Relational Concepts and the Logic of Reciprocity

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1 Introduction

Reciprocal expressions like *each other* and *one another* introduce some well-known challenges for logical semantic theories. One central problem concerns the variety of interpretations that reciprocals exhibit. Consider for instance the contrast between the following sentences.

(1) Mary, Sue and Jane know each other.

(2) Mary, Sue and Jane are standing on each other.

Expressions like *know* and *stand on* are standardly interpreted as binary relations between entities. Sentence (1) can be paraphrased by requiring that every element of the set {*Mary, Sue, Jane*} is in the *know* relation with every other element of this set. By contrast, in sentence (2) an analogous interpretation is highly unlikely. We describe the contrast in Figure 1, modeling binary relations using *directed graphs* (Tutte 2001). In sentence (1) the *know* relation constitutes a *complete directed graph* (possibly with loops) over the three entities for *Mary*, *Sue* and *Jane*. Sentence (2) is true when the graph described by the *stand on* relation is not complete but constitutes a *directed path*. Similar variations in the interpretation of reciprocal sentences have repeatedly been demonstrated in semantic studies of reciprocals.[1]

Figure 1: a complete graph (possibly with loops) vs. a directed path

Many theories analyze the semantic variability of reciprocals by assuming that they are ambiguous between different quantifiers and postulating additional semantic/pragmatic principles that regulate the ambiguity.[2] In this paper we take a different route. Developing proposals in Winter (1996, 2001*b*), Gardent & Konrad (2000) and Sabato & Winter (2005), we treat reciprocals unambiguously using a quantifier that takes semantic properties of binary relations as a parameter. For example, the difference between sentences (1) and (2) is analyzed as stemming directly from the different semantic properties of the expressions *know* and *stand on*. The different parameter values that the reciprocal quantifier receives

[1] See Fiengo & Lasnik (1973), Dougherty (1974), Langendoen (1978), Higginbotham (1980), Kański (1987), Dalrymple et al. (1994, 1998), Sternefeld (1997), Beck (2001), Filip & Carlson (2001) and Kerem et al. (2009), among others.

[2] See especially Langendoen (1978), Sternefeld (1997), Beck (2001), Dalrymple et al. (1994, 1998).

in the two cases leads to the different interpretations of the sentences. We assume that the semantics of relational expressions is associated with the mental *concepts* that they refer to (Margolis & Laurence 1999), then study some central logical properties of relational concepts and their effects on the interpretation of reciprocal sentences. For example, we analyze the difference between the interpretation of sentences (1) and (2) as closely related to the fact that the relational expression *stand on* must be interpreted as an *acyclic* relation, whereas the denotation of the verb *know* is not so restricted. We argue that a comprehensive theory of reciprocals must rely on a general taxonomy of the logical restrictions on relational concepts. Developing such a taxonomy, we propose a new principle for the interpretation of reciprocals. This principle, the *Maximal Interpretation Hypothesis* (MIH), interprets reciprocals as *partial* polyadic quantifiers. The relational domain of such quantifiers is specified using the studied logical restrictions on the interpretation of relational expressions. The MIH-based quantificational analysis of reciprocals requires a relational expression to denote a *maximal* relation given the logical restrictions on its denotation. In addition to avoiding the postulation of ambiguity in the theory of reciprocals, our partial quantifier analysis avoids much of the indeterminacy that has surrounded the choice of total quantifiers as meanings of reciprocal expressions. Relying extensively on the work of Dalrymple et al. (1998), we show that the MIH exhibits some observational improvements over Dalrymple et al's Strongest Meaning Hypothesis (SMH). While the SMH is based on selecting reciprocal meanings by informal pragmatic considerations, the MIH rigorously defines the interpretation of reciprocals using well-studied properties of binary relations. By concentrating on such logical properties of relational expressions, the MIH opens the way for a more systematic study of the conceptual and contextual parameters that regulate the semantics and pragmatics of reciprocity.

The paper is structured as follows. Section 2 introduces and illustrates our distinction between reciprocal meanings and reciprocal interpretations, and analyzes it as corresponding to the distinction between total/partial $\langle 1, 2\rangle$ quantifiers, respectively. Section 3 introduces formal details in the definition of Dalrymple et al.'s SMH and the proposed MIH, and lays out one central empirical caveat in the application of these principles to "partitioned" readings of plurals. Section 4 analyzes and exemplifies the application of the MIH to various classes of logical restrictions on relational concepts, and empirically compares it to the SMH. Section 5 briefly overviews some developments in the analysis of reciprocals in relation to typicality phenomena with relational concepts, quantificational noun phrases and collective predicates. Section 6 concludes, and Appendix A summarizes some further internet data concerning asymmetric relational concepts and their occurrences with reciprocals.

2 Reciprocal meanings and reciprocal interpretations

Simple reciprocal sentences like (1) and (2) above are standardly analyzed using generalized quantifiers of type $\langle 1, 2\rangle$. One way of describing such quantifiers is as relations between sets and binary relations. For instance, Peters & Westerståhl (2006, p.367) analyze the reciprocal expression *each other* in sentence (1) as a relation between the set denotation of the subject *Mary, Sue and Jane* and the binary relation denoted by the verb *know*. Equivalently, we here view reciprocals as denoting *characteristic functions* of relations between sets and binary relations. Accordingly, we model $\langle 1, 2\rangle$ quantifiers as functions from pairs of sets and binary relations to truth-values.

In the case of sentence (1), the relevant $\langle 1,2\rangle$ quantifier is commonly assumed to be the function SR of *strong reciprocity* that is defined in (3) below.[3] In this definition and henceforth, we standardly assume a non-empty domain E of entities and a domain $\mathbf{2} = \{0,1\}$ of truth-values. The latter is ordered by the partial order $\leq$, which corresponds to material implication between truth-values.

(3) The $\langle 1,2\rangle$ quantifier SR is the function in $(\wp(E)\times\wp(E^2))\to\mathbf{2}$, s.t. for every set $A\subseteq E$ and binary relation $R\subseteq E^2$:

$$\mathsf{SR}(A,R)=1 \;\Leftrightarrow\; \forall x,y\in A\,[x\neq y\to R(x,y)].$$

In words: R describes a *complete graph* over A, possibly with loops.

In such cases, where each pair of different elements of the set A is in the relation R, we say that R *satisfies strong reciprocity* over A.

Note that the SR function is defined as a *total* function on the domain $\wp(E)\times\wp(E^2)$. In sentences like (1), or the similar sentence (4) below, it is straightforward to use the SR function as the denotation of the reciprocal expression.

(4) The girls know each other.

For logical purposes, we can safely assume that the subject of sentence (4) may denote any set of entities with at least two members. Similarly, we assume that the verb *know* may denote any binary relation. The latter assumption reflects the intuition that there are no logically significant restrictions on the denotation of the verb *know*. For the purposes of this paper, we assume that any entity may in principle stand in the *know* relation to any entity, or to no entities at all.[4] This assumption about the free interpretation of verbs like *know* in reciprocal sentences like (4) means that the reciprocal expression *each other* in such sentences must denote a *total* $\langle 1,2\rangle$ quantifier on sets and binary relations. However, the situation is quite different in sentence (2), repeated below.

(5) Mary, Sue and Jane are standing on each other. (=(2))

Unlike the verb *know*, the expression *stand on* has obvious restrictions on its denotation. Most notably, our common sense knowledge tells us that the *stand on* expression should denote an *acyclic* relation.[5] Therefore, in cases like (5), unlike (1) or (4), the reciprocal expression does not have to be analyzed using a total function on all sets and binary relations. Furthermore, since the *stand on* relation in sentence (5) is acyclic, any analysis of sentence

[3]Some works assume that reciprocal meanings should also include a requirement that the set of entities argument contains at least two elements. In this paper we ignore this requirement. The complex relationships between plurality, reciprocity and cardinality of set arguments merit special attention. See Heim et al. (1991), Schwarzschild (1996), Winter (2002) and Zweig (2009) for relevant details.

[4]More accurately, we should note that the verb *know* requires an *animate* entity as its subject argument. However, for the logical analysis what is important is that the verb *know* may denote any of the subsets of some given cartesian product $A\times B\subseteq E^2$. For our purposes here we avoid this complication, and ignore the need to specify A and B using the selectional restrictions of binary predicates.

[5]Some Escher paintings may come to mind as contradicting such world knowledge. More generally, many of the restrictions on the interpretation of relational expressions, even relatively strong restrictions like the acyclicity of *stand on*, may be relaxed in some highly atypical contexts. For the sake of this study we ignore such exceptional scenarios. See Kerem et al. (2009) for recent experimental work on typicality and reciprocals, and remarks in section 5.1.

(5) using strong reciprocity would lead to a patently false interpretation, contrary to facts. Whatever the interpretation of the reciprocal expression in (5) may be, it must be logically weaker than strong reciprocity.

One of the main claims of this paper is that the "weak" interpretations of some reciprocals are inseparable from their partiality. In order to develop this idea, Definition 1 introduces partial $\langle 1,2\rangle$ generalized quantifiers as the semantic domain of reciprocal expressions. Accordingly, we refer to such partial quantifiers as *reciprocal functions*.

Definition 1. *Let* $\Theta \subseteq \wp(E^2)$ *be a set of binary relations over* E*. A partial* $\langle 1,2\rangle$ *quantifier* $f : (\wp(E) \times \Theta) \to \mathbf{2}$*, from subsets of* E *and binary relations in* Θ *to truth-values, is called a* RECIPROCAL FUNCTION *over* Θ*. When* $f(A,R) = 1$ *we say that* R SATISFIES f-RECIPROCITY *over* A*.*

We refer to a total reciprocal function over $\Theta = \wp(E^2)$ as RECIPROCAL MEANING.

What are the reciprocal functions that may be realized as interpretations of natural language reciprocals expressions? Two familiar constraints on the denotation of reciprocals are *conservativity* and *neutrality to identities* (Dalrymple et al. 1998, Peters & Westerståhl 2006). To exemplify these facts, let us consider the following sentence.

(6) Mary, Sue and Jane are pinching each other.

The conservativity of the reciprocal in sentence (6) is illustrated by the fact that the truth of (6) does not depend on pairs in the *pinch* relation which are outside the set of Mary, Sue and Jane.[6] Neutrality to identities is illustrated in (6) by the fact that the truth of the sentence does not depend on whether or not any of the three girls is pinching herself. In addition to these two properties, all reciprocal interpretations known to us are *monotonic* on their relation argument. For example, suppose that sentence (6) is true in a situation where the *pinch* relation describes a directed cyclic graph on the three girls. Adding another pair to this cycle by letting one of the girls pinch the two other girls simultaneously cannot make sentence (6) false. We call this property R*-monotonicity*.[7] When a reciprocal function satisfies the three properties of conservativity, neutrality to identities and R-monotonicity, we call it an *admissible reciprocal interpretation*.

The three logical properties of reciprocal interpretations are formally summarized in Definition 2, using the notation I for the identity relation $\{\langle x,x\rangle : x \in E\}$ over E.

Definition 2. *Let* $\Theta \subseteq \wp(E^2)$ *be a set of binary relations over* E*, and let* f *be a reciprocal function from* $\wp(E) \times \Theta$ *to* $\mathbf{2}$*.*

f *is* CONSERVATIVE *if for every set* $A \subseteq E$*, for all relations* $R_1, R_2 \in \Theta$*:*
$$A^2 \cap R_1 = A^2 \cap R_2 \quad \Rightarrow \quad f(A,R_1) = f(A,R_2).$$

f *is* NEUTRAL TO IDENTITIES *if for every set* $A \subseteq E$*, for all relations* $R_1, R_2 \in \Theta$*:*
$$R_1 - I = R_2 - I \quad \Rightarrow \quad f(A,R_1) = f(A,R_2).$$

[6] This conservativity of reciprocals as $\langle 1,2\rangle$ quantifiers is similar to the more familiar conservativity of $\langle 1,1\rangle$ quantifiers in natural language (Peters & Westerståhl 2006, p.138). See also section 5.2.

[7] A potential counter-example to R-monotonicity is mentioned by Kański (1987):

(i) The students followed each other (into the room).

Indeed, it is impossible to add a pair of students to any linear graph of the *follow* relation. However, as Dalrymple et al. (1998) mention, this fact is a result of the semantic restrictions on the predicate *follow*, and does not bear on the monotonicity of the reciprocal.

f is R-MONOTONIC *if for every set $A \subseteq E$, for all relations $R_1, R_2 \in \Theta$:*
$R_1 \subseteq R_2 \;\Rightarrow\; f(A, R_1) \le f(A, R_2)$.

If the reciprocal function f is conservative, neutral to identities and R-monotonic, we call it an ADMISSIBLE RECIPROCAL INTERPRETATION.

Most logical semantic work on reciprocity has concentrated on total $\langle 1, 2\rangle$ quantifiers, i.e. on reciprocal meanings. In this paper we use the more general notion of partial $\langle 1, 2\rangle$ quantifiers, which we have called 'reciprocal functions'. Definition 2 classifies some of these functions as admissible reciprocal interpretations. The reciprocal meanings among these admissible interpretations are referred to as ADMISSIBLE RECIPROCAL MEANINGS.

Below we give some examples of reciprocal sentences and admissible reciprocal meanings that have been proposed in their semantic analysis. Most of these examples are from Dalrymple et al. (1998), which is henceforth referred to as 'DKKMP'.

(7) "The captain", said the pirates, staring at each other in surprise (DKKMP).
One-way Weak Reciprocity:
$\mathtt{OWR}(A, R) = 1 \;\Leftrightarrow\; \forall x \in A\; \exists y \in A\; [x \neq y \wedge R(x, y)]$
In words: every node in the graph that R describes on A has at least one (non-loop) outgoing edge.

(8) Five Boston pitchers sat alongside each other (DKKMP).
Intermediate Reciprocity:
$\mathtt{IR}(A, R) = 1 \;\Leftrightarrow$
$\forall x, y \in A\; [x \neq y \rightarrow \exists m \exists z_0, \ldots, z_m \in A\; [x = z_0 \wedge y = z_m \wedge R(z_0, z_1) \wedge \ldots \wedge R(z_{m-1}, z_m)]]$
In words: R describes a *strongly connected graph* on A – a graph that has a directed path between any two different nodes.

(9) The third-grade students in Mrs. Smith's class gave each other measles (DKKMP).
Intermediate Alternative Reciprocity:
$\mathtt{IAR}(A, R) = 1 \;\Leftrightarrow$
$\forall x, y \in A\; [x \neq y \rightarrow \exists m \exists z_0, \ldots, z_m \in A[x = z_0 \wedge y = z_m \wedge (R(z_0, z_1) \vee R(z_1, z_0)) \wedge \ldots \wedge (R(z_{m-1}, z_m) \vee R(z_m, z_{m-1}))]]$
In words: R describes a *weakly connected graph* on A – a graph that has an undirected path between any two different nodes.

