# Huiswerk 7 - Intensionaliteit

**Please submit no later than Friday 30 March.**

**Exercise I:** Recall the tutorial exercise on *noodzakelijkerwijs*[1]. Similarly to the definition of the necessity operator $\mathbf{nec}'$ as a function of type $(st)(st)$ (from propositions to propositions), write down a definition for the possibility operator $\mathbf{pos}'$ of the same type. Show that under the definitions given in class, specifically the revised truth-conditionality criterion, the following (non-)entailments are explained:

(1) The morning star is the evening star
$\Rightarrow$ Possibly, the morning star is the evening star

(2) Possibly, the morning star is the evening star
$\not\Rightarrow$ The morning star is the evening star

For (1) show that for every intended model: $\mathbf{is}'(\mathbf{es}')(\mathbf{ms}') \subseteq \mathbf{pos}'(\mathbf{is}'(\mathbf{es}')(\mathbf{ms}'))$.

For (2) show an intended model where: $\mathbf{pos}'(\mathbf{is}'(\mathbf{es}')(\mathbf{ms}')) - \mathbf{is}'(\mathbf{es}')(\mathbf{ms}') \neq \emptyset$

The types that you should use are:

$\mathbf{es}'$ and $\mathbf{ms}'$: $se$ – individual concepts
$\mathbf{is}'$: $(se)((se)(st))$ – function from pairs of individual concepts to propositions
$\mathbf{pos}'$: $(st)(st)$ – function from propositions to propositions

**Exercise II:**

Suppose that $D_e = \{j', m', b'\}$ and $D_s = \{1, 2, 3\}$ and that we have the following functions:

$$rigid_{e(se)} = \lambda x_e.\lambda i_s.x_e$$
$$ran_{se(st)} = \lambda c_{se}.\lambda i_s.\underbrace{c(i) \in \{j', b'\}}_{t}$$
$$kissed_{(se)(se)st} = \lambda x_{se}.\lambda y_{se}.\lambda i_s.\underbrace{\langle y(i), x(i)\rangle \in \{\langle j', m'\rangle, \langle m', j'\rangle\}}_{t}$$

1. Write $rigid(j')$ explicitly as a function. (give its value for each element of the

[1] In the definition of the denotation of *necessarily*, the notation $\forall$ is the function that quantifies over all the worlds in the frame, which is defined by:
$\forall_{(st)t}$ is a function that takes an $st$ function $f$ and returns 1 if $f$ characterizes $D_s$, and 0 otherwise.
Or equivalently in other words:
$\forall_{(st)t}$ takes a $(st)$ function $f$ and checks if $f(w)$ is true for all $w \in D_s$.

domain)

2. Explain in you own words what *rigid* does.
3. How many individual concepts are there in this frame? How many of them refer to the same entity in every model?
4. Not all individual concepts refer to the same entity in every world. Give an example of an individual concept that refers to differents entities in different worlds.
   You can write it explicitly as a function.
5. Reduce $ran(\ rigid(j')\ )$ and give the set of indices/worlds it characterizes.
6. Now do the same, but use the individual concept you gave in 2. instead of $rigid(j')$
7. Reduce $(kissed(\ rigid(j')\ ))(\ rigid(m')\ )$ and give the set of indices it characterizes.

**Exercise III:** Consider the sentences below:

1. Some woman hugged every cat.
2. Every woman hugged some cat.
3. Most women hugged some cat.
4. Most women hugged every cat.
5. Every women hugged most cats.
6. some woman hugged most cats.

Write down the ONS and OWS operators we defined as $\lambda$-terms. For each of the sentences above: (a) reduce the $\lambda$-terms corresponding to the ONS-based and OWS-based analyses as far as possible; (b) tell which one entails which one, if any. (c) can you find a regularity regarding which reading entail the other one? Find a generalization when one of the determiners is *some* or *every* and the other quantifier is upward monotone, and prove it formally (hint: assume that $E$ is finite).

**Example**: For the first sentence *some woman hugged every cat*:

- We assumed the determiner functions:
  - $\textsc{some} = \lambda A_{et}.\lambda B_{et}.\exists x_e.A(x) \wedge B(x)$, or equivalently:
    $\textsc{some} = \lambda A_{et}.\lambda B_{et}.A \cap B \neq \emptyset$
  - $\textsc{every} = \lambda A_{et}.\lambda B_{et}.\forall x_e.A(x) \rightarrow B(x)$, or equivalently:
    $\textsc{every} = \lambda A_{et}.\lambda B_{et}.A \subseteq B$
- The ONS-based and OWS-based terms that we derive are:
  - $\textsc{some}(\mathbf{woman})(\textsc{ons}(\mathbf{hug})(\textsc{every}(\mathbf{cat})))$
  - $\textsc{ows}(\textsc{ons}(\mathbf{hug}))(\textsc{every}(\mathbf{cat}))(\textsc{some}(\mathbf{woman}))$
- Reduce those terms to:

- $\exists x_e.\mathbf{woman}(x) \wedge [\forall y_e.\mathbf{cat}(y) \rightarrow \mathbf{hug}(y)(x)]$, or equivalently:
  $\mathbf{woman} \cap [\lambda x.[\mathbf{cat} \subseteq \lambda y.\mathbf{hug}(y)(x)]] \neq \emptyset$
- $\forall y_e.\mathbf{cat}(y) \rightarrow [\exists x_e.\mathbf{woman}(x) \wedge \mathbf{hug}(y)(x)]$, or equivalently:
  $\mathbf{cat} \subseteq \lambda y.[\mathbf{woman} \cap [\lambda x.\mathbf{hug}(y)(x)] \neq \emptyset]$
  **Important**: in your answer, please show every step in the reduction, for this example as well! (i.e. complete the reduction steps that are missing above).
- Point out that the first term entails the second term.
- After you answer (a) and (b) in this way for all sentences, answer (c).

**Exercise IV:** Consider the following de dicto/de re analyses in the handout of the sentence *Donald looked for a cat*:

(1) Donald [[looked for] a cat].

$I^i_{\mathbf{d}'}((\mathbf{look\_for}')(\mathbf{some}^i(\mathbf{cat}^i)))$

$= \{w \in D_s : \mathbf{look\_for}'(\mathbf{some}^i(\mathbf{cat}^i))(\mathbf{d}')(w)\}$ – *de dicto* reading

(2) Donald [$\epsilon_{\text{ows}}$[looked for] a cat].

$\text{ows}^i(\mathbf{look\_for}')(\mathbf{some}^i(\mathbf{cat}^i))(I^i_{\mathbf{d}'})$

$= \{w \in D_s : \exists x[\mathbf{cat}^i(x)(w) \wedge \mathbf{look\_for}'(I^i_x)(\mathbf{d}')(w)]\}$ – *de re* reading

Let us add to our lexicon the following entry:

$$\mathbf{existed}^i = \lambda x_e.\lambda w_s.\top$$

Thus, when '$\top$' is standardly interpreted as 1 in every model, the function $\mathbf{existed}^i$ sends every entity to the set of all possible worlds.

Using the intensional version of the TCC show that our analyses above correctly expects the following (lack of) entailments:

- Donald looked for some cat (de dicto) $\not\Rightarrow$ Some cat existed (show an intensional model that does not satisfy containment between the propositions)
- Donald looked for some cat (de re) $\Rightarrow$ Some cat existed (prove why every intensional model satisfies containment between the propositions)