): The English Noun Phrase in its Sentential Aspect. PhD thesis, Massachusetts Institute of Technology.

Asher, N. (1992): 'A Default, Truth Conditional Semantics for the Progressive', *Linguistics and Philosophy* **15**, 463–508.

Asher, N. (1993): *Reference to Abstract Objects in Discourse*. Kluwer, Dordrecht.

Austin, J. (1961): Truth. In: *Philosophical Papers*. Oxford University Press, Oxford.

Bach, E. (1986*a*): 'The Algebra of Events', *Linguistics and Philosophy* **9**, 5–16.

Bach, E. (1986*b*): Natural language metaphysics. *In:* R. B. Marcus, G. Dorn & P. Weingartner, eds, *Logic, methodology and philosophy of science VII. Studies in Logic 114*. North Holland, Amsterdam.

Bonomi, A. (1997): 'The Progressive and the Structure of Events', *Journal of Semantics* **14**, 173–205.

Cantini, A. (1996): *Logical Frameworks for Truth and Abstraction*. Elsevier, Amsterdam.

Chierchia, G. (1988): *Topics in the Syntax and Semantics of Infinitives and Gerunds*. Garland Publishing, Inc., New York.

Chomsky, N. (1970): Remarks on Nominalization. *In:* A. Jacobs & P. Rosenbaum, eds, *Readings in Transformational Grammar*. Ginn and co., Waltham (Mass), pp. 184–221.

Chomsky, N. (1981): *Lectures on Government and Binding*. Foris, Dordrecht.

Cresswell, M. (1973): *Logics and Languages*. Methuen, London.

Dowty, D. (1979): *Word Meaning and Montague Grammar*. Reidel, Dordrecht.

Dowty, D. (1991): 'Thematic Proto–Roles and Argument Selection', *Language* **67**, 547–619.

Eckardt, R., ed. (1998): *Adverbs, Events, and Other Things*. Niemeyer, Tübingen.

Feferman, S. (1984): 'Toward Useful Type-Free Theories I', *The Journal of Symbolic Logic* **49**, 75–111.

Gamut, L.T.F. (1991): *Logic, Language and Meaning: Volume II*. University of Chicago Press, Chicago.

Hamm, F., A. Holzapfel & S. Schüle (1998): 'Nominalisierungstypen im Akatek Maya und im Japanischen', *STUF* **51**, 228–255.

Hamm, F. & M. van Lambalgen (2002): Intensionality and Coercion. To appear in R. Kahle (ed.), *Intensionality*. ASL Lecture Notes in Logic.

Higginbotham, J. (1983): 'The Logic of Perceptual Reports: An Extensional Alternative to Situation Semantics', *Journal of Philosophy* **80**, 100–127.

Hodges, W. (1993): *Model Theory*. Cambridge University Press, Cambridge.

Jespersen, O. (1933): *Essentials of English Grammar*. Allen & Unwin, London.

Kamp, H. & U. Reyle (1993): *From Discourse to Logic*. Reidel, Dordrecht.

Kleene, S. (1952): *Introduction to Metamathematics*. Van Nostrand, Princeton.

Kowalski, R. A. & M. Sergot (1986): 'A logic-based calculus of events', *New Generation Computing* **4**, 65–97.

Krifka, M. (1989): *Nominalreferenz und Zeitkonstitution*. Fink, München.

Landman, F. (1992): 'The Progressive', *Natural Language Semantics* **1**, 1–32.

Lasersohn, P. (1995): *Plurality, Conjunction and Events*. Kluwer, Dordrecht.

Lifschitz, V. (1994): Circumscription. *In:* D. Gabbay, C. Hogger & J. Robinson, eds, *Handbook of Logic in Artificial Intelligence and Logic Programming: Vol 3*. Clarendon Press, Oxford.

Link, G. (1987): Algebraic Semantics for Event Structures. *In:* J. Groenendijk, M. Stokhof & F. Veltman, eds, *Proceedings of the Sixth Amsterdam Colloquium*. .

M.B.Kalsbeek & Y. Jiang (1995): A vademecum of of ambivalent logic. *In:* K. Apt & F. Turini, eds, *Meta-logics and logic programming*. MIT Press, pp. 27–56.