(10) He and scores of other inmates slept on foot-wide wooden planks stacked atop each other (Kański 1987, DKKMP).
Inclusive Alternative Ordering:
$\mathtt{IAO}(A, R) = 1 \;\Leftrightarrow\; \forall x \in A\; \exists y \in A\; [x \neq y \wedge (R(x, y) \vee R(y, x))]$
In words: every node in the graph that R describes on A has at least one (non-loop) outgoing or incoming edge.

(11) John, Bill, Tom, Jane and Mary had relations with each other (Dougherty 1974, Langendoen 1978).

Symmetric Reciprocity:
$\mathtt{SmR}(A, R) = 1 \;\Leftrightarrow\; \forall x \in A\; \exists y \in A\; [x \neq y \wedge R(x, y) \wedge R(y, x)]$
In words: every node in the graph that R describes on A has at least one (non-loop) bi-directional edge.

The total $\langle 1, 2\rangle$ quantifiers in (7)-(11) have all been proposed as the meanings of the reciprocal expressions in the respective sentences. As we shall see, it is not always easy to support such proposals. One of the complicating factors is that the semantic restrictions on the denotation of relational expressions often leave some possibilities open regarding the meaning of the reciprocal expression. For example, DKKMP doubt the usefulness of the SmR quantifier for analyzing sentence (11), pointing out that, given the symmetry of the binary relation *had relations with*, both the SmR and the IAO meanings lead to identical truth-conditions. In formula: for every set $A \subseteq E$ and *symmetric* binary relation $R \subseteq E^2$, $\mathtt{SmR}(A, R) = \mathtt{IAO}(A, R)$. Using our terminology, we say that when the total quantifiers SmR and IAO are restricted to the domain of symmetric binary relations, they yield the same reciprocal interpretation. This example shows a general problem for assessing empirical claims about reciprocal meanings using truth-conditional evidence about natural language sentences. We will avoid this problem by concentrating on reciprocal interpretations rather than reciprocal meanings. Reciprocal meanings will only be used here in order to compare our results to previous ones. This leaves us with our main question: what are the origins of variability in the interpretation of reciprocal sentences?

3 Accounting for reciprocal interpretations

As we saw above, different reciprocal meanings have been proposed for analyzing reciprocal interpretations in different sentences and contexts. DKKMP analyze reciprocals as ambiguous quantificational expressions and propose an informal principle, the *Strongest Meaning Hypothesis* (SMH), for selecting between their different meanings. The SMH selects a reciprocal meaning based on contextual information that is assumed *ad hoc* for each analyzed reciprocal sentence. We propose a more formal analysis of the quantificational variability of reciprocals, replacing the SMH by a principle that we call the *Maximal Interpretation Hypothesis* (MIH). Unlike the SMH, the MIH does not assume ambiguity of reciprocals between different meanings. Rather, under the MIH all reciprocals denote one operator that takes meaning postulates on relational concepts as a parameter. Our definition of reciprocity derives a maximal interpretation with respect to the logical constraints that the meaning postulates on the relation impose. In section 4 we will show that the results of using the MIH are compatible in many cases with DKKMP's informal SMH-based analysis. However, a central difference between our proposed analysis and many previous proposals, including DKKMP's, concerns the question of connectivity or partitioning in graphs that are induced by reciprocal interpretations. We argue that reciprocal expressions impose a connectivity requirement on their arguments. Potential counter-examples involving partitioned interpretations of reciprocal sentences are argued to be derived by general quantificational mechanisms with plurals, which are independent of the interpretation of reciprocal expressions. This section introduces the SMH, the MIH and our assumptions about partitioning with plurals.

3.1 Dalrymple et al's Strongest Meaning Hypothesis

DKKMP's theory is based on six reciprocal meanings: SR, OWR, IR, IAR and IAO, which were defined above, and an additional meaning, *Strong Alternative Reciprocity*, which is defined below.

(12) *Strong Alternative Reciprocity*:
$\mathsf{SAR}(A,R) = 1 \Leftrightarrow \forall x,y \in A\,[x \neq y \rightarrow (R(x,y) \vee R(y,x))]$
In words: the graph that R describes on A has a complete underlying (undirected) graph, possibly with loops.[8]

Having assumed this six-way ambiguity,[9] DKKMP further propose a disambiguation strategy that governs it. The denotation of a reciprocal expression in a given sentence is selected using an informal principle that DKKMP call the *Strongest Meaning Hypothesis* (SMH). We paraphrase the SMH below.

Strongest Meaning Hypothesis (SMH): *Let S be a natural language sentence containing a reciprocal expression* RECIP. *For any reciprocal meaning Π, let φ_Π be the proposition derived for S by letting* RECIP *denote Π. The occurrence of* RECIP *in S denotes the strongest meaning $\Pi \in \{\mathsf{SR}, \mathsf{OWR}, \mathsf{IR}, \mathsf{IAR}, \mathsf{IAO}, \mathsf{SAR}\}$ such that φ_Π is consistent with the context of S's utterance.*

We standardly say that a reciprocal meaning Π_1 is *stronger than* a meaning Π_2 if for every $A \subseteq E$ and $R \subseteq E^2$: $\Pi_1(A,R) \leq \Pi_2(A,R)$. For the logical ordering of the six meanings proposed by DKKMP, see Figure 2.

$$\begin{tikzcd}
 & & \mathsf{SR} \arrow[dl] \arrow[dr] & \\
 & \mathsf{IR} \arrow[dl] \arrow[dr] & & \mathsf{SAR} \arrow[dl] \\
\mathsf{OWR} \arrow[dr] & & \mathsf{IAR} \arrow[dl] & \\
 & \mathsf{IAO} & &
\end{tikzcd}$$

Figure 2: DKKMP's six reciprocal meanings and their logical ordering

As an example for DKKMP's use of the SMH, let us consider sentence (9), restated below.

(13) The third-grade students in Mrs. Smith's class *gave* each other *measles* (=(9)).

As DKKMP point out, according to common world knowledge, people can only be given measles once. In other words, if the expression *give measles* in (13) denotes a relation R, then its inverse relation R^{-1} is a function. The function that R^{-1} describes may be partial, since some people may not get measles at all. In addition, giving measles is only possible

[8]For further discussion of the SAR meaning see Sabato & Winter (2005), where we argued that this meaning is unlikely to be attested as a reading of natural language reciprocals. See also footnote 17.

[9]DKKMP argue for these six meanings as the *a priori* available denotations of reciprocals by showing that they are all derived using three basic meanings. Each of these basic meanings is applied either of the denotation R of the relational expression in the sentence, or to its symmetric closure $R^\vee$. For more details on this analysis see Dalrymple et al. (1998, pp.187-8).

after getting it. This means that the graph described by the relation R does not contain circles, and we say that the relation R has to be *acyclic*.

Formally, we define the following sets of binary relations over a domain E:

(14) $\mathtt{FUN}^{-1} = \{R \subseteq E^2 : \forall x, y_1, y_2 \in E\ [(R(y_1, x) \wedge R(y_2, x)) \rightarrow y_1 = y_2]\}$

In words: $\mathtt{FUN}^{-1}$ is the set of relations over E whose *inverse* is a *function*, possibly a partial one.

(15) $\mathtt{ACYC} = \{R \subseteq E^2 : \forall n \forall x_1, \dots, x_n \in E$
$\neg[R(x_1, x_2) \wedge R(x_2, x_3) \wedge \dots \wedge R(x_{n-1}, x_n) \wedge R(x_n, x_1)]\ \}$

In words: $\mathtt{ACYC}$ is the set of *acyclic* relations over E.

We rephrase DKKMP's assumption about sentence (13) by requiring that the binary relation R for the expression *give measles* must be in the set $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$. Conversely, any relation in $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$ is a possible denotation for the relational expression *give measles*.[10] We say that the set $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$ is the DOMAIN for interpreting the relational expression *give measles*. Let us officially state this terminological convention.

Terminology: *Let* REL *be a relational expression, and let* $\Theta \subseteq \wp(E^2)$ *be a set of binary relations over* E*. If every relation in* Θ *is a possible denotation of* REL *over* E*, and any possible denotation of* REL *over* E *is in* Θ*, we say that* Θ *is* REL*'s* DOMAIN *of interpretation over* E*, and denote* $\Theta_{\mathrm{REL}} = \Theta$.

Summarizing, we express DKKMP's assumption on the relational expression *give measles* by denoting:

$$\Theta_{\textit{give measles}} = \mathtt{ACYC} \cap \mathtt{FUN}^{-1}.$$

Let $A \subseteq E$ be the set of entities denoted by the subject of sentence (13), where $|A| \geq 2$. And let $R \in \mathtt{ACYC} \cap \mathtt{FUN}^{-1}$ be a denotation of the relational expression *give measles*. Given our assumptions, it is easy to verify that $\mathtt{IAR}$ is the strongest reciprocal meaning $\Pi \in \{\mathtt{SR}, \mathtt{OWR}, \mathtt{IR}, \mathtt{IAR}, \mathtt{IAO}, \mathtt{SAR}\}$ that satisfies $\Pi(A, R) = 1$. To see that, note that since R is acyclic, $\mathtt{SR}(A, R) = 0$ and $\mathtt{IR}(A, R) = 0$. Since R is also in $\mathtt{FUN}^{-1}$, we have $\mathtt{OWR}(A, R) = 0$, and further $\mathtt{SAR}(A, R) = 0$ for any A s.t. $|A| \geq 3$. Assuming that the expression *give measles* can denote any relation in $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$, we are left with two reciprocal meanings in DKKMP's account that are consistent with $\Pi(A, R) = 1$: $\mathtt{IAR}$ and $\mathtt{IAO}$.[11] The $\mathtt{IAR}$ meaning is stronger than $\mathtt{IAO}$. Hence, the SMH selects $\mathtt{IAR}$ as the denotation of the reciprocal expression in sentence (13). This meaning, together with the acyclicity and $\mathtt{FUN}^{-1}$ properties of the predicate, entail that the relation *give measles* in (13) describes a *directed*

[10] Although DKKMP do not explicitly state this assumption, it seems to directly follow from their informal notion of "relevant context": if the denotation of *give measles* were contextually restricted to be a proper subset of $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$, this would have to be taken into account when using the SMH. As we shall see below, the SMH might have derived absurd results if only some of the relations in $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$ were used as possible denotations of the relational expression.

[11] As mentioned in footnote 10, the assumption $\mathtt{ACYC} \cap \mathtt{FUN}^{-1} \subseteq \Theta_{\textit{give measles}}$ is crucial for DKKMP's analysis. Without this (plausible) assumption, it would not be guaranteed that even $\mathtt{IAR}$ and $\mathtt{IAO}$ are consistent with $\Pi(A, R) = 1$. As an extreme example, note that all analyses of (13) using the SMH must make sure that the domain for the expression *give measles* is not empty, i.e. that somebody *could* have given somebody measles.

tree on the third-graders.[12] Ignoring at this stage some empirical complications,[13] we note that this result basically agrees with speaker intuitions about the truth conditions of sentence (13).

3.2 The Maximal Interpretation Hypothesis

In our presentation of the SMH above, we have considered the relational domain of interpretation as the only parameter that affects the selection of a reciprocal meaning. However, by appealing to contextual information, the SMH strives to be more general than that, and take into account further pragmatic parameters beyond the semantics of the relation to which the reciprocal applies. DKKMP do not elaborate on this point beyond saying that the contextual information appealed to by the SMH should be "relevant to the reciprocal interpretation" (Dalrymple et al. 1998, p.193). Although possibly useful, DKKMP's general statement does not explain further the notion of "relevance" that it appeals to. Consequently, it becomes hard to test the expectations of the SMH against the interpretation of simple reciprocal sentences. Consider for instance the reciprocal sentence (16b) below, uttered in the context of (16a).

(16) a. Mary likes John, but John doesn't like Mary.
b. Mary and John like each other.

The context (16a) contradicts SR, OWR and IR as possible meanings of the reciprocal in sentence (16b). Therefore, using the SMH we may expect sentence (16b) to be true in context (16a), with SAR as the meaning of the reciprocal expression in (16b). This is because SAR is the strongest among the three remaining reciprocal meanings in DKKMP's account. However, the context (16a) flatly contradicts sentence (16b). Thus, DKKMP's analysis requires the contextual information that sentence (16a) conveys to be defined as irrelevant for the interpretation of the reciprocal in (16b). We are unaware of any well-defined notion of relevance that does that.

This and similar problems lead us to abandon the notion of "relevant context" as the key factor in determining reciprocal interpretations, while retaining the "logical maximality" idea that underlies the SMH. Of course, we do not deny that general contextual factors affect the interpretation of reciprocals.[14] However, we believe that for the purpose of studying the basic principles and mechanisms underlying the interpretative variability of reciprocals, it is useful to concentrate on what seems like the key contextual factor governing it – the interpretation of the relational concept with which the reciprocal combines. As we shall

[12] A relation R describes a directed tree, or an *arborescence* (Tutte 2001, p.126), if the undirected version of R (its symmetric closure) is a tree (a connected acyclic undirected graph) and in addition, there is a node r (root) such that for each other node x, there is a directed path in R from r to x. To see that an acyclic and weakly connected graph that has the FUN^{-1} property is an arborescence, consider the following procedure. Select any node, and follow the edge that point to it if there is such an edge (there is at most one such edge because of FUN^{-1}). Repeat this process until reaching a node r that has no edges pointing to it (such a node exists because of acyclicity). The node r has a directed path to any other node because: (i) r has an undirected path with any other node (weak connectivity), and (ii) no node x in such an undirected path has more than two incoming edges (FUN^{-1}).

[13] As DKKMP mention, sentence (13) can also be true if the relation *give measles* describes a *collection* of directed trees on the third-graders. In this case there is more than one third grader who got measles from outside the group of third grades. See section 3.3 below.

[14] See some remarks in footnote 36

see, the interpretation of relational expressions, while itself being quite sensitive to general contextual factors, is often logically stable, and this stability helps to test hypotheses about the semantics/pragmatics of reciprocals.

Our proposal is based on two general assumptions, which develop previous work in Winter (1996, 2001*b*), Gardent & Konrad (2000) and Sabato & Winter (2005). First, as said above, instead of considering the whole context of utterance, we focus on the meaning of the relational expression as the main parameter that determines the semantics of reciprocity. Second, unlike DKKMP's ambiguity based approach, we do not adopt any assumption on the *a priori* possible meanings of reciprocal expressions.[15] Rather, our account directly derives a reciprocal interpretation using the domain in which the relational expression is interpreted. These two assumptions lead us to an alternative to the SMH, which we call the *Maximal Interpretation Hypothesis*. This principle is informally stated below.

