Homework 5 – Boolean Semantics

Deadline: **Wednesday 14 March**.

Exercise I:

1. Construct the structure of the following sentence and give a type for the word very:

   Yoda is very short

2. Do the same for the following sentence, where *quietly* is assumed to of type $(et)(et)$. What is the type of very in this case?

   Yoda talked very quietly

3. Note the entailments: *Yoda is very short* $\Rightarrow$ *Yoda is short* and *Yoda talked very quietly* $\Rightarrow$ *Yoda talked quietly*. Given the types that you gave for the word *very* in the two sentences, what should be the restrictions on its meanings that would account for these entailments?
4. Could you suggest a generalization for the two restrictions you proposed? Hint: think of a property of functions of type $\tau\tau$, where τ is a boolean type.

Exercise II: Consider the following pair of sentences:

A. (a) Yoda is short and Yoda is wise or not wise
   (b) Yoda is short

B. (a) Yoda is short or Yoda is wise and not wise
   (b) Yoda is short

1. By writing down the λ-terms we assume for these sentences, and simplifying them, prove that in all models we considered[1], the two sentences in (A) have the same truth-value, and the two sentences in (B) have the same truth-value.
2. Show two similar equivalences but using only predicate coordinations.

Exercise III: In this question you're asked to verify that the type-theoretical construct $\sqcap$ for $e(et)$ functions corresponds to set intersection for the relations characterized by these functions. Thus, let R and Q be two functions of type $(et)t$. The binary

1. (all models in which is denotes the identity function and and , or and not denote the polymorphic boolean operators)

relations $R', Q' \subseteq D_e \times D_e$ are characterized by R and Q as follows – for all $x, y \in D_e$: $\langle x, y\rangle \in R'$ iff $(R(y))(x) = 1$ and $\langle x, y\rangle \in Q'$ iff $(Q(y))(x) = 1$. Show that $R \cap Q$ characterizes the intersection of R' and S': thus, prove that for all $x, y \in D_e$: $((R \sqcap Q)(y))(x) = 1$ iff $\langle x, y\rangle \in R' \cap Q'$.

Exercise IV:

When a word like *and* or *or* is missing as in the following sentences in (A), the meaning is equivalent to repeating the *and*/*or* as in the respective sentences in (B).

(A) a. Alex ate, drank and relaxed.
b. Alex, Sana or Tatiana relaxed.
c. Alex ate, Sana drank and Tatiana relaxed.
(B) a. Alex ate and drank and relaxed.
b. Alex or Sana or Tatiana relaxed.
c. Alex ate and Sana drank and Tatiana relaxed.

1. Define all the types and λ-terms of *and* and *or* as ternary connectives that are needed to explain the equivalences between the sentences in (A) and the respective sentences in (B).
2. Show that the following sentences are not equivalent:
   (C) Alex ate, drank or danced and relaxed.
   (D) Alex ate or drank or danced and relaxed.
   Do that by showing a sentence (S) that satisfies one of the following:
   - there is a reading of (C) that entails (S) but there is no reading of (D) that entails (S).
   - there is a reading of (D) that entails (S) but there is no reading of (C) that entails (S).
   - (S) entails one of (C)'s readings but entails no reading of (D).
   - (S) entails one of (D)'s readings but entails no reading of (C).

   Which of these conditions does your (S) satisfy?
3. Give suitable structures of (C) and (D), and show (by simplifying the λ-terms for these structures) that given those structures and the Truth-Conditionality Criterion, the entailment and non-entailments you have shown for (C),(D) and (S) are expected.