McCarthy, J. & P. Hayes (1969): Some Philosophical Problems from the Standpoint of Artificial Intelligence. *In:* D. Michie & B. Meltzer, eds, *Machine Intelligence 4*. Edinburgh University Press, Edinburgh.

Mendelson, E. (1987): *Introduction to Mathematical Logic*. Wadsworth & Brooks, Pacific Grove.

Naumann, R. & Ch. Piñón (1997): Decomposing the Progressive. *In:* P. Dekker, M. Stokhof & Y. Venema, eds, *Proceedings of the 11th Amsterdam Colloquium*. University of Amsterdam, Amsterdam.

Parsons, T. (1990): *Events in the Semantics of English*. MIT University Press, Cambridge MA.

Portner, P. (1991): Situation Theory and the Semantics of Propositional Expressions. PhD thesis, University of Massachusetts at Amherst.

Poutsma, H. (1923): *The Infinitive, the Gerund and the Particles of the English Verb*. Noordhoff, Groningen.

Pullum, G. (1991): ‘English Nominal Gerund Phrases as Noun Phrases with Verb-Phrase Heads’, *Linguistics* **29**, 763–799.

Reichenbach, H. (1947): *Elements of Symbolic Logic*. Dover Publications, Inc., New York.

Richards, B. (1974): ‘A point of view’, *Synthese* **28**(4), 431–445.

Sato, T. (1992): Meta-programming through a truth predicate. *In:* K. Apt, ed., *Proceedings JICSLP*. MIT Press, pp. 526–540.

Scalise, S. (1984): *Generative Morphology*. Foris, Dordrecht.

Schachter, P. (1976): ‘A Nontransformational Account of Gerundive Nominals in English’, *Linguistic Inquiry* **7**, 1–55.

Shanahan, M. (1990): Representing Continous Change in The Event Calculus. In: *Proceedings ECAI 90*. pp. 598–603.

Shanahan, M. (1995): ‘A Circumscriptive Calculus of Events’, *Artificial Intelligence* **77**, 249–287.

Shanahan, M. (1996*a*): A Logical Account of the Common Sense Informatic Situation for a Mobile Robot. London.

Shanahan, M. (1996*b*): Robotics and the Common Sense Informatic Situation. In: *Working Notes of Common Sense 96, The Third Symposium on Logical Formalisation of Commonsense*. pp. 186–198.

Shanahan, M. (1997): *Solving the Frame Problem – A Mathematical Investigation of the Common Sense Law of Inertia*. MIT Press, Cambridge, MA.

Spencer, A. (1991): *Morphological Theory*. Blackwell, Oxford.

Stassen, L. (1997): *Intransitive Predication*. Clarendon Press, Oxford.

Steedman, M. (1997): Temporality. *In:* J. van Benthem & A. ter Meulen, eds, *Handbook of Logic and Language*. Elsevier, Amsterdam.

van Lambalgen, M. & F. Hamm (2001): Moschovakis' Notion of Meaning as Applied to Linguistics. To appear in M. Baaz, J. Krajicek (eds.), *Logic Colloquium 2001*. ASL Lecture Notes in Logic.

Vendler, Z. (1967): *Linguistics in Philosophy*. Cornell University Press, Ithaca, New York.

Vendler, Z. (1968): *Adjectives and Nominalizations*. Mouton, The Hague.

Verkuyl, H. (1993): *A Theory of Aspectability. The Interaction between Temporal and Atemporal Structure*. Cambridge University Press, Cambridge.

Westerståhl, D. (1989): Quantifiers in Formal and Natural Languages. *In:* D. Gabbay & F. Guenthner, eds, *Handbook of Philosophical Logic, Vol IV*. Reidel, Dordrecht.

Zucchi, A. (1993): *The Language of Propositions and Events*. Reidel, Dordrecht.

Zucchi, S. (1999): 'Incomplete Events, Intensionality and Imperfective Aspect', *Natural Language Semantics* **7**, 179–215.

Fritz Hamm
Seminar für Sprachwissenschaft
der Universität Tübingen
Wilhelmstr. 113
72074 Tübingen
e–mail: friedrich.hamm@uni-tuebingen.de

Michiel van Lambalgen
FGW / ILLC
Universiteit van Amsterdam
Nieuwe Doelenstraat 15
1012 CP AMSTERDAM
e–mail: vanlambalgen@hum.uva.nl