Maximal Interpretation Hypothesis (MIH): *Let* REL *be a relational expression composing with a reciprocal expression in a natural language sentence. Reciprocity requires* REL *to denote a relation in* REL*'s domain of interpretation that is not properly contained in any other relation in* REL*'s domain. In this case we say that* REL *denotes a* <u>*maximal*</u> *relation in* REL*'s domain.*

The MIH employs logical semantic domains for interpreting relational expressions. Thus, we assume that meanings of relational expressions, or relational *concepts* (Margolis & Laurence 1999), impose logical restrictions on their possible denotations. These logical restrictions are often classified as *meaning postulates* (Carnap 1952, Montague 1973, Zimmermann 1999). The meaning postulates on the denotation of the expression *give measles* were expressed above by requiring that the domain for interpreting it is the set $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$. With this assumption, the MIH boils down to requiring that the expression *give measles* in sentence (13) denotes a maximal relation in $\mathtt{ACYC} \cap \mathtt{FUN}^{-1}$. As we shall see below, this MIH-based interpretation of (13) agrees with the $\mathtt{IAR}$ meaning that is derived for (13) in the SMH-based analysis.

When formally stating the MIH, we adopt the following notation for restricting binary relations $R \subseteq E^2$ and relational domains $\Theta \subseteq \wp(E^2)$ using a set $A \subseteq E$:

$R\|_A$	=	$R \cap A^2$	–	R restricted to A
$\Theta\|_A$	=	$\{R\|_A : R \in \Theta\}$	–	Θ restricted to A

For disregarding identities in relations and relational domains, we use the notation:

$R{\downarrow}$	=	$R - I$	–	R, disregarding identities
$\Theta{\downarrow}$	=	$\{R{\downarrow} : R \in \Theta\}$	–	Θ, disregarding identities

Combining the two notations we get:

$R{\downarrow}_A$	=	$R\|_A - I$	–	R restricted to A, disregarding identities
$\Theta{\downarrow}_A$	=	$\{R{\downarrow}_A : R \in \Theta\}$	–	Θ restricted to A, disregarding identities

Using this notation, we define *MIH-based reciprocal functions* as follows.

[15] The admissibility of reciprocal interpretations (cf. Definition 2) follows as a direct corollary of our account, rather than being a separate assumption. However, in section 3.3 we will see that a *connectivity* assumption on reciprocal interpretations has to be added in order to make our approach empirically coherent.

Definition 3. *Let $\Theta \subseteq \wp(E^2)$ be a set of binary relations over E. The* MIH-BASED *reciprocal function* $\text{RECIP}^{\text{MIH}}_{\Theta}$ *is defined for all sets $A \subseteq E$ and relations $R \in \Theta$ by:*

$\text{RECIP}^{\text{MIH}}_{\Theta}(A,R) = 1$ *iff for all $R' \in \Theta{\downarrow_A}$: $R{\downarrow_A} \subseteq R' \Rightarrow R{\downarrow_A} = R'$.*

In words: a relation $R \in \Theta$ satisfies MIH-based reciprocity over a set $A \subseteq E$ with respect to Θ if $R{\downarrow_A}$ is maximal on $\Theta{\downarrow_A}$.

Note that by definition, the reciprocal function $\text{RECIP}^{\text{MIH}}_{\Theta}$ is conservative, neutral to identities and R-monotonic for every set Θ of binary relations. Thus, in our terminology, every reciprocal function $\text{RECIP}^{\text{MIH}}_{\Theta}$ is an admissible reciprocal interpretation, independently of Θ.

Let us reconsider example (13) above. For the expression *give measles*, we have assumed $\Theta_{\textit{give measles}} = \texttt{ACYC} \cap \texttt{FUN}^{-1}$. For this set Θ and a relation R in Θ, we observe that the reciprocal function $\text{RECIP}^{\text{MIH}}_{\Theta}$ satisfies $\text{RECIP}^{\text{MIH}}_{\Theta}(A,R) = 1$ if and only if R describes a weakly connected graph on A.[16] Thus, we note the following fact.

Fact 1. *Let Θ be the set of binary relations $\texttt{ACYC} \cap \texttt{FUN}^{-1} \subseteq E^2$. For every set $A \subseteq E$ and relation $R \in \Theta$: $\text{RECIP}^{\text{MIH}}_{\Theta}(A,R) = 1 \Leftrightarrow \texttt{IAR}(A,R) = 1$.*

We see here that for the domain $\Theta = \texttt{ACYC} \cap \texttt{FUN}^{-1}$ of binary relations, the reciprocal interpretation $\text{RECIP}^{\text{MIH}}_{\Theta}$ and the SMH-based reciprocal meaning IAR agree with one another. Thus, as in the SMH-based analysis above, the MIH analyzes the relation *give measles* in sentence (13) as describing a *directed tree* on the third-graders. However, our reliance on the notion of 'reciprocal interpretation' gives no special status to the IAR meaning in the analysis of sentence (13). The interpretation that the MIH derives is consistent with the IAR meaning, but also with stronger meanings. Consider for instance the following reciprocal meaning ROOT, which is stronger than IAR and requires that, on top of weak connectivity, the graph described by the relation contains at least one node that has a directed path to any other node.

(17) $\texttt{ROOT}(A,R) = 1 \Leftrightarrow$
$\exists r \in A\ \forall x \in A\ [x \neq r \rightarrow \exists m \exists z_0, \ldots, z_m \in A\ [r = z_0 \wedge x = z_m \wedge R(z_0,z_1) \wedge \ldots \wedge R(z_{m-1},z_m)]]$

In words: R describes a graph on A with at least one root r – a node that has a directed path to every other node.

We have seen that the MIH-based interpretation of the reciprocal in (9) agrees with both IAR and ROOT. The following standard definition of *consistency* between a partial function and a total function, formalizes this notion of 'agreement' between reciprocal functions and reciprocal meanings.

Definition 4. *Let $\Theta \subseteq \wp(E^2)$ be a set of binary relations over $E \neq \emptyset$, and let $f : (\wp(E) \times \Theta) \rightarrow \mathbf{2}$ be a reciprocal function. Let $\Pi : \wp(E) \times \wp(E^2)$ be a reciprocal meaning over E. We say that f is* CONSISTENT *with Π on E if for every set $A \subseteq E$ and relation $R \in \Theta$: $f(A,R) = \Pi(A,R)$.*

---

[16] *Proof 'only if'*: assume that $R{\downarrow_A}$ is maximal on $\Theta{\downarrow_A}$, and assume for contradiction that $R{\downarrow_A}$ is not weakly connected. Then there are two non-empty weakly connected components $C_1 \subseteq A$ and $C_2 \subseteq A - C_1$. The acyclicity and $\texttt{FUN}^{-1}$ properties of R entail that C_1 and C_2 are both directed trees (cf. footnote 12). Thus, we can add an edge to R, connecting the trees C_1 and C_2 and leaving the acyclicity and $\texttt{FUN}^{-1}$ properties of R intact. This contradicts to $R{\downarrow_A}$'s maximality on $\Theta{\downarrow_A}$. *Proof 'if'*: if $R{\downarrow_A}$ is weakly connected, then $R \in \texttt{ACYC} \cap \texttt{FUN}^{-1}$ entails that $R{\downarrow_A}$ is a directed tree (cf. footnote 12). By definition of directed trees, adding any edge to $R{\downarrow_A}$ would create either a a non-acyclic or a non-$\texttt{FUN}^{-1}$ relation. Hence $R{\downarrow_A}$ is maximal on $\Theta{\downarrow_A}$.

This notion of consistency will be useful for our analysis of concrete examples in section 4.[17]

3.3 MIH-based connectivity and partitioning

When analyzing reciprocal sentences we should be careful to distinguish general plurality phenomena from the quantificational semantics of reciprocals. One especially relevant property of plurals concerns their *partitioning* effects (Schwarzschild 1996, Winter 2000, Beck & Sauerland 2001). These are cases where a plural argument is interpreted by dividing its denotation into two or more sets. Consider the simple example (18a).

(18) a. The Indians and the Chinese are numerous.

b. $\mathbf{numerous}(I) \wedge \mathbf{numerous}(C)$

The likely interpretation of sentence (18a) that is formalized in (18b) claims that there are many Indian people as well as many Chinese people. Thus, while the surface argument of the predicate *be numerous* in sentence (18a) is one plural subject, the sentence can be interpreted as involving predication over two sets. A similar effect also appears with plural sentences containing reciprocal expressions. Consider the following simple example of such "partitioned reciprocity".

(19) a. Mary and John and Sue and Bill are married to each other.

b. $\mathbf{married}(\{\mathbf{mary}, \mathbf{john}\}) \wedge \mathbf{married}(\{\mathbf{sue}, \mathbf{bill}\})$

The likely interpretation of sentence (19a) involves *two* sets (of married couples), as formalized in (19b).

The reason we have dubbed examples (18a) and (19a) "simple" is because semantic theory has a ready explanation for their partitioning effects. As stressed in Winter (2001*a*), the boolean analysis of the conjunction *and* in complex noun phrases directly derives the partitioning effects in (18) and (19). Of course, boolean conjunction of noun phrases does not require any partitioning mechanism in the semantics of collective predicates, reciprocal expressions or plural predicates in general. Therefore, one likely source of the partitioning in sentences (18a) and (19a) is *external* to the predicate.

In other examples, however, it is less clear that partitioning can be a predicate-external process. Consider for instance the following familiar example Gillon (1987).

(20) Rodgers, Hammerstein and Hart wrote musicals together.

[17] In Sabato & Winter (2005) we introduced a notion of *congruence* between reciprocal functions and reciprocal meanings. A reciprocal meaning Π is congruent with a reciprocal function f if Π is consistent with f, and furthermore Π is the strongest reciprocal meaning consistent with f. We consider congruence as a formal correlate to the intuition that a certain reciprocal meaning is "attested" in a given sentence: when a sentence interpretation is congruent with a meaning Π, we may reasonably claim that Π is attested. As shown in Sabato & Winter (2005), the SAR meaning is only congruent with the reciprocal interpretation $\text{RECIP}^{\text{MIH}}_{\texttt{ASYM}}$, where ASYM is the set of asymmetric relations. As will be mentioned in section 4.3 below, we are not aware of any relational expression in natural language whose domain contains all and only the asymmetric relations. As a result we expect the SAR meaning not to be easily attested. Another meaning that was proposed in the literature for reciprocals is *weak reciprocity* (WR, see Langendoen (1978)):

$\texttt{WR}(A, R) = 1 \;\Leftrightarrow\; \forall x \in A\, \exists y, z \in A\, [x \neq y \wedge x \neq z \wedge R(x, y) \wedge R(z, x)]$.

In words: each node in the graph described by R over A has a (non-loop) incoming edge as well as a (non-loop) outgoing edge. In Sabato & Winter (2005) we show that for every set E s.t. $|E| \geq 6$, there is no relational domain Θ over E s.t. WR is congruent with the reciprocal interpretation $\text{RECIP}^{\text{MIH}}_{\Theta}$. For empirical arguments against WR as an "unattested" reciprocal meaning, see DKKMP (p.176).

This sentence may be true even though the three writers never collaborated as a trio. As things were, the sentence is true, but only due to the collaborative work of the two duos *Rodgers & Hammerstein* and *Rodgers & Hart*. This example shows that we need some semantic/pragmatic principles on top of NP structure to account for partitioning effects. An on-going debate in the semantic study of plurals concerns these principles, their account and their theoretical implications.

This debate on partitioning effects with plurals is highly relevant for our understanding of reciprocity. To see that, let us first reconsider DKKMP's *measles* example, which is repeated below.

(21) The third-grade students in Mrs. Smith's class *gave* each other *measles* (=(13)).

As mentioned by DKKMP, this sentence can be true if a few third graders got measles from people outside Mrs. Smith's class. In this case, there were different origins for the disease in the class, and the *give measles* relation describes a *collection* of directed trees on the third-graders. This interpretation illustrates a partitioning of the class into mutually disjoint sets, which is consistent with the `IAO` meaning, but not with the `IAR` meaning that the SMH derives for (21) (section 3.1). DKKMP suggest that the reciprocal in (21) indeed means `IAR`, and that the partitioning effect is a result of "vagueness in the meaning" of this sentence (Dalrymple et al. 1998, p.192). Thus, DKKMP take partitioning to be a reciprocal-independent effect. This assumption is consistent with many accounts of partitioning effects (e.g. (20)) in the literature on plurality.[18] However, in their analysis of sentence (10), restated below, DKKMP adopt a different approach to the choice between `IAR` and `IAO`.

(22) He and scores of other inmates slept on foot-wide wooden planks stacked atop each other (=(10)).

DKKMP claim that it is impossible for `IAR` to hold in (22), since "it would not be possible for scores of sleeping inmates to fit in a single stack of wooden planks" (Dalrymple et al. 1998, p.195). Acordingly, DKKMP claim that the SMH selects `IAO` as the meaning of the reciprocal in (22).

We see that DKKMP explain the partitioning effect in (21) as a vagueness effect on top of the `IAR` meaning of the reciprocal. Also with some other examples with reciprocals, DKKMP propose that vagueness plays a role in allowing partitions (Dalrymple et al. 1998, pp.177-179). However, when analyzing the partitioning effect in (22), DKKMP do not appeal to vagueness, but base their account on the `IAO` reciprocal meaning, which allows partitioning. We are not sure what the justification for this analytic discrepancy may be: reasonably, the same principles that allow partitioning through vagueness in the measles example (21) may allow it in the plank example (22) as well. We thus propose that partitioned interpretations uniformly follow from mechanisms that are external to the interpretation of the reciprocal expression. Accordingly, we adopt the following unifying principle (Sabato & Winter 2010).

Connectivity Principle: *The graph that a reciprocal interpretation describes on a set must be weakly connected (i.e. consistent with* `IAR`*).*

According to this principle, the `IAR` meaning, defined in (9), is the weakest possible meaning that is consistent with reciprocal functions in natural language. Implementing this connectivity requirement must be done on top of Definition 3 of MIH-based reciprocal functions. Thus, we adopt the following definition of *MIH-based connected reciprocity*.

[18] See Schwarzschild (1996), Winter (2000), Beck & Sauerland (2001).

Definition 5. *Let* $\Theta \subseteq \wp(E^2)$ *be a set of binary relations over* E. *The* MIH-BASED <u>*connected*</u> *reciprocal function* $\text{RECIP}_{\Theta}^{\text{MIH-C}}$ *is defined as follows for all sets* $A \subseteq E$ *and relations* $R \in \Theta$*:*

$\text{RECIP}_{\Theta}^{\text{MIH-C}}(A, R) = 1$ *iff* $\text{RECIP}_{\Theta}^{\text{MIH}}(A, R) = \texttt{IAR}(A, R) = 1$.

In words: a relation $R \in \Theta$ satisfies MIH-based connected reciprocity over a set $A \subseteq E$ with respect to Θ if $R{\downarrow}_A$ is maximal on $\Theta{\downarrow}_A$ and $R|_A$ is weakly connected.

Basing ourselves on the connectivity principle, we assume that partitioning follows from the general semantics of plurality, as implied by DKKMP's informal discussion of sentence (21) and other cases in Dalrymple et al. (1998, pp.177-179). We retain a connected interpretation of the reciprocal expression in (21), but assume that the set argument of the reciprocal function may be different than the denotation of the subject due to a partitioning mechanism independent of reciprocal quantification. For instance, consider the following analysis of sentence (21).

