On Intensional Semantics

Topics: intensional expressions and their (lack) of entailments, extensional/intensional semantics, extension (reference) vs. intension (sense), possible world semantics, indices, typing with indices, individual concepts, propositions, properties, intensional models, revised truth-conditionality criterion

Reading: L. T. F. Gamut, *Logic, Language and Meaning*, vol. II, chapter 5, The University of Chicago Press, 1991.

Substitution property of compositional model-theoretic semantics: Let S_1 be a structure for a well-formed sentence. Let S_2 be the structure of another well-formed sentence that we obtain when replacing an expression *exp*$_1$ in S_1 by another expression *exp*$_2$. Suppose that *exp*$_1$ and *exp*$_2$ have the same denotation in a model M. It follows that S_1 and S_2 must have the same denotation in M.

IF $[\![exp_1]\!] = [\![exp_2]\!]$ **THEN** $[\![S_1]\!]$ (X $[\![exp_1]\!]$ Y) = $[\![S_2]\!]$ (X $[\![exp_2]\!]$ Y)

Substitution problem of extensional semantics: The system we have developed is based on types e for simple entities in the model and t for truth-value denotations of sentences. In many cases, the substitution property leads to undesired results with this system.

Examples:

(1) Tina smiles, and
Mary dances,
and John *believes* Tina smiles
$\not\Rightarrow$ John *believes* Mary dances

(2) Lewis Carroll is C. L. Dodgson, and
Mary *believes* that Lewis Carroll wrote *Alice*
$\not\Rightarrow$ Mary *believes* that C. L. Dodgson wrote *Alice*

More examples:

(3) the evening star is the morning star, and
necessarily, the evening star is the evening star
$\not\Rightarrow$ *necessarily*, the evening star is the morning star

(4) every manager is a board member, and every board member is a manager, and
Mary met a *former* manager
$\not\Rightarrow$ Mary met a *former* board member

(5) every maid is a cook, and every cook is a maid, and
Mary is *looking for* a maid
$\not\Rightarrow$ Mary is *looking for* a cook

(6) Every knife is a diamond and every diamond is a knife, and
this is a fake diamond
$\not\Rightarrow$ This is a fake knife

Intensional expressions: *believe, necessarily, former, look for, alleged, fake...*
All these expressions are called *intensional*: they create an *intensional context*, where replacing expressions with equal denotations in our system may lead to an (unexpected) change in entailment relations.
Non-intensional expressions are called *extensional*.
A system like ours, which only deals with entailments involving extensional expressions, is called an *extensional semantics*.

Extension vs. Intension:

Extension = reference (*Bedeutung*) = the object (entity, set, function) to which an expression refers.

Intension = sense (*Sinn*) = the algorithm/concept leading to identifying this object.

Possible world semantics: In addition to $D_e = E$ (domain of entities) and $D_t = \{0,1\}$ (domain of truth-values), let us add a primitive domain $D_s = W$ of *indices*, with the corresponding type s. An index can be thought of as a *possible world* or a *world-time* pair.

Expression	Example	E-type	I-type	I-denotation name
proper name	Tina	e	se	*individual concept*
sentence	Tina smiles	t	st	*proposition*
1-place predicate	smile	et	$(se)(st)$	1-place *property* of i-concepts

In general: in the typing function, we replace any e by se and every t by st.
Remark: sometimes more "economical" typings are used, e.g. $e(st)$ or $s(et)$ for 1-place properties.

Intensional model – the former definition, with the additional primitive type s and the corresponding domain.

Definition 1 (types) *The set of (intensional)* types *is defined as the smallest set $\mathcal{T}$ that satisfies: (i) e, s and t are types in $\mathcal{T}$; (ii) If τ and σ are types in $\mathcal{T}$ then $(\tau\sigma)$ is also a type in $\mathcal{T}$.*

Definition 2 (domains) *$D_e = E$, $D_s = W$ are arbitrary non-empty sets. $D_t = \{0,1\}$, with the numerical order $\leq$. If τ and σ are types then $D_{\tau\sigma} = D_\sigma^{D_\tau}$, the set of functions from D_τ to D_σ.*

Definition 3 (frame) *An intensional* frame *$F^{E,W}$ over non-empty sets of entities E and indices W is the collection $\bigcup_{\tau\in\mathcal{T}} D_\tau$.*

Definition 4 (model) *Let Σ be a non-empty finite set of words, and let* $\text{TYPE} : \Sigma \to \mathcal{T}$ *be a* typing function *over Σ. A* model *M over Σ is a pair $\langle F^{E,W}, I\rangle$, where $F^{E,W}$ is an intensional frame and $I : \Sigma \to F^E$ is an* interpretation function *that satisfies $I(w) \in D_{\text{TYPE}(w)}$ for each word $w \in \Sigma$.*

The truth-conditionality criterion (intensional version): *Let S_1 and S_2 be sentences of type st. Then S_1 entails S_2 if and only if for every intended model M: $[\![S_1]\!]^M \leq [\![S_2]\!]^M$.*

Note: The relation $\leq$ is domination relation for type st – subset between sets of indices.