(23) $\forall A \in \textit{PART}(S)\, [\text{RECIP}_{\Theta}^{\text{MIH-C}}(A, R)]$, where:

S	=	the set of students in E
$\textit{PART}(S)$	=	a set of subsets of S, s.t. $\bigcup \textit{PART}(S) = S$
Θ	=	$\Theta_{\textit{give measles}} = \texttt{ACYC} \cap \texttt{FUN}^{-1}$
R	=	the binary *give measles* relation in Θ

In words: for each set A in a given partitioning of the students, the *give measles* relation describes a connected graph on A that satisfies the acyclicity and $\texttt{FUN}^{-1}$ properties, and which is a maximal graph on A that satisfies those properties.

This analysis of sentence (21) is consistent with the IAO meaning. However, the proposition $\text{RECIP}_{\Theta}^{\text{MIH-C}}(A, R)$ within it is consistent with IAR for each set A in the collection $\textit{PART}(S)$. Similarly, but unlike DKKMP's account, our analysis of the reciprocal expression in (22) is consistent with IAR, but the sentence itself is analyzed as involving an external partitioning mechanism (see section 4.3).

As DKKMP remark, when the number of elements in the subject denotation is small, partitioning of the subject becomes pragmatically unlikely.[19] For instance, DKKMP mention that in the example *those six children gave each other measles*, the sentence prefers a connected interpretation. In agreement with this empirical caveat, we summarize our informal assumptions on partitioning below.

Partitioning: *Partitioned predication over a plural argument must be pragmatically triggered. It is more likely to occur when the set that the argument denotes is relatively big.*

This approach to partitioning is shared by many works, although the exact way of implementing it remains controversial. The choice between the available semantic accounts of partitioning is not trivial and will not be addressed here. At the same time, we note that

[19]This claim may seem to be contradicted by sentence (20), which gives the impression of partitioning with a subject that denotes a small set. However, as claimed by Winter (2000), the partitioning impression in (20) is misleading, and appears due to the plurality of the object *musicals*. When this object is replaced by a singular object like *a musical*, the partitioning effect vanishes. See Winter (2000) for further discussion of this empirical point.

our assumption on the connectivity of reciprocal interpretations is an integral part of our MIH-based proposal. Consider for instance the following unacceptable sentence.

(24) #Mary, Sue and Bill are married to each other.

Assuming a ban on polygamy, a person can only be married to one other person at a time. Thus, consider a situation where Bill is married to one of the two women in (24). Such a situation describes a maximal non-polygamous marriage relation among the three individuals. Therefore, without the connectivity principle, the MIH would expect sentence (24) to be true in this situation. This expectation is problematic, since sentence (24) is clearly unacceptable in this situation. With the addition of the connectivity principle, our analysis requires that all three individuals partake in the relation, and thus expects sentence (24) to be necessarily false. This accounts for the infelicity of (24) in monogamous contexts. Similarly, the connectivity principle rules out any acceptable interpretation of the following sentence.

(25) #Mary, Sue, Bill and John are married to each other.

The unacceptability judgement in (25) is similar to the one in (24). Here again, the MIH without the connectivity principle would expect an acceptable interpretation. Furthermore, also the SMH might incorrectly expect a similarly coherent reading, using the `OWR` meaning of the reciprocal. We conclude that DKKMP's postulation of the reciprocal meaning `OWR`, and the weaker `IAO` meaning, which allow partitioned interpretations, is not empirically supported.

Let us reconsider sentence (7), restated below.

(26) "The captain", said the pirates, staring at each other in surprise (=(7)).

Sentence (26) underspecifies the number of the pirates, and therefore readily allows partitioning effects. For instance, it is possible that with eight pirates, the *stare at* relation in (26) forms two circles of four pirates each. However, this kind of partitioning is no longer readily possible in the following sentence.

(27) Mary, Sue, Bill and John are staring at each other.

The preferable interpretation of sentence (27) requires connectivity. To see that, consider sentence (27) in a partitioned situation as in Figure 3, where Mary and Sue are staring at each other, and so do Bill and John, but there is an opaque wall separating between the two pairs. In this situation the speakers we consulted hesitate to consider sentence (27) as true.

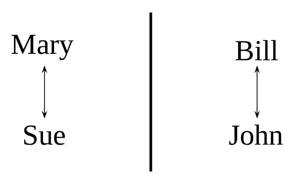

Figure 3: two *staring at* pairs separated by a wall

As argued by Winter (2000), conjunctions as in the subject of (27) do not easily license external partitioning. As a result, our connectivity principle about reciprocals expects the marked status of sentence (27) in Figure 3.

4 MIH and the logical typology of relational concepts

In section 3 we introduced the MIH as an alternative principle to the SMH, which takes the logical properties of binary relations as its only parameter when specifying reciprocal interpretations. In this section we take a closer look on the logical typology of relational concepts and its implications for the interpretation of reciprocal expressions.

4.1 Strong reciprocity with unrestricted and symmetric relations

Reconsider sentence (4), which is reproduced in (28) below.

(28) The girls *know* each other (=(4)).

We noted that the interpretation of (28) is consistent with strong reciprocity. The same holds for the following sentences, with symmetric predicates.

(29) John, Bill and Tom are *similar* to each other.

(30) a. These three paintings *are identical to* each other.
b. These three lines *run parallel to* one another.

These facts are expected by both the SMH and the MIH using natural assumptions on the relevant meaning postulates for the relational expressions in these sentences. Let us illustrate this point and elaborate on it.

As noted above, the predicate *know* in (28) shows no logical restrictions on its denotation. This lack of logical restrictions is described by assuming that the domain Θ_{know} for this predicate is the whole domain $\wp(E^2)$ of binary relations. The situation is similar with many other relational expressions, some of which are illustrated below.

(31) Relational expressions with $\Theta = \wp(E^2)$:
to know, to like, to admire, to see, to refer to, to mention, to hear, to hate, to forget, to praise, to understand, to listen to, to compliment

We say that relational expressions as in (31) have an *unrestricted* interpretation, and denote it by the assumption $\Theta = \wp(E^2)$.

Symmetry of relational expressions like *be similar to* in sentence (29) is standardly defined in (32) below using the domain SYM.

(32) $\mathsf{SYM} = \{R \subseteq E^2 : \forall x, y \in E\ [R(x,y) \rightarrow R(y,x)]\}$

In words: SYM is the set of *symmetric* relations over E.

When saying that a relational expression REL is 'symmetric', we assume that the domain Θ_{REL} for its interpretation is contained in SYM. Normally this containment is proper: most symmetric relational expressions that we considered have further restrictions on their denotations besides symmetry. For instance, consider the relational expression *be far from*. In addition to its symmetry, this expression is also *irreflexive*. Therefore the domain for its interpretation is a proper subset of SYM. Importantly for our purposes, however, further reflexivity or irreflexivity restrictions on the domains of relational expressions do not affect the SMH-based and the MIH-based analyses of reciprocals. Following the basic observation about the neutrality of reciprocal interpretations to identities (Definition 2), both the SMH-based and the MIH-based approaches properly ignore identities in denotations of relational expressions. Considering this point, we may consistently ignore identities when classifying the domains of relational expressions for the sake of studying reciprocity. For instance, instead of characterizing the domain of the expression *be far from* as the domain of all *irreflexive symmetric* relations, we only stress that this predicate satisfies $\Theta_{\textit{be far from}}{\downarrow} = \mathsf{SYM}{\downarrow}$. In words: when identity pairs are subtracted from the relations in the domain of the expression *be far from* and the domain of all symmetric relations, we get the same set of relations. Some more examples of symmetric relational expressions of this sort are given below.

(33) Relational expressions with $\Theta{\downarrow} = \mathsf{SYM}{\downarrow}$:
to be dis/similar to, to be adjacent to, to be far from, to overlap, be outside of, to be a neighbor/cousin/relative of, to have relations/contact/an affair with.

Some of these predicates, like the predicate *be far from*, are irreflexive. Others, like *be similar to*, may be reflexive. Whether any symmetric relational expressions are 'purely symmetric' with no reflexivity or irreflexivity restriction, is a question that we ignore for the purposes of this paper.[20]

The symmetric reflexive expression *be identical to* in sentence (30a) is of course also transitive, as standardly defined below.

(34) $\mathsf{TR} = \{R \subseteq E^2 : \forall x, y, z \in E\ [(R(x,y) \wedge R(y,z)) \rightarrow R(x,z)]\}$

In words: TR is the set of *transitive* relations over E.

More examples for symmetric transitive relational expressions are summarized below.

(35) Relational expressions with $\Theta{\downarrow} = (\mathsf{SYM} \cap \mathsf{TR}){\downarrow}$:

a. Sameness predicates: *be identical/equal to, be the same as*

b. Equality comparatives: *be as tall/smart as, be equally tall/smart as*

c. Kinship terms: *be sibling, brother, sister of*

d. Other predicates: *be equivalent to, run parallel to*

The predicates in (35a-b) are clearly reflexive; the kinship terms in (35c) are clearly irreflexive. The reflexivity properties, if any, of the predicates in (35d) are unclear to us.

As we saw in (28)-(30), the three types of predicates illustrated in (31), (33) and (35) are consistent with strong reciprocity.[21] The MIH captures this fact, as formally stated below.

Fact 2. *Let $\Theta \subseteq \wp(E^2)$ be a set of binary relations over E that satisfies $\Theta = E^2$, $\Theta{\downarrow} = \mathsf{SYM}{\downarrow}$ or $\Theta{\downarrow} = (\mathsf{SYM} \cap \mathsf{TR}){\downarrow}$. The MIH-based reciprocal function $\textsc{recip}^{\textsc{mih}}_{\Theta}$ is consistent with the* SR *meaning over E.*

When $\Theta = E^2$, the interpretation $\textsc{recip}^{\textsc{mih}}_{\Theta}$ is a total function, hence it is furthermore identical to the meaning SR.

[20]The relational expressions *have relations with* or *have contact with* may be examples for such purely symmetric relations. It is possible that sentences like *John has relations with himself* is contingent. For some intricacies concerning the possibly collective interpretation of sentences like (11), which contains this relation, see some remarks in section 5.2.

[21]Another class of symmetric relational expressions that lead to SR readings of reciprocals are expressions like *unequal to, different than, inequivalent to* or *unparallel to*, which are further restricted in having a transitive complement (cf. (35)).

A property similar to Fact 2 also holds for the SMH. Since $\mathtt{SR}(A, R)$ is contingent for the three Θ domains in Fact 2, the SMH also expects SR to be the realized reciprocal meaning in sentences like (28)-(30).[22] We conclude that for the most common types of strong reciprocity, the MIH and the SMH agree with each other and with the facts.[23]

4.2 Functional relational expressions

A simple distinction between the SMH and the MIH is observed in the analysis of DKKMP's example (26), restated below.

(36) "The captain", said the pirates, staring at each other in surprise (=(26)).

The relational expression *stare at* is quite special among the natural language predicates that we have examined, in having the set of *partial functions* as its entire interpretation domain. As DKKMP mention, a person is likely to stare at only one object at a time.[24] The definition of this relational domain follows.

(37) $\mathtt{FUN} = \{R \subseteq E^2 : \forall x, y_1, y_2 \in E\ [(R(x, y_1) \wedge R(x, y_2)) \rightarrow y_1 = y_2]\}$

In words: FUN is the set of relations over E that describe a *function* on their first argument, possibly a partial one.

Dalrymple et al. (1998, p.196) note that, given the FUN restriction on the domain of the relation *stare at*, the SMH expects the meaning of the reciprocal in (36) to be IR (intermediate reciprocity). This reciprocal meaning requires that the *stare at* graph is strongly connected, i.e. there is a directed path between any two different pirates in (36). Such strong connectivity can only be realized with a functional relation if the graph that it describes is a directed circle. This interpretation is stronger than what is intuitively required in sentence (36), which is true as long as each pirate stares at some pirate or another. Thus, interpreting sentence (36) is an open challenge for the SMH.

The MIH-based analysis does not face this problem. According to our analysis, any functional relation denoted by the expression *stare at* that is maximal on the set of pirates, is expected to lead to an acceptable interpretation of sentence (36). Such maximal interpretations agree with DKKMP's claim that sentence (36) is consistent with the OWR meaning, which requires an outgoing edge from each node. This is stated in the following fact.

Fact 3. *Let* FUN *be the set of functional binary relations over E. The MIH-based reciprocal function* $\mathrm{RECIP}^{\mathtt{MIH}}_{\mathtt{FUN}}$ *is consistent with the* OWR *meaning over E.*

[22] Unlike DKKMP, we assume here that the only 'contextual information' relevant for the SMH is the domain Θ of the relational expression. As claimed in sections 3.1 and 3.2, without this assumption it is hard to formally evaluate the SMH.

[23] In one case the speaker judgements we got on reciprocity with symmetric predicates were mixed. This involves sentences like *Mary, Sue and Jane are cousins of each other*. Some speakers consider this sentence as possibly true if Mary and Sue, as well as Sue and Jane, are first cousins, but Mary and Jane are only second cousins. We believe that this possibility reflects strong reciprocity with some vagueness of the relation *cousin*. First, the sentence *Mary, Sue and Jane are first cousins of each other* is false in this situation, as far as we were able to check. Second, as we shall see in section 4.3, many other kinship terms clearly do not allow reciprocal interpretations that are weaker than SR.

[24] The object that is stared at may be composed of smaller objects. As a result, one may also stare at a group of people. This brings up some of the issues discussed in section 3.3, but it is does not affect too much the relevant interpretation of sentence (36).

As we proposed in section 3.3, reciprocals require weak connectivity. This is also expected to be the case in sentences like (27) or (36) (=(26)). With the connectivity principle, the MIH (definition 5) expects the reciprocal interpretation with functional relations to be consistent with the reciprocal meaning $\mathtt{OWR} \cap \mathtt{IAR}$. This meaning is stronger than both $\mathtt{OWR}$ and $\mathtt{IAR}$, but weaker than the strong connectivity meaning $\mathtt{IR}$ that is expected by the SMH.

DKKMP give another example for a functional relational expression, using the following example.[25]

(38) The children followed each other around the Maypole.

The relation *follow around the Maypole* is likely to be interpreted functionally, because it is hard to directly follow two or more people around a Maypole.[26] Similarly, *follow around* is likely to have the $\mathtt{FUN}^{-1}$ property: it is hard for two or more people to directly follow another person around the Maypole, unless they act as as a group (see footnote 26). Another transitive verb that behaves similarly to *follow* in this respect is the verb *chase*. The net result of the two requirements $\mathtt{FUN}$ and $\mathtt{FUN}^{-1}$ is that the MIH expects the relation in sentence (38) to describe a circular graph over the children, which is consistent with the $\mathtt{IR}$ reciprocal meaning. With external partitioning, sentence (38) can be true if the children were divided into some subgroups, where each subgroup forms a circle of children around the Maypole.

4.3 Asymmetry (1) – intransitive relational expressions

In section 3.3 we analyzed sentence (21), with the acyclic relational expression *give measles*. Logically, the class of acyclic relations is a proper subset of the larger class of *asymmetric* relations, as standardly defined below.

(39) $\mathtt{ASYM} = \{R \subseteq E^2 : \forall x, y \in E\, [R(x,y) \rightarrow \neg R(y,x)]\}$

In words: $\mathtt{ASYM}$ is the set of *asymmetric* relations over E.

Many of the asymmetric relations in natural language are also transitive. By definition of asymmetry and transitivity, these relations are also acyclic. By contrast, due to its $\mathtt{FUN}^{-1}$ property, the acyclic relational meaning of the expression *give measles* is *intransitive* in the following sense.

(40) $\mathtt{INTR} = \{R \subseteq E^2 : \forall x, y, z \in E\, [(R(x,y) \wedge R(y,z)) \rightarrow \neg R(x,z)]\}$

In words: $\mathtt{INTR}$ is the set of *intransitive* relations over E.

All asymmetric relational expressions that we are aware of are either transitive or intransitive. Before moving on to the big class of transitive asymmetric relations in natural language, which will be discussed in section 4.4, let us first consider some more intransitive

[25] As mentioned by DKKMP (p.194), the predicate *follow* in sentence (38) is quite hard to classify semantically when appearing without modifier or a very specific context. Specifically, it is often unclear if specific uses of *follow* are transitive, or whether they mean *indirectly follow*. And similarly for possible acyclic usages of *follow*, as in *the boys are following each other*. For this reason we only concentrate in this paper on modified occurrences of this verb, as in sentences (38) and (46) below. Other relations expressions similar to the verb *follow* in the relevant respect are *to precede, be predecessor of, to succeed* and *be successor of*.

[26] The children in (38) may have been following each other in pairs, for instance. This sort of "group partitioning" involves collective individuals (e.g. pairs) as the units of predication, which is rather independent of the problem of reciprocity.

relations like *give measles to*, and their interactions with reciprocity. All intransitive asymmetric relational expressions known to us satisfy both acyclicity and the $\mathtt{FUN}$ or $\mathtt{FUN}^{-1}$ properties.[27] In (41) below we summarize the three classes of asymmetric intransitive relational expressions that we found. Note that by asymmetry, all these relations are irreflexive, hence their domain Θ is characterized without our habit of ignoring identities.

(41) Intransitive asymmetric relational expressions:

a. $\Theta = \mathtt{ACYC} \cap \mathtt{FUN}^{-1}$:
give measles to, bury, be mother of, give birth to, procreate

b. $\Theta = \mathtt{ACYC} \cap \mathtt{FUN}$:
get measles from, be buried by, be born to

c. $\Theta = \mathtt{ACYC} \cap \mathtt{FUN} \cap \mathtt{FUN}^{-1}$:
be stacked atop, follow into the treehouse, inherit the shop from, bequeath the shop to

Let us now consider the behavior of these relational expressions with reciprocals. Beck (2001) mentions the following reciprocal sentence, with the verb *bury*.

(42) The settlers have buried each other on this hillside for centuries.

Like the predicate *give measles*, the verb *bury* is acyclic and has the $\mathtt{FUN}^{-1}$ property, since a person is only likely to be buried once. Indeed, similarly to sentence (21), sentence (42) can be interpreted as true when the relation *bury* describes a collection of directed trees on the set of settlers, which is analyzed in (23) using the MIH and external partitioning.

The relational expressions *get measles from*/*be given measles by* and *be buried by* are the inverse relations of *give measles to* and *bury*. Therefore they are acyclic and functional. As a result, when they combine with a reciprocal expression, the MIH expects these relations to describe a directed graph with a unique 'sink' s: a node that has a unique directed path from any other root. This requirement is symmetric to the requirement of path from the root that with $\mathtt{FUN}^{-1}$ acyclic relations. Thus, MIH-based interpretations with acyclic functional relations are inverse relations of directed trees (arborescences, see footnote 12). Such interpretations are consistent with the following meaning, which is the correlate of the meaning $\mathtt{ROOT}$ in (17).

(43) $\mathtt{SINK}(A,R) = 1 \Leftrightarrow$
$\exists s \in A\ \forall x \in A\ [x \neq s \rightarrow \exists m \exists z_0, \ldots, z_m \in A\ [x = z_0 \wedge s = z_m \wedge R(z_0, z_1) \wedge \ldots \wedge R(z_{m-1}, z_m)]]$

In words: R describes a graph on A with at least one sink s – a node that has a directed path from every other node.

Together with our assumptions on external partitioning (section 3.3), the MIH expects acyclic functional relations to lead to reciprocal interpretations describing collections of "arborescence inverses". This expectation agrees with speaker intuitions on reciprocal sentences with the relational expressions *get measles from*/*be given measles by* and *be buried by*.

[27] In the sentence *the bricks are laid on top of each other*, the acyclic relation *be laid on top of* seems an exception to this rule. This relational expression does not seem to satisfy either $\mathtt{FUN}$ or $\mathtt{FUN}^{-1}$, since a brick may have more than one brick laid on top or below it. However, the collective interpretation of the predicate complicates the analysis in this case (cf. section 5.2).

Other acyclic relational concepts that have the $\texttt{FUN}^{-1}$ property are the kinship relations *be mother of*, *give birth to* and *procreate*. The kinship relations *be given birth by* or *be born to*, which are inverses of *give birth to*, are therefore acyclic and functional. With most kinship relations of this kind, reciprocals are unacceptable, as in the following sentences.

(44) #These women are each other's mother(s).
#These women are mothers of one another.
#These women gave birth to each other.
#These women were born to one another.

Both the SMH and the MIH incorrectly expect sentences as in (44) to be acceptable. We have no general explanation to offer here for their unacceptability, but see section 4.4 for some more remarks on this problem and attempts to solve it within current theories of reciprocity.

Consider next the predicates *stacked atop* and *follow into the treehouse*, as they appear in examples (45) and (46) by DKKMP.

(45) He and scores of other inmates slept on foot-wide wooden planks stacked atop each other (=(22)).

(46) The children followed each other into the treehouse.

Like *give measles to*, these two relational expressions are clearly acyclic.[28] These relations are also likely to be interpreted as having the $\texttt{FUN}^{-1}$ property: it is hard to directly stack more than one wooden-plank atop another one or to have two or more people directly following another person into a treehouse (entrances of treehouses are normally too small for that). In addition, these relations are often interpreted as functional: it is hard to directly stack a wooden plank atop more than one other wooden plank, or to directly follow two or more people into a treehouse (cf. footnote 26). Because of their acyclicity, $\texttt{FUN}$ and $\texttt{FUN}^{-1}$ properties, the MIH expects the graphs in sentences (45) and (46) to describe simple directed paths. This interpretation is in agreement with speaker intuitions, and consistent with the $\texttt{IAR}$ meaning of weak connectivity. In addition, speakers can also interpret the sentence as supported by a collection of such path graphs, which is consistent with our assumptions in section 3.3 on the partitioning mechanism with plurals. Consider for instance the following partitioned analysis of sentence (45).

(47) $\forall A \in \mathit{PART}(S)\,[\mathrm{RECIP}_{\Theta}^{\text{MIH-C}}(A,R)]$, where:

$$
\begin{array}{lcl}
S & = & \text{the set of planks in } E \\
\mathit{PART}(S) & = & \text{a set of subsets of } S \text{, s.t. } \bigcup \mathit{PART}(S) = S \\
\Theta & = & \Theta_{\textit{stacked atop}} = \texttt{FUN} \cap \texttt{ACYC} \cap \texttt{FUN}^{-1} \\
R & = & \text{the } \textit{stacked atop} \text{ relation in } \Theta
\end{array}
$$

By its definition, the relation $\mathrm{RECIP}_{\Theta}^{\text{MIH-C}}(A,R)$ requires R to describe a directed path on each set A in the partition of the set S. This interpretation of the sentence is consistent with DKKMP's $\texttt{IAO}$ meaning, similarly to our analysis (23) of sentence (21) above.

The following reciprocal sentence, with the asymmetric verb *inherit from*, is another example from Beck (2001).

[28] Note that acyclicity is a property of the complex relational expression *follow into NP*. As we saw in section 4.2, in other cases with the verb *follow*, acyclicity is not guaranteed.

(48) The members of this family have inherited the shop from each other for generations.

The relation *inherit the shop from* is acyclic. In addition it is likely to be interpreted as both FUN and FUN^{-1}, since a shop can only be inherited from one person, or one group of people, and the inherited shop can only go to one person or one group of people. Indeed, sentence (48), similarly to sentences (45) and (46), is interpreted as true when the inheritance relation forms a directed path on the family members or groups thereof. This is a relatively simple way in which reciprocals can apply with potentially collective predicates like *inherit from*. For more complex cases of collectivity and reciprocity, see section 5.2.[29]

4.4 Asymmetry (2) – transitive relational expressions

As we mentioned above, many of the asymmetric relations in natural language are also transitive. Thus, such predicates denote *strict partial orderings* (SPOs).[30] Due to their transitivity, such asymmetric orders are acyclic. Some of the SPO relational concepts are clearly *not total*.[31] For instance, consider the asymmetric transitive relation *be ancestor of*, which obviously does not hold of many pairs of non-identical entities. Similarly, the prepositions *in* and *inside* and the verb *contain* (in its spatial sense) denote SPOs that are not total on their domains. Another important subclass of SPO relations are *comparative expressions*, most notably comparative adjectival constructions such as *be taller than* and verbs of comparison like *outrate* or *exceed*. These SPO relations are not total as well.[32] For instance, there may be many pairs of distinct entities x and y of the same height, so that neither *x is taller than y* nor *y is taller than x* hold. However, such comparative relational concepts are "almost total", because they do not distinguish entities that they render incomparable. For instance, if John is not taller than Mary and Mary is not taller than John, there can be no entity that is taller than John but not taller than Mary, and vice versa (van Rooij 2010). The domain of "almost total" relations is defined below.

(49) $$\text{ATOT} = \{R \subseteq E^2 : \\ \forall x, y \in E\, [(\neg R(x,y) \wedge \neg R(y,x)) \rightarrow \\ \forall z \in E\, ((R(x,z) \leftrightarrow R(y,z)) \wedge (R(z,x) \leftrightarrow R(z,y)))]\,\}$$

[29] Beck (2001) also considers the unacceptability of the following sentences.

(i) #These three settlers have buried each other on this hillside.

(ii) #These three members of the family have inherited the shop from each other.

We do not have an account of the contrasts (42)-(i) and (48)-(ii), and we refer the reader to Beck (2001) and Mari (2006) for relevant discussion.

[30] A relation R is *antisymmetric* iff $R(x,y)$ and $R(y,x)$ entail $x = y$. An antisymmetric, transitive and reflexive relation is a (non-strict) *PO*. If R is a (non-strict) PO then $R - I$ is an SPO. Conversely, if R is an SPO and $I' \subseteq I$ is a (non-empty) set of identity pairs, then $R \cup I'$ is a (non-strict) PO. As mentioned below, some of the SPO (hence asymmetric) relational expressions have non-strict (hence non-asymmetric) correlates.

[31] A (non-strict) PO is *total* if for all x and y: $R(x,y)$ or $R(y,x)$ (or both) hold. An SPO R is total if for all x and y: $R(x,y)$, $R(y,x)$ or $x = y$. Thus, similarly to footnote 30, we can move back and forth between a total SPO and a total (non-strict) PO by subtracting/unioning the identity pairs. The notion of *total relation* should not be confused with the notion of "total functions" that we have used above to distinguish standard $\langle 1, 2 \rangle$ quantifiers from our use of "partial" quantifiers.

[32] In certain usages of comparatives they may not even seem asymmetric, as in *John outrates Mary (in swimming) and Mary outrates John (in running)* or *John is quicker than Mary (in swimming) and Mary is quicker than John (in running)*. For the sake of our discussion here, we ignore such qualified uses of comparatives, and tentatively assume their asymmetry. For more relevant examples see appendix A.

In words: ATOT is the set of relations over E that do not distinguish between elements that they leave incomparable.

The ATOT property follows from the natural assumption that dimensional adjectives like *tall* and their comparative forms are associated with a totally ordered set of *degrees*, in this case height degrees. We refer to SPOs that have the ATOT property as *strict weak orderings* (SWOs).[33] In addition to comparative expressions, some spatial and temporal prepositions like *be above*, *below*, *before* and *after* also behave in many contexts as "almost total", similarly to comparatives.[34]

The two order-based classes of relational expressions are summarized below.

(50) Strict partial-order (SPO) relational expressions – $\Theta = \mathtt{ASYM} \cap \mathtt{TR}$:

a. Kinship relations: *be ancestor*/*descendant of*, *descend from*

b. Some spatial relations: *be in*/*inside*, *to contain*, *to be contained in*

(51) Strict weak-order (SWO) relational expressions – $\Theta = \mathtt{ASYM} \cap \mathtt{TR} \cap \mathtt{ATOT}$:

a. Inequality comparative adjectives: *be taller*/*smarter than*, *be less tall*/*less smart than*

b. Comparative verbs: *outdo, outperform, outrank*, *outrate, outreach, outnumber, outrun, excel, exceed, surpass*

c. Comparative nouns: *be senior*/*junior of*

d. "Pointal" usages of some spatial and temporal terms: *be above*/*below*/*before*/*after*, *antecede*, *be antecedent of*

Some of these relational expressions give rise to odd sentences when appearing with reciprocals, like the following examples from Mari (2006) (see also Beck & von Stechow 2007).

(52) #The two trees are taller than each other.

(53) #The two sets outnumber each other.

These examples involve SWO relations and are clearly unacceptable. However, it would be too hasty to conclude that all SPO and SWO predicates resist appearance in reciprocal

[33] For an SPO R, a requirement equivalent to the ATOT property is the requirement that R be *almost connected*: $\forall x, y[R(x,y) \to \forall z(R(x,z) \vee R(z,y))]$. Still equivalently, an SPO R is an SWO if the relation $\neg R(x,y) \wedge \neg R(y,x)$ is transitive. These equivalent definitions all boil down to assuming an order-preserving mapping from the set of entities to a totally ordered set. Thus, for any non-empty set E and function $f : E \to D$, we assume $x <_E y$ iff $f(x) <_D f(y)$. If $<_D$ is a total SPO on D, then $<_E$ is an SWO on E. Conversely, if $<_E$ is an SWO on E, then there is a set D (of cardinality $|D| \leq |E|$) and a function $f : E \to D$, s.t. D is totally ordered by $<_D$. Thus, by using a totally ordered set of degrees, we can define the domain of comparative relations over entities without appealing to the ATOT property or to SWOs. See Kennedy (1999) and references therein for degree-based works on the semantics of adjectives and their comparative forms. Degrees are only implicitly assumed in *vagueness-based* approaches to comparatives such as Klein (1980). Here we remain neutral between these theoretical assumptions on adjectives, as the characterization of comparatives as SWOs is sufficient for our purposes.