Example:

(7) John believes Tina smiles.

believe:
type: $(st)(e(st))$ – a two-place property relating propositions and entities
denotation: $\mathbf{believe}$ (arbitrary)

smile:
type: $e(st)$ – a one-place property of entities
denotation: $\mathbf{smile}$ (arbitrary)
and similarly for *dance*.

Prove now: There is an intensional model where:

$\mathbf{smile}(\mathbf{tina}) \cap \mathbf{dance}(\mathbf{mary}) \cap \mathbf{believe}(\mathbf{smile}(\mathbf{tina}))(\mathbf{john}) \not\subseteq \mathbf{believe}(\mathbf{dance}(\mathbf{mary}))(\mathbf{john})$

Proof: we need a world w_1 in which Tina smiles and Mary dances, and where John believes the proposition for *Tina smiles*, but he does not believe the proposition for *Mary dances*. For instance:
$\mathbf{smile}(\mathbf{tina}) = \{w_1\}$
$\mathbf{dance}(\mathbf{mary}) = \{w_1, w_2\}$
$\mathbf{believe}(\{w_1\})(\mathbf{john}) = \{w_1\}$
$\mathbf{believe}(\{w_1, w_2\})(\mathbf{john}) = \{w_2\}$
In this case: $w_1 \in \mathbf{smile}(\mathbf{tina}) \cap \mathbf{dance}(\mathbf{mary}) \cap \mathbf{believe}(\mathbf{smile}(\mathbf{tina}))(\mathbf{john})$
But $w_1 \notin \mathbf{believe}(\mathbf{dance}(\mathbf{mary}))(\mathbf{john})$.

More examples:

(8) Necessarily, the morning star is the evening star.

necessarily:
type: $(st)(st)$ – function from propositions to propositions
denotation – modal necessity operator: $\mathbf{nec}'_{(st)(st)}(f_{st})(i_s) = \begin{cases} 1 & f \text{ characterizes } D_s \\ 0 & \text{otherwise} \end{cases}$

the evening star:
type: se – individual concept
denotation: $\mathbf{es}'_{se}$ (arbitrary)

the morning star:
type: se – individual concept
denotation: $\mathbf{ms}'_{se}$ (arbitrary)

is:
type: $(se)((se)(st))$ – a two-place property

denotation – extensional identity: $\mathbf{is}'_{(se)((se)(st))}(x_{se})(y_{se})(i_s) = 1$ iff $x(i) = y(i)$.

Prove now:

$\mathbf{nec}'(\mathbf{is}'(\mathbf{es}')(\mathbf{ms}')) \subseteq \mathbf{is}'(\mathbf{es}')(\mathbf{ms}')$

but there are models where $\mathbf{is}'(\mathbf{es}')(\mathbf{ms}') \nsubseteq \mathbf{nec}'(\mathbf{is}'(\mathbf{es}')(\mathbf{ms}'))$

and in addition $\mathbf{nec}'(\mathbf{is}'(\mathbf{es}')(\mathbf{es}'))$ is a tautology, just like $\mathbf{is}'(\mathbf{es}')(\mathbf{es}')$.

(9) This is a fake diamond.

this:
type: se – individual concept
denotation: $\mathbf{t}'_{se}$ (arbitrary)

is a:
type: $((se)(st))((se)(st))$ – modifier of 1-place properties
denotation: $\mathbf{is\_a}'_{((se)(st))((se)(st))}(P) = P$

is not a:
type: $((se)(st))((se)(st))$ – modifier of 1-place properties
denotation: $\mathbf{is\_not\_a}'_{((se)(st))((se)(st))}(P) = \overline{P}$

fake:
type: $((se)(st))((se)(st))$ – modifier of 1-place properties
denotation – a *co-restrictive* modifier: $\mathbf{fake}'_{((se)(st))((se)(st))}(P) \subseteq \overline{P}$

every:
type: $((se)(st))(((se)(st))(st))$ – intensional determiners
denotation: $\mathbf{every}'_{((se)(st))(((se)(st))(st))}(A_{(se)(st)})(B_{(se)(st)})(i_s) = 1$ iff
for every x_{se}, if $A(x)(i) = 1$ then $B(x)(i) = 1$

Prove now:
This is a fake knife/diamond $\Rightarrow$ This is not a knife/diamond
What's the problem here with a standard extensional semantics? Prove now:
Every knife is a diamond and every diamond is a knife and this is a fake diamond $\not\Rightarrow$ This is a fake knife

(10) John believes that a witch arrived.
de dicto reading: John has a belief "a witch arrived"
de re reading: There is a person, say Mary, which is a witch, and John has a belief "Mary arrived"

believe:
type: $(st)((es)(st))$ – a two-place property relating propositions and i-concepts
denotation: $\mathbf{believe}'$ (arbitrary)