[34] In some contexts totality is relaxed with these four prepositions. For instance, a bird B that is flying alongside a plane P may fail to be either *above* or *below* P, but it may be questioned whether the altitudes of B and P are indistinguishable: some other bird B' may fly above or below B, but, just like B, fail to be in either the *above* or *below* relation to P. Still, in many contexts these prepositions treat the spatial or temporal location of objects as *points* (Zwarts & Winter 2000), in which case they behave like comparatives.

sentences. Quote (54) from a book by Charles Darwin uses a reciprocal with the SPO relation *descend from* to describe an evolutional hypothesis. The text in (55) describes behaviors of stock exchanges using a reciprocal sentence with the SWO verb *outperform*, or perhaps the compound *outperform as expected*, which in the given context is reasonably an SPO.

(54) The simplest answer seemed to be that the inhabitants of the several islands *had descended from each other*, undergoing modification in the course of their descent.
Charles Darwin, *The Variation of Animals and Plants Under Domestication*, Vol. 1. Kessinger Publishing, 2009, page 10

(55) To counter this theory, Greenblatt divided the stock universe (in his study) into deciles. He found that the deciles *outperformed each other* exactly as expected. In other words, the 4th ranked decile outperformed the 5th ranked decile, the 5th ranked decile outperformed the 6th ranked decile etc.
`http://seekingalpha.com/article/167120-the-little-book-that-beats-the-market-chapters-1-7`
(retrieved January 2011)

Appendix A shows more data retrieved from the internet concerning SPO and SWO predicates that appear in reciprocal sentences.

The variation in acceptability between cases like (52)-(53) and cases like (54)-(55) does not exhaust the interpretational effects in reciprocal sentences containing asymmetric predicates. In many cases, reciprocals sanction a non-asymmetric interpretation of the predicate, overriding its usual asymmetric meaning. Consider for instance the following example.

(56) As usual our politicians have *outperformed each other* with facts and figures about what a marvellous country we live in (or lack thereof) and how they are going to make Sri Lanka even better place to live in.
`http://perambara.org/featured/2010/05/`
`putting-entrepreneurship-at-the-heart-of-economic-revival-in-the-north-east-and-beyond`
(retrieved January 2011)

In sentence (56), unlike sentence (55), the verb *outperform* is interpreted as non-asymmetric, and the reciprocal is interpreted as consistent with strong reciprocity, entailing that *every politician outperforms every other politician*.

Let us summarize the three effects that we have seen when reciprocals appear with asymmetric predicates:

A. The sentence is interpreted using SR and the predicate retains its asymmetry, which leads to semantic/pragmatic infelicity: (44), (52), (53), footnote 29.

B. The sentence receives an interpretation weaker than SR, consistent with the asymmetry of the predicate: (21), (48), (42), (45), (46), (54), (55).

C. The sentence is interpreted using SR but the predicate's interpretation is weaker than its standard asymmetric meaning: (56).

These effects illustrate three different strategies that language uses to handle the conflict between strong reciprocity and asymmetric relational expressions: leaving the conflict unresolved (A), or weakening the interpretation of one of the expressions (B,C). Both the

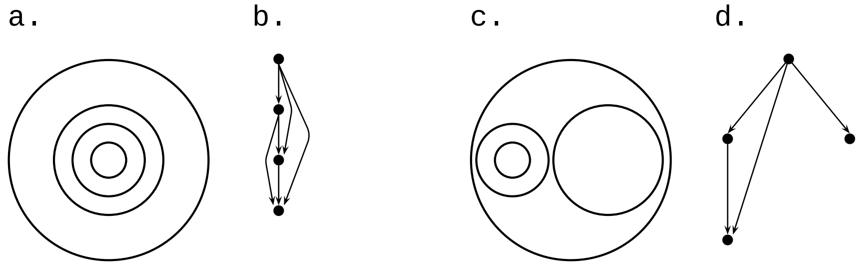

Figure 4: containment in transitively closed directed path and tree

SMH and the MIH are specifically designed to account for strategy B, in which SR is replaced by a weaker reciprocal interpretation. Cases of unresolved interpretational conflicts (A) or where the predicate "ironically" changes its normal meaning (C) are not treated here, and require further study. We refer the reader to Beck (2001), Beck & von Stechow (2007), Mari (2006), Dotlačil & Nilsen (2008) for works that attempt to account for this variation.

When SPO relational expressions are standardly interpreted with reciprocals, some differences appear between the expectations of the SMH and the MIH. Consider for instance the following example with the SPO verb *contain*.

(57) The four circles contain each other.

Sentence (57), when acceptable, most readily describes a linear containment situation as in Figure 4a, similarly to the situation described in example (55), with the SWO verb *outperform*.[35] Because of the transitivity of the *contain* relation, the graph described by the containment Figure 4a is a *transitive closure of a path*, as described in Figure 4b. By contrast, a situation as in Figure 4c, where the *contain* relation does not describe such a graph (cf. Figure 4d), is hardly acceptable for sentence (57).

This difference between the acceptability of sentence (57) in Figures 4a and 4c is not accounted for by the SMH. The IAR meaning (weak connectivity) is the strongest reciprocal meaning in DKKMP's proposal that is consistent with SPO relations like *contain*, and this meaning leads to a true interpretation of (57) in both Figures 4a and 4c. By contrast, the MIH expects a difference between these two situations for sentence (57). This is because the graph in Figure 4b is a maximal situation for an SPO relation whereas the graph in Figure 4d is not. As a result, the MIH rules out the situation in Figure 4c for sentence (57), but accepts the situation in Figure 4a. A reciprocal meaning consistent with this interpretation of sentence (57) is the following meaning, which we call TPR, for *transitive path reciprocity*.

(58) *Transitive Path Reciprocity*:
TPR$(A, R) = 1 \Leftrightarrow$
there is an indexing $\{x_1, \ldots, x_n\}$ of A s.t. $\forall i, j \in [1..n]\ [i < j \rightarrow R(x_i, x_j)]$

In words: the graph that R describes on A contains a transitive closure of a directed path passing through all of its nodes.

The fact that we have observed above is formally summarized as follows.

[35] For some reciprocal examples from the internet with the verb *contain*, see appendix A.

Fact 4. *Let* $\mathtt{SPO} = \mathtt{ASYM} \cap \mathtt{TR}$ *be the set of strict partial orders over* E*. The MIH-based reciprocal function* $\text{RECIP}^{\text{MIH}}_{\mathtt{SPO}}$ *is consistent with the* TPR *meaning over* E*.*

Among the five classes of asymmetric relations that we have considered in (41), (50) and (51), only SPO relations like *contain* show a distinction between the interpretations expected by the SMH and the MIH. For acyclic relations with one of the properties $\mathtt{FUN}^{-1}$ or FUN, like the relations *give measles to* and *get measles from*, both the SMH and MIH expect a *directed tree* interpretation, consistent with IAR. For acyclic relations with both properties $\mathtt{FUN}^{-1}$ and FUN, like the relation *be stacked atop*, both the SMH and MIH expect a *directed path* interpretation, which for those predicates is consistent with IAR. For SWO relations like *outperform*, both the SMH and the MIH expect an interpretation that describes a transitive closure of a directed path, which for such SWO predicates is consistent with both IAR and TPR. See Table 1 for a summary of these facts.

Concluding remarks on asymmetry Asymmetric relational expressions introduce a remarkable challenge for theories of reciprocity. On the one hand, as we have seen, asymmetric relational concepts may be compatible with reciprocal expressions and lead to reciprocal interpretations weaker than SR. This fact is expected by both the SMH analysis and the MIH analysis, which only differ in their treatment of SPO asymmetric relations. However, with many of the asymmetric relational concepts, reciprocals are unacceptable, which is unexpected by either the SMH or the MIH. Below we summarize some of the factors that we believe affect this unacceptability.

1. *Temporal/modal effects*. Some examples, like (91) and (92) in Appendix A, require asymmetry in each given point in time, or in each given situation, but also strong reciprocity when considering the whole temporal/modal context as a whole. This interesting complex combination of strong reciprocity with temporality/modality and asymmetry has been extensively addressed by Alda Mari (Mari 2006 and further unpublished work). However, at present we are not sure that the restrictions on such effects are fully specified. See some remarks in appendix A.5.

2. *Pragmatic weakening*. This is the possibility illustrated in (56), of "ironically" extending the domain of typically asymmetric relational concepts to also include non-asymmetric relations. The pragmatic principles underlying such atypical interpretations are currently ill-understood.

3. *The SPO/SWO distinction*. In some cases, such as (55) above, an SPO relation (*outperformed as expected*) seems more acceptable with reciprocals than a corresponding SWO (*outperformed*). One possible reason for this alternation may be that the combination of an SWO relation with a reciprocal should result, according to both the SMH and the MIH, in a statement that is "almost tautological". For instance, according to the SMH and the MIH, a sentence like *Mary and John outperform each other* can only be true if Mary's and John's performances are not of equal excellence. The simplicity of this claim may be a pragmatic reason for blocking its complex semantic derivation and preferring an SR reading of the reciprocal with a weakening of the semantic restrictions on the predicate.

Given these complexities, we believe that the behavior of asymmetric relational expressions with reciprocal requires more in-depth research, with more general formal hypotheses on the factors that affect their interpretation.

4.5 A note on total preorders

Many SWO comparative expressions have natural reflexive (hence not asymmetric) correlates. For instance, the *equative* comparative expression *be at most as tall as* denotes the complement of the SWO comparative *be taller than*, whereas the equative *be at least as tall as* denotes the complement of the SWO comparative *be less tall than*. These equative expressions (Rett 2011) denote reflexive transitive relations, or *preorders*. Furthermore they denote *total preorders*: for instance, for every two entities x, y that have any height, x is at least as tall as y or y is at least as tall as x (or both). In this paper we do not further discuss total preorder expressions because as far as we know, their behavior with reciprocals is as recalcitrant as that of their correlate comparative forms. For instance, we agree with Langendoen (1978) and DKKMP about the oddity of examples like *they are at least as heavy as one another*. As with other comparatives, accounting for this unacceptability is an open challenge for theories of reciprocals.

4.6 Maximal patient/agent cardinality

In section 4.2 we have seen a couple of relational concepts with the $\mathtt{FUN}$ and $\mathtt{FUN}^{-1}$ properties. These properties require that the maximal number of patients per agent ($\mathtt{FUN}$) or agents per patient ($\mathtt{FUN}^{-1}$) be one. These requirements are generalized in the following relational domains, which we call *maximal patient cardinality* ($\mathtt{MPC}$) and *maximal agent cardinality* ($\mathtt{MAC}$).

(59) $\mathtt{MPC}_n \ = \ \{R \subseteq E^2 : \forall x \in E\ [|\{y \in E : R(x,y)\}| \leq n]\}$

In words: $\mathtt{MPC}_n$ is the set of relations over E that map each agent to at most n patients.

(60) $\mathtt{MAC}_n \ = \ \{R \subseteq E^2 : \forall y \in E\ [|\{x \in E : R(x,y)\}| \leq n]\}$

In words: $\mathtt{MAC}_n$ is the set of relations over E that map each patient to at most n agents.

For the set of relations $\mathtt{FUN}$ and $\mathtt{FUN}^{-1}$ we have: $\mathtt{FUN} = \mathtt{MPC}_1$ and $\mathtt{FUN}^{-1} = \mathtt{MAC}_1$.

Symmetric predicates that have one of the properties $\mathtt{MPC}_n$ or $\mathtt{MAC}_n$, also have the other property (with the same n). In section 3.3 we considered the behavior of the symmetric $\mathtt{FUN}$ and $\mathtt{FUN}^{-1}$ predicate *be married to* in reciprocal sentences. Whenever the denotation of a noun phrase *NP* includes more than two entities, the MIH expects reciprocal sentences of the form *NP are married to each other* to be interpreted using graphs that are not connected. When adding the connectivity requirement ($\mathtt{IAR}$) to the MIH, this explains the unacceptability of such sentences in cases that do not allow external partitioning (cf. section 3.3). A similar predicate is the relational expression *look into the eyes of*. Like the relation *stare at*, this relation is functional, and like the relation *be married to*, it is symmetric. Consequently, the expectations of the MIH is that the reciprocal sentences with the predicate *look into the eyes* behave similarly to sentences with the predicate *be married*. The expectation is borne out, as observed by comparing the following sentences to sentences (19a), (24) and (25) respectively.

(61) In this picture, Mary and John, and Sue and Bill, are looking into each other's eyes.

(62) #In this picture, Mary, Sue and Bill are looking into each other's eyes.

(63) #In this picture, Mary, Sue, Bill and John are looking into each other's eyes.

Sentence (61) is acceptable, but relies on an partition of the subject denotations into two couples. This is much harder in (63). In sentence (62), furthermore, no external partitioning can make the sentence true. These facts are expected by the MIH and our connectivity and partitioning principles of section 3.3.

A slightly more interesting class of symmetric predicates are relational expressions like *sit alongside* or *hold/shake hands with*. Because people have two sides and two hands, these symmetric expressions also have the $\texttt{MPC}_2$ and $\texttt{MAC}_2$ properties. Consider now the following reciprocal sentences.

(64) The five pitchers are sitting alongside each other. (cf. (8))

(65) The five pitchers are holding hands with each other.

Sentence (64), like DKKMP's example (8), is true when the pitchers are sitting in a circle, or when they are sitting in a line. Similarly, sentence (65) can be true when the pitchers' hands close a circle, but also when they only form a line. DKKMP's SMH allows both possibilities, since the $\texttt{IR}$ meaning, which requires strong connectivity, is the strongest reading in DKKMP's proposal that is consistent with the $\texttt{SYM}$ and $\texttt{MPC}_2$ (or $\texttt{MAC}_2$) properties of the predicates. This meaning allows both linear and circular configurations. By contrast, the MIH only expects circular configurations to support sentences like (64) and (65), consistent with the following reciprocal meaning.

(66) $\texttt{CIRC}(A,R) = 1 \Leftrightarrow$
there is an indexing $\{x_1,\ldots,x_n\}$ of A s.t. $R(x_1,x_2)\wedge\ldots\wedge R(x_{n-1},x_n)\wedge R(x_n,x_1)$

In words: the graph that R describes on A contains a circle passing through all of its nodes.

This incorrect behavior of the MIH appears because the circular configuration, but not the linear configuration, is maximal relative to $\texttt{SYM}$ and $\texttt{MPC}_2$ (or $\texttt{MAC}_2$). Thus, in this case the SMH describes the facts better than the MIH.[36]

Another class of relational expressions that put cardinality restrictions on patients or agents are asymmetric predicates like *tie up* or *handcuff*. A person tying up another person is normally not being tied up himself at the same time, nor can he be tying up another person simultaneously. Thus, each entity may be assumed to participate in the relation only once, as either agent or patient. Formally, this is the following requirement on a relation R.

[36]This problem for the MIH is currently studied experimentally, by checking subjects' judgements on reciprocal sentences with various predicates in circular and linear configurations (E. Poortman, master thesis in prep., Utrecht University). In this work it is hypothesized that background knowledge about a geometrical configuration may prime a proper subset of the reciprocal interpretations that the MIH considers. For instance, as DKKMP (p.195) point out, the distances allowed between the locations in the following example may depend on contextual knowledge about the geometrical path that the inspector might have formed in his search.
(i) The inspector found peach fruit flies at four different locations within a mile of each other.

(67) $\forall x \in E\ [\,|\{y \in E : R(x,y)\}| + |\{y \in E : R(y,x)\}| \leq 1]$

This requirement, similarly to the predicates *be married to* or *look into the eyes*, does not allow reciprocal sentences with more than two agents to be interpreted without partitioning. This is expected by both the SMH and the MIH. What is not expected (by both principles) is the unacceptability of sentences like #*the two policemen are handcuffing each other* (cf. footnote 29).

4.7 Summary

Table 1 summarizes the main classes of relational expressions that we have characterized, with the expectations of the MIH regarding their (connected) interpretations. For each relational expression, the domain of interpretation is specified by the '+' signs, marking sets of binary relations. The actual domain of the relational expression, ignoring identities, is the intersection of these sets. For instance, the domain Θ for the relational expression *follow around the Maypole* (cf. section 4.2) satisfies: $\Theta{\downarrow} = (\mathtt{FUN} \cap \mathtt{FUN}^{-1}){\downarrow}$.

Relational expression		**Domain of interpretation**[*1] SYM	ASYM	TR	INTR	ACYC	FUN	FUN^{-1}	other	MIH-C	Graph
know, like, see	(31)	–	–	–	–	–	–	–	–	SR	complete
similar to, cousin of	(33)	+	–	–	–	–	–	–	–	SR	complete
equal to, as tall as, sibling of	(35)	+	–	+	–	–	–	–	–	SR	complete
stare at		–	–	–	–	–	+	–	–	OWR∩IAR[*2]	con.+out.e.[*3]
follow around Maypole		–	–	–	–	–	+	+	–	IR/CIRC	circular
sit alongside, hold hands of		+	–	–	–	–	–	–	+[*4]	CIRC[*5]	circular
give measles to	(41a)	–	(+)	–	(+)	+	–	+	–	IAR/ROOT	dir.tree
get measles from	(41b)	–	(+)	–	(+)	+	+	–	–	IAR/SINK	dir.tree
stacked atop, follow into house	(41c)	–	(+)	–	(+)	+	+	+	–	IAR	dir.path
descend from, contain	(50)	–	+	+	–	(+)	–	–	–	TPR[*6]	tr.clos.path
taller than, outrank	(51)	–	+	+	–	(+)	–	–	+[*7]	IAR/TPR	tr.clos.path
be married to, look into eyes of		+	–	–	–	–	+	(+)	–	IAR[*8]	pairs

Legend: MIH-C meaning consistent with connected MIH-based interpretation; partitions external-only
(+) property is entailed by other properties
*1 the specification of the domain ignores identities (see sections 4.1, 4.7)
*2 the SMH incorrectly expects the IR meaning in this case (see section 4.2)
*3 a weakly connected graph where each node has an outgoing edge (see section 4.2)
*4 these symmetric relations furthermore have the $\mathtt{MPC}_2$ ($\mathtt{MAC}_2$) properties
*5 incorrectly, unlike the SMH, the MIH only expects circular interpretations (see section 4.6)
*6 the SMH incorrectly expects the IAR meaning in this case (see section 4.4)
*7 these strict partial orders are "almost total" (cf. (49)), and are thus strict *weak* orderings
*8 (external) partitioning is required for coherence with more than two entities

Table 1: Reciprocal meanings and relational domains

5 Further problems of reciprocity

In this section we briefly discuss further challenges to the theory of reciprocity, especially in connection to its behavior as analyzed by the SMH and the MIH.

5.1 Kerem et al. – the Maximal Typicality Hypothesis

One challenge for both the SMH and the MIH comes from examples like the following.

(68) Mary, Sue and Jane are pinching each other (=(6)).

Sentence (68) can be interpreted as true if each girl is only pinching one other girl (Figure 5b). However, it is also physically possible for each of the three girls to be pinching each of the other two (Figure 5a).

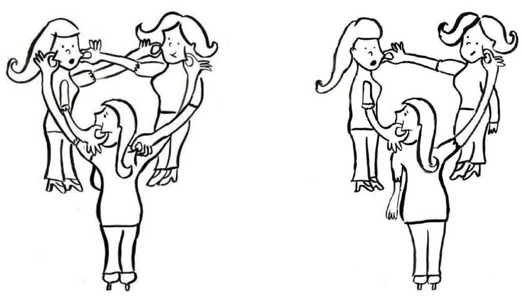

Figure 5: instances of the relational concept for *pinch* (drawings by R. Noy Shapira)

Because of this physical possibility, both the SMH and the MIH expect strong reciprocity in sentence (68) and similar ones involving verbs of physical contact like *tickle, push, touch, paint* etc. As sentence (68) illustrates, these expectations are clearly not borne out. To solve this problem for the SMH and MIH, Kerem et al. (2009) propose that interpretation domains of relational expressions should be replaced by *typicality functions* (see e.g. Smith 1988, Smith et al. 1988, Kamp & Partee 1995): functions from binary relations to real numbers in $[0,1)$. This captures the intuition that certain binary relations, e.g. ones in which people pinch two other people simultaneously, are not ruled out from the relational concept's domain Θ, but have low typicality relative to other relations in Θ. When a relation R is outside the domain Θ of a relational expression, we assume that R's typicality is zero. Using typicality functions, Kerem et al. generalize the MIH into the following principle, which they call the *Maximal Typicality Hypothesis* (MTH).

Definition 6. *Let* $\mathtt{tp} : \wp(E^2) \to [0,1)$ *be a typicality function for the binary relations over* E*. The* MTH-BASED *reciprocal function* $\mathrm{RECIP}^{\mathrm{MTH}}_{\mathtt{tp}}$ *is defined for all sets* $A \subseteq E$ *and relations* $R \subseteq E^2$ *s.t.* $\mathtt{tp}(R{\downarrow_A}) > 0$ *by:*

$\mathrm{RECIP}^{\mathrm{MTH}}_{\mathtt{tp}}(A,R) = 1$ *iff for all* $R' \subseteq E^2{\downarrow_A}$: $R{\downarrow_A} \subseteq R' \wedge \mathtt{tp}(R{\downarrow_A}) \leq \mathtt{tp}(R') \Rightarrow R{\downarrow_A} = R'$.

In words: a relation $R \subseteq E^2$ of non-negative typicality $\mathtt{tp}(R)$ (i.e. R is in the domain for the relational concept described by $\mathtt{tp}$) satisfies MTH-based reciprocity over a set $A \subseteq E$ with respect to the typicality function $\mathtt{tp}$, if $R{\downarrow_A}$ has maximal typicality among the supersets of $R{\downarrow_A}$ contained in $E^2{\downarrow_A}$ (= $A^2{\downarrow}$).

For example, in sentence (68) let us assume that the binary relation $R_0 = \{\langle a,b\rangle, \langle b,c\rangle, \langle c,a\rangle\}$ attains maximal typicality for the relational expression *pinch* over the set $\{a,b,c\}$. Formally:

(69) For all $R' \subseteq E^2{\downarrow_{\{a,b,c\}}}$: $R_0 \subseteq R' \wedge \mathtt{tp}_{\mathit{pinch}}(R_0) \leq \mathtt{tp}_{\mathit{pinch}}(R') \Rightarrow R_0 = R'$.

Assumption (69) is plausible, because a non-identity pair can only be added to R_0 by requiring one of the elements in $\{a,b,c\}$ to stand in the pinching relation to both other elements. Given this assumption, the MTH correctly describes the truth of sentence (68) in Figure 5b. Also (68)'s truth in Figure 5a is explained by the MTH. Although a complete graph is not of globally maximal typicality, the MTH, in conformity with the R-monotonicity of reciprocals (cf. definition 2), only requires local "upward monotone" maximality of a relation R: maximal typicality with respect to all other relations that contain R in the relevant domain. This is trivially the case in such a complete graph as in Figure 5a, since there is no way to add a non-identity pair to it.

Kerem et al. experimentally study typicality effects of relational concepts, as well as their correlations with reciprocal interpretation, showing initial support for the MTH.

5.2 Reciprocals with quantificational noun phrases and collective predicates

So far we have only considered reciprocal sentences with simple plural noun phrases like *the girls* or *Mary, Sue and Jane*. One of the complicating factors in treating reciprocals is their appearance with quantificational noun phrases. Consider for instance the following examples by DKKMP.

(70) At most five people hit each other.

(71) Many people at the party yesterday are married to each other.

(72) Exactly thirty people know each other.

(73) Exactly thirty people are waltzing with each other.

(74) Few (members) have spoken to each other.

(75) No one even chats to each other.

In order to be able to consider the interpretation of such sentences using the SMH, Dalrymple et al. propose an operator that combines reciprocal expressions with quantificational expressions. DKKMP call this operator *Bounded composition* (BC). The BC operator takes four arguments – a determiner, a reciprocal meaning, a one-place relation and a two-place relation – and derives a truth-value. For instance, using the BC operator, sentence (70) is analyzed as follows.

(76) $BC(\mathbf{at\_most\_5}, A, \text{RECIP}^{\text{SMH}}, R)$

In this analysis, the denotation **at_most_5** of the determiner *at most five* in (70) is the standard relation between subsets of E, satisfying for all $B, C \subseteq E$: $|B \cap C| \leq 5$. The set $A \subseteq E$ and the binary relation $R \subseteq E^2$ are the denotations of the noun *people* and verb *hit* in (70), respectively. The reciprocal meaning $\text{RECIP}^{\text{SMH}}$ is selected by the SMH. We will not repeat here the definition of the BC operator, which is rather involved, or study its interaction with the SMH, which is also quite complex. A detailed empirical evaluation of DKKMP's claims and various alternative proposals in this area (Ben-Avi & Winter 2003, Szymanik 2010) goes beyond the scope of this paper.

Two general remarks are in place, however. First, the question of quantificational NPs and reciprocity is inseparable from the more general question of collective quantification (Scha 1981, van der Does 1992, 1993, van den Berg 1996, Winter 2001*a*). Consider the following examples:

(77) At most five people gathered.

(78) Many people at the party yesterday are friends.

(79) Exactly thirty people surrounded the castle.

It is reasonable (and common) to treat verb phrases like *gathered, are friends* and *surrounded the castle* in (77)-(79) similarly to reciprocal verb phrases (e.g. *hit each other*), as denoting collections of sets. Peters & Westerståhl (2006, p.370) use this analysis, and replace DKKMP's BC operator by a similar operator, called *CQ*, which can interpret sentences like (77)-(79) similarly to DKKMP's treatment of (70)-(75). A simpler alternative to Peters and Westerståhl's CQ operator is Scha's (1981) "neutral" operator, defined below (cf. van der Does 1993, Ben-Avi & Winter 2003).

(80) Let $D \subseteq \wp(E)^2$ be a binary relation between subsets of E. The *neutral lifting* of D is the function $\mathbf{N}(D) : (\wp(E) \times \wp(\wp(E))) \to \mathbf{2}$, which describes a relation between subsets of E and sets of subsets of E. This function is defined s.t. for all sets $A \subseteq E$ and $\mathcal{B} \subseteq \wp(E)$:

$$\mathbf{N}(D)(A)(\mathcal{B}) = 1 \Leftrightarrow \langle A, \cup(\mathcal{B} \cap \wp(A))\rangle \in D.$$

In words: $\mathbf{N}(D)$ holds of a set A of entities and a set $\mathcal{B}$ of sets of entities, if D holds of A and of the union of the sets in $\mathcal{B}$ that are subsets of A.

For instance, sentence (77) is interpreted as follows:

$$\mathbf{N}(\mathbf{at\_most\_5})(P)(\mathcal{G}) = 1 \Leftrightarrow |P \cap \cup(\mathcal{G} \cap \wp(P))| \leq 5 \Leftrightarrow |\cup(\mathcal{G} \cap \wp(P))| \leq 5.$$

In words: the collection of all sets of people who gathered is composed of not more than five entities.

This strategy of dealing quantification with collective predicates leads to intuitive results in cases like sentence (77). For the sentence *at most five people hit each other* (=(70)), we assume that the denotation of the verb phrase *hit each other* is $\text{RECIP}^H = \{A \subseteq E : \text{RECIP}(A, H) = 1\}$, where RECIP is a reciprocal function and $H \subseteq E^2$ is a binary relation over entities. Using a similar analysis to the analysis of sentence (77) above, we obtain the following analysis of sentence (70).

$$\mathbf{N}(\mathbf{at\_most\_5})(P)(\text{RECIP}^H) = 1 \Leftrightarrow |P \cap \cup(\text{RECIP}^H \cap \wp(P))| \leq 5$$

$$\Leftrightarrow |\cup(\text{RECIP}^H \cap \wp(P))| \leq 5.$$

In words: the collection of all sets of people who hit each other is composed of not more than five entities.

As said above, for our purposes here we ignore the processes (e.g. the SMH or the MIH) that determine the reciprocal interpretation RECIP in quantificational reciprocal sentences like (70)-(75). However, it is important to note that there is a clear connection between this problem and the problem of reciprocity with collective transitive predicates, also with non-quantificational subjects. Consider for instance the following examples.

(81) The three forks are propped against each other. (DKKMP)

(82) The gravitation fields of the Earth, the Sun and the Moon cancel each other out. (DKKMP)

(83) Mary, John, Sue and Bill played doubles tennis against each other.

(84) John, Bill, Tom, Jane and Mary had relations with each other (=(11)).

(85) These four people fought each other.

(86) The bricks are laid on top of each other.

(87) Mutual assistance on hard rocks takes all manner of forms: two, or even three, people *climbing on one another's shoulders*, or using an ice axe propped up by others for a foothold.
`http://en.wikipedia.org/wiki/Mountaineering`
(retrieved April 2011)

In all those cases, the reciprocal expression combines with a binary relation that should be analyzed as holding between collections, rather than simple entities (cf. Sternefeld 1997). For instance, in (81), each of the forks is propped against the other two as a whole pair, not simply against each of the other forks.

A definition of the meaning of reciprocals as a function that applies to such collective relations, can be based on an extension of the treatment of quantificational NPs as in (70)-(75) and (77)-(79). To see that, let us revise some notation. For a set of entities $A \subseteq E$, a collection of sets of entities $\mathcal{B} \subseteq \wp(E)$, and a binary relation over such collections $\mathcal{R} \subseteq \wp(E)^2$, we denote:

$$\begin{array}{lcll} \mathcal{B}|_A & = & \mathcal{B} \cap \wp(A) & \text{– } \mathcal{B} \text{ restricted to } A \\ \mathcal{R}|_A & = & \mathcal{R} \cap \wp(A)^2 & \text{– } \mathcal{R} \text{ restricted to } A \\ *\mathcal{B} & = & \cup\mathcal{B} = \{x \in E : \exists A \in \mathcal{B}[x \in A]\} & \text{– union of the sets in } \mathcal{B} \\ *\mathcal{R} & = & \{\langle x, y\rangle \in E^2 : \exists \langle A, B\rangle \in \mathcal{R}[x \in A \wedge y \in B]\} & \text{– “union” for binary relations} \end{array}$$

Note that restricting collective one-place predicates ($\mathcal{B}|_A$) and two-place predicates ($\mathcal{R}|_A$) is perfectly consistent with the *conservativity* of distributive quantification (Winter 2001*a*). Using this notation, the neutrality operator **N** in (80) can be rewritten as follows:

$$\mathbf{N}(D)(A)(\mathcal{B}) = 1 \;\Leftrightarrow\; \langle A, *(\mathcal{B}|_A)\rangle \in D.$$

And along similar lines, when RECIP is a reciprocal interpretation defined for relations over entities, we define RECIP^N as the corresponding reciprocal interpretation for relations over sets of entities:

$$\text{RECIP}^N(A, \mathcal{R}) = 1 \;\Leftrightarrow\; \text{RECIP}(A, *(\mathcal{R}|_A)) = 1.$$

For instance, in sentence (81) assume that the forks are the set of entities $F \subseteq E$ and that the relational expression *propped against* denotes a binary relation $\mathcal{P} \subseteq \wp(E)^2$ between sets of entities. Supposing RECIP = SR, we get the following analysis of sentence (81):

$\mathsf{SR}^N(F,\mathcal{P}) = 1 \Leftrightarrow \mathsf{SR}(F,*(\mathcal{P}|_F)) = 1 \Leftrightarrow$

$\forall x,y \in F\ [x \neq y \rightarrow \exists\langle A,B\rangle \in \mathcal{P}[x \in A \wedge y \in B]]$.

In words: every two different forks in F belong to two sets of forks that are propped against each other. This is an intuitively correct analysis of sentence (81).

There is obviously much further study that is needed on the interactions of reciprocity (and the SMH or MIH) with collectivity (81)-(87) and quantification (70)-(75). At the same time, as the analysis sketched above implies, we believe that the two kinds of interactions involve one and the same problem: the interaction of distributive quantifiers – NP quantifiers and reciprocals alike – with collective predicates.

6 Conclusions

We started out this paper by reviewing Dalrymple et al's account of reciprocals using the Strongest Meaning Hypothesis, which was proposed as a general theory of reciprocal meanings and their selection by contextual factors. Despite the attractiveness of Dalrymple et al's approach, we have given reasons to doubt its generality. First, we doubt that a theory that relies on total $\langle 1,2\rangle$ quantifiers might be able to enumerate in a principled manner all *a priori* possible meanings of reciprocals. As we saw, in many cases different reciprocal meanings lead to the same sentential interpretation. This fact, together with the heterogeneous effects on the interpretation of reciprocal sentences, make it hard to use purely logical considerations for proposing an "optimal" set of hypothesized reciprocal meanings regulated by general formal pragmatic principles. Second, we have shown that an informal notion of "context", as used by the Strongest Meaning Hypothesis, does not only reduce the clarity of the theory, it also leads to some unintuitive complications. Instead of these two ingredients of Dalrymple's et al's analysis, we proposed a new principle, the Maximal Interpretation Hypothesis, which generates an interpretation of a reciprocal expression based on meaning postulates about the interpretation domain of relational expressions. In this way, in our proposal the general notion of reciprocal *meaning* loses its theoretical centrality, and a more sentence-specific notion of reciprocal *interpretation* takes its place. Our study of contextual effects on reciprocal interpretation focused on those effects that come from the relational concept. We believe that this theoretical change of focus has more to offer than some improvements in empirical coverage or formal rigor. The interplay that it attempts to capture between logical operations, conceptual knowledge and the contextual factors on both of them, is central to semantic theory. We believe that by focusing on the first two elements, the Maximal Interpretation Hypothesis may improve our understanding of the relations between logic and concepts in natural language semantics, and help in developing a more adequate understanding of contextual effects on logical interpretation.

Appendix

A Internet examples with asymmetric predicates (retrieved January-April 2011)

A.1 The verbs 'outperform', 'outdo', 'outrank' and 'outnumber'

Google hits:

outperformed each other:	55,000
outdid each other:	74,400
outnumber each other:	25,000
outrank each other:	20,000

Examples – reasonably not asymmetric:

(88) Between Raja and Toshi, there have been days when they *outperformed each other*.
`http://starvoiceofindiashow.com/toshi-sings-dard-e-disco`

(89) Clients and volunteers were split into two teams which *outdid and outperformed each other* with their acting skills at skits, cracked their heads looking for clues at the treasure hunt, and were extremely good at charades.
`http://www.spd.org.sg/volunteers/volunteerism/vivian.html`

Examples – asymmetric:

(90) Even during the last decade, when U.S. and developed foreign markets tended to move in the same direction, they *outperformed each other* by at least ten percent in six of those ten years. For example, while the Wilshire 5000 – which represents most of the publicly traded stocks in America – returned 29 percent in 2003, the Dow Jones World Stock Index – which excludes the United States – rose 38.6 percent. For the same year, Morgan Stanley Capital International reported emerging markets returning 42 percent.
`http://www.rockwoodfinancial.com/cgi-bin/cginews.pl?record=11`

(91) Figure 6.1 demonstrates how US and international markets *outperformed each other* during certain time periods.[37]
The Investing Revolutionaries: How the World's Greatest Investors Take on Wall Street and Win in Any Market, by James N. Whiddon and Nikki Knotts, McGraw-Hill Professional, 2009, p.149.

(92) Kaer had a census from Sep 20th, and Frostwolf was 47% alliance and 53% horde. So it is the most balanced of all molten's realms. However as stated before, factions do *outnumber each other* on certain times. Right around 11:00AM-2:00PM though the balance is virtually perfect.
`http://forum.molten-wow.com/showthread.php?t=36642`

(93) Whether or not two competing clients *outrank each other* is determined more by the search engine algorithms, age of the client's site, frequency of product turnover, popularity of the site based on naturally occurring external links, etc. Once we put

[37] Figure 6.1 in Whiddon and Knotts' book illustrates 17 consecutive years in which U.S. markets outperformed foreign markets or vice versa.

our plan in place for each client, we often see them flip-flopping between first and second position for the same exact keywords.
`http://www.flyteblog.com/flyte/2010/03/can-you-work-with-clients-who-compete-with-each-other.html`

(94) If all qualities are equally valued (beta=gamma, for any delta) then market share can easily be divided between any two brand clusters which *mutually outrank each other* in one quality dimension each (i.e. trade-off collectively).
`http://marketing-bulletin.massey.ac.nz/V16/MB_V16_A2_Schley.pdf`

(95) Search engines use algorithms to determine how websites *outrank each other* and climb to the top of the (much coveted) search query results list.
`http://www.articlesbase.com/link-popularity-articles/increasing-website-traffic-part-one-82569.html`

(96) Personnel of equivalent-level ranks *outrank each other* by department on the chart below from left-to-right. That is, Naval ranks outrank Intelligence ranks, Intelligence ranks outrank Marine ranks, and so on. Personnel of equivalent rank and department outrank one another by seniority.
`http://aurigae.qblix.com/index.html`

(97) They are the best in what they offer...dont judge a school if u r nt interested in the courses they offer...I think this cluster thing is good since u cant really distinguish between a no.8 nd no.9 in one or the parameters they *outrank each other*....
`http://www.pagalguy.com/forum/cat-and-related-discussion/50452-pagalguy-2010-rankings-national-regional-17.html`

A.2 The verb 'contain'

Google hits:

contain each other:	875,000
contained within one another:	13,200,000

(98) Circles may touch, overlap or *contain each other*.
`http://acm.tju.edu.cn/acm/showp2385.html`

(99) Intersection of infinite sets that *contain each other*. If each A_i is a set containing infinite elements, and A_1 contains A_2 contains A_3 contains ... on and on, then is the intersection of all these sets infinite?
`http://www.mathhelpforum.com/math-help/f37/intersection-infinite-sets-contain-each-other-85541.html`

(100) The simplest of all methods for detecting intersections between objects is a simple bounding sphere test. Essentially, this represents objects in the world as circles or spheres, and test whether they *touch, intersect or completely contain* each other.
`http://devmag.org.za/2009/04/13/basic-collision-detection-in-2d-part-1`

(101) Does anyone know (giving a URL is obviously o.k.) which of the C++ classes *contain each other*? (For example, <fstream> contains <iostream> [I think]).
`http://www.velocityreviews.com/forums/t456346-containment-of-standard-c-classes.html`

(102) Two XML instances that *contain each other*.[38]
Mario A. Nascimento (ed.), *Proceedings of the Thirtieth International Conference on Very Large Data Bases, Toronto, Canada.* Morgan Kaufmann 2004, page 136.

[38] A figure shows a structure and a substructure of it.

(103) As mentioned in the document, “Setting your Watch Folder the same as your Music Management folder will create duplicates in your Library.” That should be the reason that management folder and watch folder can’t *contain each other*.
http://getsatisfaction.com/songbird/topics/how_to_set_up_file_management

(104) It is possible, in some profile types, for terms to be *contained within one another* and be nested, which is suited to the expression of hierarchical vocabularies.
http://en.wikipedia.org/wiki/IMS_VDEX

(105) Block if statements can be nested that is, *contained within one another*.
http://ol.cadfamily.com/CATIA/English/online/kwxug_C2/kwxugat0018.htm

(106) The given circles must not be tangent to each other, overlapping, or *contained within one another*.
http://mathforum.org/mathimages/index.php/Problem_of_Apollonius

(107) Yin and yang not only oppose but also *contain each other*.
http://susansayler.wordpress.com/2011/03/19/the-science-of-yin-and-yang

A.3 The nouns ‘ancestor (of)’ and ‘descendant (of)’ and the verbs ‘descend (from)’ and ‘ascend (from)’

Google hits:

descendants of each other:	98,500
ancestors of each other:	56,000
descend(*ed*)*from each other*	34,000
ascend(*em ed*)*from each other*	3

(108) In Hesiod’s version the members of the chain of divine rulers are father, son, grandson, ie, *descendants of each other*, while in the Hurro-Hittite myth...
Geoffrey W. Bromiley, *The international standard Bible encyclopedia.* Wm. B. Eerdmans Publishing 1995, page 81.

(109) By definition, items in an itemset cannot be *ancestors or descendants of each other*.
Xue Li, Osmar Zaïane, Zhanhuai Li, *Advanced data mining and applications*, Springer, 2006, page 66.

(110) If there is a conflict between “include” and “exclude” links pointing to features on different levels of the feature tree (i.e. if the features pointed to are *descendants and ancestors of each other*), the link pointing to the lower level feature has priority with respect to this feature and all it descendants.
Henk Obbink and Klaus Pohl Birkhäuser (eds.), *Software product lines: 9th international conference, SPLC 2005, Rennes, France, September 26-29*, 2005, page 27.

(111) It is understood today that species which are presented as *ancestors of one another* are actually different races that lived at the same period.
http://www.evidencesofcreation.com/tellme25.htm

(112) Scientists who support evolution give examples within a family that appear to be *ancestors of each other*.
https://cafewitteveen.wordpress.com/tag/the-grand-experiment-chapter-8-the-fossil-record-record-of-fish

(113) Maybe its like saying: Folk of Hador, Northmen, Ethoed, Rohirrim: they were not the same, but *ancestors of each other*.
`http://www.terrainguild.com/thelastalliance/viewtopic.php?f=17&t=2486`

(114) those hominids are not contemporary, and thus we can situate them according to the oldness, but that doesn't mean that the science could prove they are *ancestors of each other*, since they didn't find enough fossils.
`http://dodona.proboards.com/index.cgi?board=genetics&action=print&thread=6749`

(115) The haplogroups *descend from each other*. It's a genetic family tree of the human race.
`http://answers.yahoo.com/question/index?qid=20110116162017AA1at9U`

(116) The line of succession can be straight or direct, consisting of people who *ascend or descend from each other* (grandparents, parents, children, grandchildren), or collateral, consisting of people who come from one common trunk (brothers, uncles, cousins) .
`http://pfasociados.es/en/inheritance`

A.4 Comparatives and the prepositions 'above' and 'below'

Google hits:

than each other:	17,200,000
above each other:	21,800,000
below each other:	16,800,000

(117) To see if two numeric values are *greater than each other*, we use the comparison operator >. To see if two string values are *greater than each other*, we use the comparison operator *gt* (Greater Than).
`http://perl.about.com/od/perltutorials/a/perlcomparison_2.htm`

(118) We're only checking to see if the two variables are either *Less Than (<) each other*, or *Greater Than (>) each other*. We need to check if they are the same (as they now are).
`http://www.homeandlearn.co.uk/php/php3p8.html`

(119) Makin' kids *older than each other*: Okay I'm just wondering, when you're in the 'Create a family' mode and your creating family relationships is there any way to have two or more teens, for example, in the family but have them at different stages of life? Coz otherwise its like they're twins or triplets or whatever. Anyone know how to do this without actually playing through the game and having children...?
`http://www.neoseeker.com/forums/5606/t441708-makin-kids-older-than-each-other/#9`

(120) Do different liquids evaporate *slower than each other*?
`http://wiki.answers.com/Q/What_liquids_other_than_water_evaporate`

(121) I think it does not look nice when two figures on one page are positioned *above each other*.
`http://www.latex-community.org/forum/viewtopic.php?f=45&t=7598`

(122) Basically I would like to have two charts below each other like you can see it on any stock chart including an indicator on various websites.
`http://www.excelbanter.com/showthread.php?t=37015`

A.5 Remark on stage-level comparatives

Alda Mari (Mari 2006 and further unpublished work) has suggested that many asymmetric relational require strong reciprocity when all times or situations are taken into account, but tolerate times or situations without strong reciprocity. This claim seems to be supported by some of the examples above. For instance, in sentence (91) above, US markets outperform international markets in some time periods, and international markets outperform US markets in other time periods. This is described by the writer using the sentence *US and international markets outperformed each other during certain time periods*. By contrast, also on the internet it is hard to find cases where a speaker refers to one situation where one entity outperforms another as a "reciprocal situation". This kind of observations may help to explain why individual-level[39] SPO/SWO relations like *mother of each other* are ruled out with reciprocals – it is probably hard to think of changes over times or worlds with such predicates. However, also with classic stage-level comparatives like *fuller/emptier/sicker than* and others, reciprocity does not seem to be licensed, unlike the relations *outnumber, outperform, outrank* etc. which were shown above in stage-level usages. This fact may indicate that in addition to the factors considered by Mari, there might be additional factors that block comparative forms of adjectives from appearing with reciprocals.

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[39] For the individual-level/stage-level distinction see Carlson (1977).

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